

Trigonometry

Trigonometry is a very useful branch of mathematics for applied physics and engineering.

Special angle ratios:

Ratio	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Trigonometric ratios:

- $\sin \theta = \frac{y}{r}$
 - $\tan \theta = \frac{y}{x}$
 - $\cos \theta = \frac{x}{r}$
- where $r = \sqrt{x^2 + y^2}$

Reciprocal functions:

- $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
- $\cot \theta = \frac{1}{\tan \theta}$
- $\sec \theta = \frac{1}{\cos \theta}$

Trigonometry

Compound angle formulae:

- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
- $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

Double angle formulae:

- $\sin 2A = 2 \sin A \cos A$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A$
 $= 2 \cos^2 A - 1$
 $= 1 - 2 \sin^2 A$

Half angle formulae:

- $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$
- $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$
- $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$
- $\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

Quotient identities:

- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Square identities:

- $\sin^2 \theta + \cos^2 \theta = 1$
- $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
- $1 + \tan^2 \theta = \sec^2 \theta$

Reduction formulae using 180° and 360°:

Quadrant 2 180° - θ	Quadrant 3 180° + θ	Quadrant 4 360° - θ
$\sin(180^\circ - \theta) = \sin \theta$ $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec} \theta$	$\sin(180^\circ + \theta) = -\sin \theta$ $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec} \theta$	$\sin(360^\circ - \theta) = -\sin \theta$ $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec} \theta$
$\cos(180^\circ - \theta) = -\cos \theta$ $\sec(180^\circ - \theta) = -\sec \theta$	$\cos(180^\circ + \theta) = -\cos \theta$ $\sec(180^\circ + \theta) = -\sec \theta$	$\cos(360^\circ - \theta) = \cos \theta$ $\sec(360^\circ - \theta) = \sec \theta$
$\tan(180^\circ - \theta) = -\tan \theta$ $\cot(180^\circ - \theta) = -\cot \theta$	$\tan(180^\circ + \theta) = \tan \theta$ $\cot(180^\circ + \theta) = \cot \theta$	$\tan(360^\circ - \theta) = -\tan \theta$ $\cot(360^\circ - \theta) = -\cot \theta$

Reduction formulae for a negative angle:

- $\sin(-\theta) = -\sin \theta$
- $\tan(-\theta) = -\tan \theta$
- $\sec(-\theta) = \sec \theta$
- $\cos(-\theta) = \cos \theta$
- $\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$
- $\cot(-\theta) = -\cot \theta$

Cofunction pairs:

- $\cos(90^\circ - \theta) = \sin \theta$
- $\tan(90^\circ - \theta) = \cot \theta$
- $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$
- $\sin(90^\circ - \theta) = \cos \theta$
- $\cot(90^\circ - \theta) = \tan \theta$
- $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

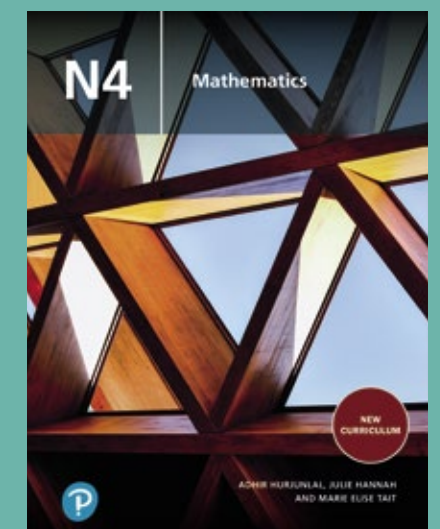
KEYWORDS

identity (in algebra)

an equation that is true for all values of the variable(s)

cofunctions

in trigonometry, cofunctions apply to complementary angles



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