N6

Electrotechnics

ALFRED MWAMUKA
DEON KALLIS
WILFRED FRITZ

Pearson

4th Floor, Auto Atlantic, Corner Hertzog Boulevard and Heerengracht Boulevard,
Cape Town, South Africa
za.pearson.com

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Module
1

DC Machines

What is covered?

Although not as common as their AC counterparts, DC motors are found in the areas where their torque-speed characteristics provide distinct advantages. The speed of DC motors can be varied over a large range, whilst still maintaining the same relatively high torque and efficiency

This first module focuses on speed control calculations, load sharing of generators working in parallel, and efficiency testing of DC machines using direct and indirect methods.

Learning Outcomes

After studying this module, you should be able to:

Unit 1

- Explain normal speed, below normal speed and above normal speed of DC motors
- Explain how speed control of a DC motor is achieved by:
  - varying the voltage across the armature
  - using the series-parallel control method
  - using the Ward-Leonard control system
  - varying the value of the main magnetic flux/ field current.
- Perform calculations with regards to speed control of shunt and series motors. Speed control being achieved by inserting a resistor:
  - in series with the shunt field
  - in series with the armature
  - in parallel with the series field.
- Interpret the magnetisation curve of a DC motor
- Determine the speed, current, EMF and torque of a DC motor (after inserting the resistor) by using the formulae
- Interpret the following conditions:
  - sudden changes in flux
  - the current drawn from the mains is to remain unchanged
  - the ohmic voltage drops are negligible.

Unit 2

- Explain, with the aid of a diagram, the procedure to parallel two generators
- Explain how the total load is shared by the two generators
- Perform calculations to determine the portion of total load supplied by each generator
- Show, by means of circuit diagrams, how the following generators are used to supply a common load:
  - two shunt generators
  - two series generators
  - two compound generators.

Unit 3

- List, classify and calculate the different losses that occur in DC machines
- Determine the efficiency of any small DC machine by using the following direct methods:
  - rope brake
  - Prony brake.
- Determine the efficiency of large shunt- and compound-wound machines by using the following indirect methods:
  - the summation of losses or Swinburne method
  - the regenerative or Hopkinson back-to-back method.
- State the condition for maximum efficiency and also calculate maximum efficiency.

Unit 1: Speed control of DC motors

LEARNING OUTCOMES

- Explain normal speed, below normal speed and above normal speed of DC motors
- Explain how speed control of a DC motor is achieved by:
  - varying the voltage across the armature
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  - using the Ward-Leonard control system
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Introduction

This section covers the popular methods used for the control of DC motor speed. The circuit configuration and simple calculations provided will give further insight into the relative advantages and disadvantages of each of these methods.

1. Operating speeds of a DC motor

This section describes three operating speeds of DC motors, namely, normal speed, below normal speed, and above normal speed.

1.1 Normal speed

The normal speed of a DC motor is the speed at which the motor will run when operated at rated electrical parameters. These parameters can be found on the nameplate of the motor, as shown in the example in Figure 1.1.

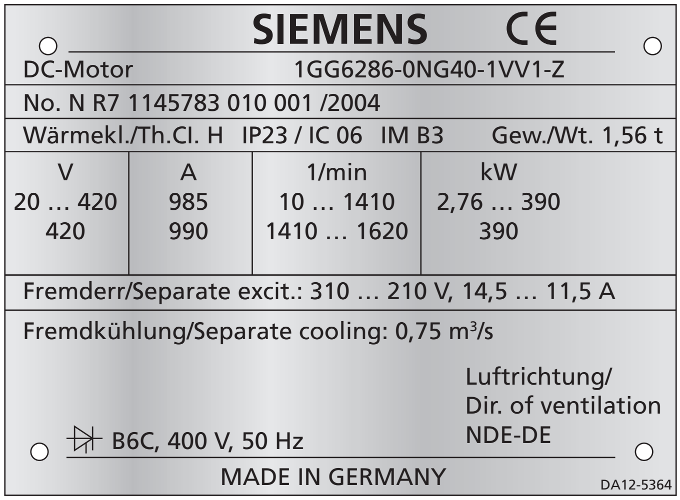

Figure 1.1 Nameplate on a commercial DC motor

In the nameplate shown above, the speed, given in the SI unit 1/min or m^{-1} covers a range of values, namely, 10 … 1 410 and 1 410 … 1 620 m^{-1}. Also note that the common unit for speed (rpm) is the same as m^{-1}. The ranges listed on the motor's nameplate refers to the speed control used for the respective range.

The normal or rated speed of this motor is interpreted as being 1 410 m^{-1} (1 410 rpm) for an armature voltage of 420 V and an armature current of 990 A. The output power at this point being 390 kW.

The speed-output graph in Figure 1.2 shows the relationship between speed, voltage, current, torque and output power. The normal or rated speed in indicated at n_N.

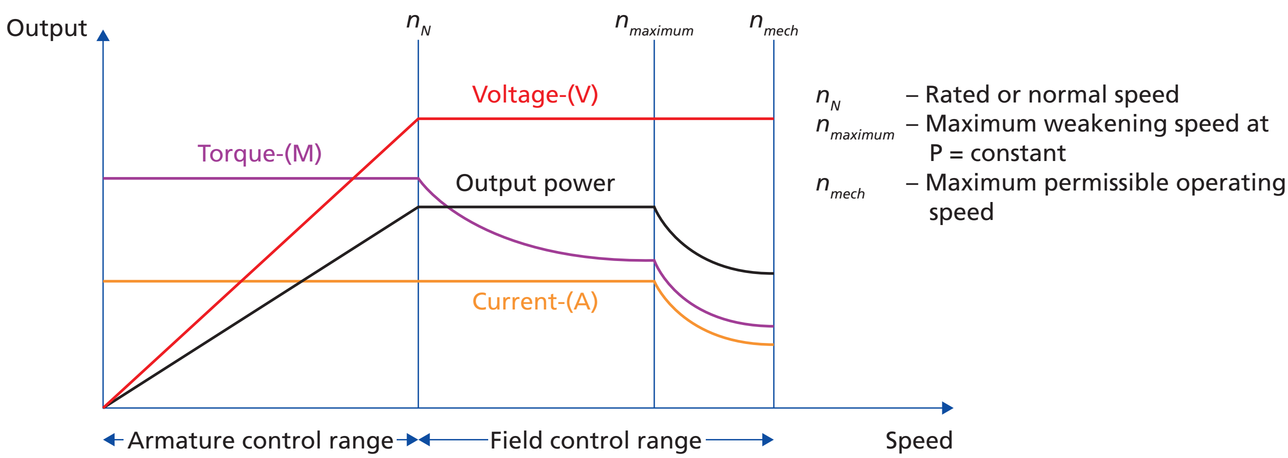

Figure 1.2 Speed vs output of a DC motor

1.2 Below normal speed

An advantage of a DC motor is that speeds below its rated speed can be easily controlled by varying the voltage, since speed ∝ V (with flux φ being constant). The example motor in Figure 1.1 can be operated between 10 and 1 410 m^{-1} (from 10 rpm up to its normal speed of 1 410 rpm) through armature control. Also note that the torque (M) is constant over this range. Section 2 discusses speed control in detail.

1.3 Above normal speed

A DC motor can be operated above normal speeds, but the range is not as large as that below normal speed. For the example DC motor shown in Figure 1.1, the standard design for field weakening speeds gives a range of 1 410 to 1 620 m^{-1} or from n_N *to* 1,15 · n_N, a much smaller range than below normal speed. This range is indicated as the field control range in Figure 1.2, from n_N *to* n_{Fmax}. Within this range, the torque decreases and the output power is constant. Motors supplied by this manufacturer can also be operated at speeds higher than the quoted n_{Nmax}. The maximum permissible speed n_{mech} is the maximum speed at which the motor can be safely operated. A DC motor should never be operated at speeds higher than this value.

2. Speed control of a DC motor

There are a few methods available to control the speed of a DC motor, for example, by varying:

- the supply voltage
- the flux or the field winding current
- the armature voltage
- the armature resistance.

We shall cover the methods of varying the voltage across the armature, connecting motors in series-parallel, the Ward-Leonard control system, and varying the value of the main magnetic flux or field current.

2.1 Varying the voltage across the armature

In the fundamental motor equations, the motor speed is inversely proportional to the armature voltage drop and directly proportional to the supply voltage. Hence, by adjusting the armature resistance, the voltage across the armature ($E_b = V - I_a R_a$) is varied, where E_b is the motor armature voltage, V is the terminal voltage, I_a is the armature current and R_a the armature resistance (see Figure 1.3). This is achieved by adjusting a variable resistor in series with the armature. If we slowly increase this series resistance value, then the voltage across the armature is decreased. A decrease in armature voltage drop results in a decrease in motor speed. So, the higher the resistance in series with the armature, the slower the motor speed.

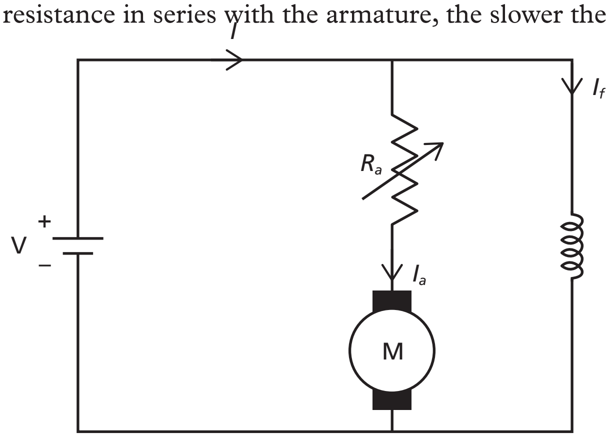

Figure 1.3 Adjusting armature voltage drop

2.2 Using the series-parallel control method

The mechanical coupling of two motors can be used to either lower the motor speed or to increase the speed. In order to decrease the speed, the two motors are connected in series. With the two motors connected in series, the same current flows through both motors. Since the supply voltage is split between the two motors, the lower motor voltage results in a slower speed. This is because the motor speed is directly proportional to the supply voltage.

In order to increase the motor speed, the two motors can be connected in parallel. In this case, each motor now has the same supply voltage, with the current splitting amongst them. As we know the motor speed is inversely proportional to the armature voltage. So, if we decrease the armature current, the armature voltage E_b will increase, according to $E_b = V - I_a R_a$, since V and R_a are constant. Hence the motor speed will increase.

2.3 Using the Ward-Leonard control system

A Ward-Leonard control system is used for precision speed control. A good example is its use in elevators. It can be seen in Figure 1.4 that it is a combination of a motor (M_1) that is mechanically coupled with a generator (G), connected in parallel with the DC motor (M_2) in question, whose speed needs to be controlled. M_1 has constant speed and can either be an AC motor or a DC motor. The generator output is connected to the armature of motor M_2. By varying the output voltage of the generator through its field regulator, the armature voltage of motor M_2 is changed accordingly. This results in a very smooth speed control.

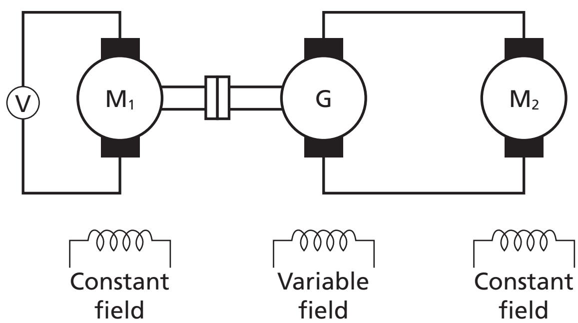

Figure 1.4 Ward-Leonard control system

2.4 Varying the value of the main magnetic flux/field current

By now you know that speed is inversely proportional to the flux per pole of a DC motor. Therefore, by increasing the flux, the motor speed can be decreased and vice versa. So, the speed of a DC motor can be controlled by adjusting the magnetic flux through the field windings. This is called the flux control method.

The magnetic flux is created by the flow of the field winding current. Hence, the magnitude of the magnetic flux can be altered by adjusting the magnitude of the field winding current. The adjustment of a variable resistor connected in series with the field windings can be used to achieve this flux control (see Figure 1.5). With an increase in resistance, the field winding current is decreased, resulting in a decrease in the produced flux with an increase in speed. The field current is comparably small in shunt motors. This results in small acceptable $I_{sh}^2 R$ total losses.

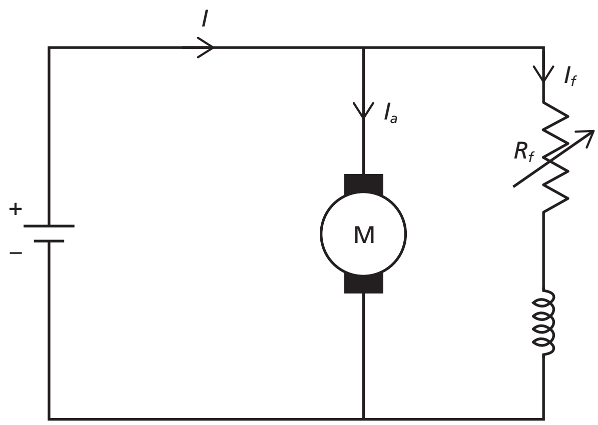

Figure 1.5 Adjusting field winding current of a DC motor

We can also adjust the motor series inductance in the case of series DC motors by:

- tapped field control, where different values of flux can be selected with different selection of number of turns of the field coil
- paralleling field coils, where likewise, different speeds can be chosen by selecting different parallel coil combinations.

This is illustrated in Figure 1.6.

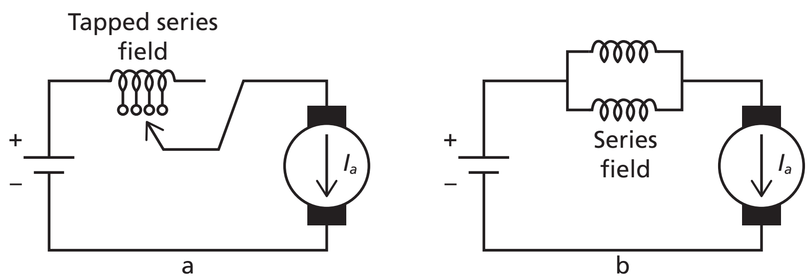

Figure 1.6 Adjusting field coils through a) tapped field and b) paralleling field coils

3. Methods of calculating the speed control of shunt and series DC motors

This section shows different methods of calculating the speed control of shunt and series motors.

3.1 Insert a resistor in series with the shunt field

The circuit at Figure 1.7 shows a DC motor with a variable resistor (connected as a rheostat) connected in series with the field winding, and the entire field branch is connected in parallel (shunt) with the armature. In this method of speed control (also known as flux control), the armature voltage is held constant.

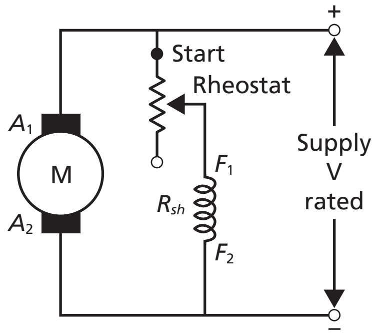

Figure 1.7 Flux speed control method – resistor in series with a shunt field

The rated parameters are: V_{supply} (V_T) = 250 V, input current (I_T) = 14 A, number of poles (P) = four, total number of armature conductors (A) = 500, number of parallel paths (A) = 2, flux per pole (Ø) = 20 mWb, resistance of the field winding (R_f) = 3 Ω, resistance of the field control resistor (R_{fc}) = 50 Ω, and resistance of the armature winding (R_a) = 0,25 Ω.

We can determine the rated speed of the motor using the equation:

$$E_a = \frac{\Phi \cdot Z \cdot N \cdot P}{60\ A}$$

Therefore, $N = \dfrac{E_a \cdot 60A}{\Phi \cdot Z \cdot P}$

where N is the rated speed in m^{-1} or rpm.

$$E_a = V_T - I_a R_a$$

Using KCL, $I_a = I_T - I_f$

From the given conditions:

$$I_f = \frac{V_T}{(R_f + R_{fc})}$$

$$= \frac{250\ V}{(3 + 50)\ \Omega}$$

$$= 4{,}72\ A$$

Therefore, $I_a = 14\ A - 4{,}72\ A$
$= 9{,}28\ A$

Therefore, $E_a = 250\ V - (9{,}28\ A \times 0{,}25\ \Omega)$
$= 247{,}68\ V$

We can now determine the rated speed N:

$$N = \frac{247{,}68\ V \times 60 \times 2}{20 \times 10^{-3} \times 500 \times 4}$$

$$= 743{,}04\ m^{-1}$$

If the field control resistance R_{fc} is increased, the field current I_f will decrease. Since flux φ is proportional to field current, a decrease in the field current will result in a decrease in flux. And since speed is inversely related to flux,

$$N = \frac{E_a \cdot 60 \cdot A}{\Phi \cdot Z \cdot P}$$

Therefore, $N \propto \frac{1}{\Phi}$

A decrease in the flux will result in an increase in the speed. This method of speed control is known as field weakening and is used to operate the motor above the rated speed. Care has to be taken that the motor does not exceed the mechanical limitations of the motor, as shown in Figure 1.2.

3.2 Insert a resistor in series with the armature

Now consider a shunt connected DC motor with a control resistor connected in series with the armature winding as shown in Figure 1.8 below.

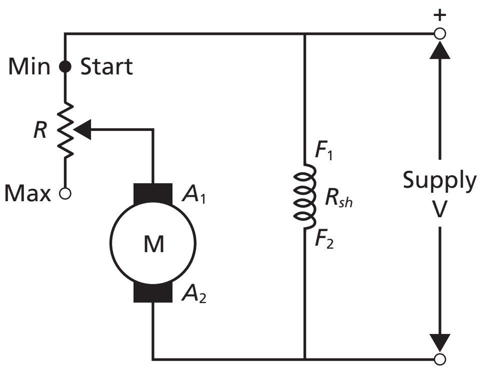

Figure 1.8 Resistor in series with armature

Assume that the supply voltage (V_T) = 220 V, the armature resistance (R_a) = 0,5 Ω, the armature control resistance (R_{ac}) = 2 Ω and the armature current is 10 A for a speed of 800 m^{-1}. Determine then determine the speed when the armature control resistor is changed to 1,36 Ω with a subsequent increase in armature current to I_a = 20 A.

For a speed of 800 m^{-1} (N_1):

$$E_a = 220\text{ V} - 10\text{ A}(0,5 + 2)$$
$$= 195\text{ V}$$

For unknown speed (N_2):

$$E_a = 220\text{ V} - 20\text{ A}(0,5 + 1,36)$$
$$= 182,8\text{ V}$$

Since $N = \frac{E_a - (R_a + R_{ac}) I_a}{k \cdot \Phi}$ and the flux can be assumed to be constant due to the constant field current, then,

$$N = E_a - \left(R_a + R_{ac}\right) I_a$$

Therefore,

$$\frac{N_2}{N_1} = \frac{182,8\text{ V}}{195\text{ V}}$$
$$= 0,937\text{ V}$$

Therefore,

$$N_2 = 0,937 \times 800$$
$$= 749,6\text{ m}^{-1}$$

Note that by changing the armature current, in this case, by a change in resistance R_{ac}, the speed of the motor can be changed. Note also that the armature current will increase if the load is increased and this will also result in a reduction in speed.

3.3 Insert a resistor parallel with the series field

Consider the DC motor connected as shown in Figure 1.9.

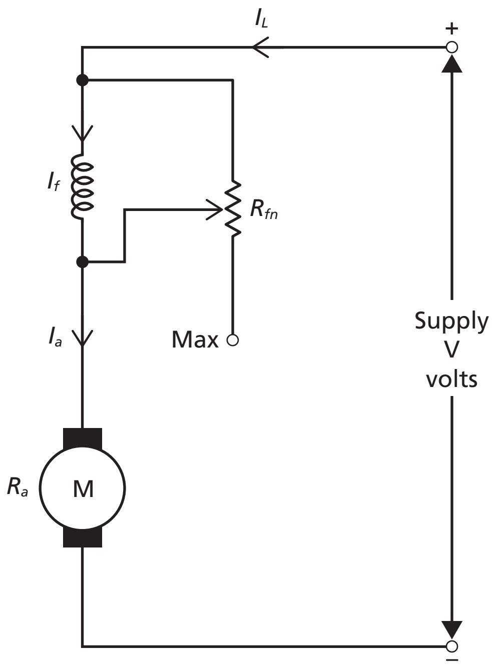

Figure 1.9 Resistor connected in parallel with the series field

The field winding is in series with the armature and a variable resistor is connected in parallel across the field winding. The variable resistor is known as a diverter since it diverts or shunts current away from the field winding. If the shunt resistor is decreased in value, less current will flow though the field and armature windings resulting in less flux and a higher speed. The motor is essentially connected in series as opposed to the two previous cases that were shunt-connected.

Assume that we have a series-connected DC-motor with a variable resistor connected in shunt with the field winding as shown in Figure 1.9 above. If the motor is running and developing a torque of 7,96 N.m, has a supply voltage (V_T) = 310 V, a supply current (I_L) = 10 A, a resistance of the field winding (R_f) = 2 Ω, a parallel resistor (R_{fp}) = 10 Ω, and a resistance of the armature winding (R_a) = 0,5 Ω, determine the speed of the motor.

The armature voltage (E_a) can be determined by:

$$E_a = V_T - I_a\left(R_a + R_f \| R_{fp}\right)$$

$$E_a = 310 \text{ V} - 10 \text{ A} \cdot \left(0{,}5 + \frac{2 \times 10}{2 + 10}\right)$$

$$= 288{,}33 \text{ V}$$

The speed is therefore:

$$\omega = \frac{E_a \cdot I_a}{T}$$

$$= \frac{288{,}33 \text{ V} \times 10 \text{ A}}{7{,}96 \text{ N.m}}$$

$$= 362{,}22 \text{ rad/s}$$

Converting to revolutions per minute:

$$N = \frac{\omega \cdot 60}{2\pi}$$
$$= 3\ 458{,}95\ m^{-1}$$

If the parallel resistor is now decreased to a value of 5 Ω, and the output torque decreases 6,82 N.m, with all the other parameters held constant, the new speed will be:

$$E_a = 310\ \text{V} - 10\ \text{A} \cdot \left(0{,}5 + \frac{2 \times 5}{2 + 5}\right)$$
$$= 290{,}71\ \text{V}$$
$$\omega = \frac{290{,}71\ \text{V} \times 10\ \text{A}}{6{,}82\ \text{N.m}}$$
$$= 426{,}26\ \text{rad/s}$$
$$N = \frac{426{,}26 \times 60}{2\pi}$$
$$= 4070{,}48\ \text{m}^{-1}$$

The diverter resistor has shunted current away from the field winding, resulting in a decrease in flux, a decrease in the output torque and an increase in the speed. This method is used for above rated speed control.

4. Interpret the magnetisation curve of a DC motor

The magnetic circuit illustration of the motor in Figure 1.10 will help to explain the magnetisation characteristics of a DC motor.

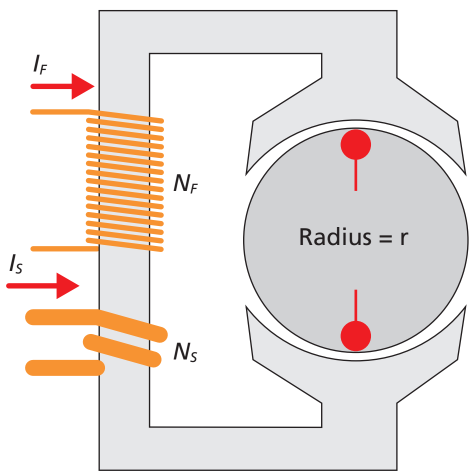

Figure 1.10 Magnetic circuit of a DC motor

The flux in the air gap of the motor is created by the currents I_s and I_f of the field coil.

Figure 1.11 shows the general magnetisation curve of a DC motor. The curve gives us an indication of how much flux (Ø) appears in the air gap of a DC motor at a given value of the MMF.

$$\text{MMF} = N_f I_f + N_s I_s$$

where,
MMF is the magnetomotive force
NI is the ampere-turns (A_t)
N is the number of turns
I_f and I_s are the currents.

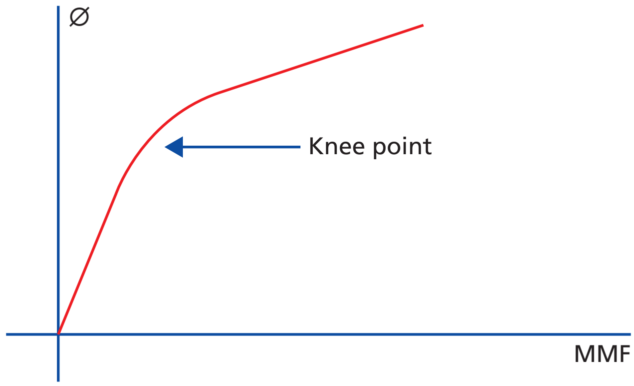

Figure 1.11 Magnetisation curve of a DC motor

The magnetisation curve is formed due to the influence of the field coil currents and represents the magnetisation characteristic of the machine. The relationship is non-linear. The point where the curve makes a bend is called the knee point. Saturation takes place at higher values after the knee point.

If we multiply the flux Ø on the y-axis with kω then it represents E_a, the induced armature voltage (kØω).

Likewise, if we divide the MMF on the x-axis of Figure 1.11 with N_f, then it represents the field current (I_f) as shown on the x-axis in Figure 1.12.

$$I_f = \frac{N_f I_f + N_s I_s}{N_f}$$

The magnetisation curve represents the voltage induced in the armature $\left(E_a = k\text{Ø}\omega\right)$ for a given value of field current I_f, when the machine rotates at a certain test speed, ω_{test} (see Figure 1.12). If we change the speed of the motor, then the gradient of the curve will either increase or decrease depending on whether it is a higher or lower motor speed.

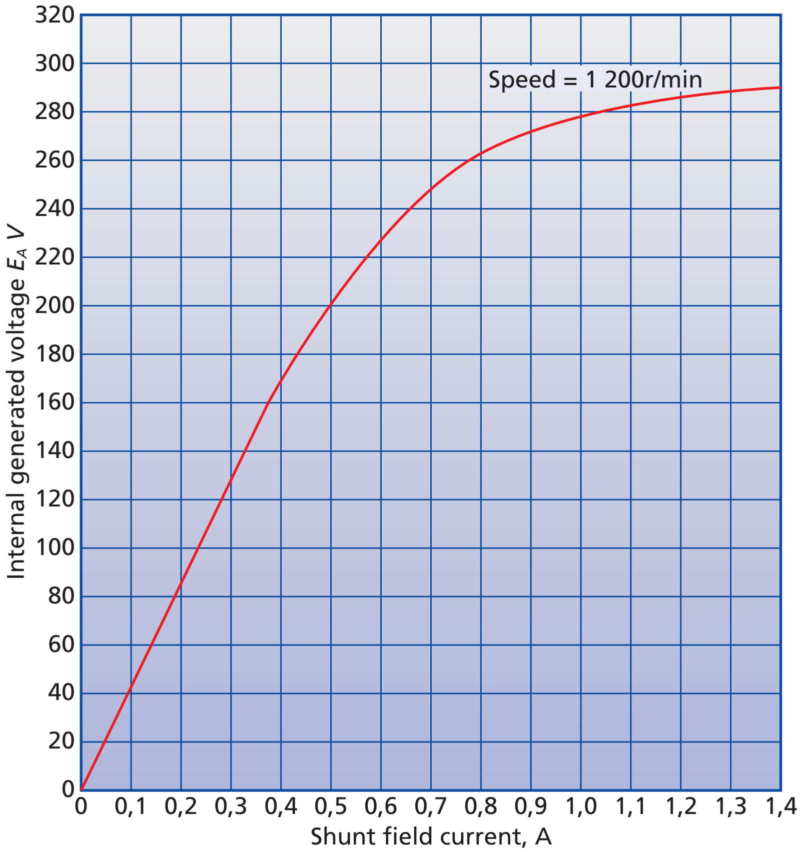

Figure 1.12 Armature voltage vs field current curve of a DC motor

If we now multiply the field current value on the x-axis with that of $kØ$, then we can read the torque ($kØI_a$) of the machine from the graph.

Worked example 1.1 **Rotor speed and torque?**

A DC motor with an armature voltage-field current curve shown in Figure 1.12, has a field coil with 4 000 turns and an additional coil with 25 turns. The field current is 0,7 A and the other coil current is 67 A. The armature induced voltage (E_a) is 200 V.

1. What is the actual speed of the motor?
2. What is the torque when the armature draws 72 A at a speed of 1 200 rpm?

Solution

1. We calculate the field current I_f as

$$I_f = \frac{N_f I_f + N_s I_s}{N_f}$$

$$= \frac{(4\,000)(0{,}7) + (25)(67)}{4\,000}$$

$$= 1{,}12 \text{ A}$$

We can interpret the following from the magnetisation curve given in Figure 1.12:

If the motor rotates at 1 200 rpm, then we can read off the curve that at 1,12 A its induced voltage E_a would be 288 V.

The actual armature voltage $E_a^{\text{actual}} = 200$ V. This would result in a speed that is less than 1 200 rpm.

$$E_a^{\text{curve}} = k\varnothing\omega_{test}$$

$$E_a^{\text{actual}} = k\varnothing\omega_{actual}$$

$$n = (1\ 200)\frac{200}{288}$$

$$= 833 \text{ rpm}$$

2. At 1 200 rpm:

$$k\varnothing = \frac{E_a}{\omega}$$

$$= \frac{288}{\frac{2\pi(1\ 200)}{60}}$$

$$= 2.29$$

Torque:

$$T = k\varnothing I_a$$

$$= (2.29)(72)$$

$$= 165 \text{ Nm}$$

ACTIVITY 1.1 — Magnetic characteristic curve of a DC motor

Refer to Figure 1.13.

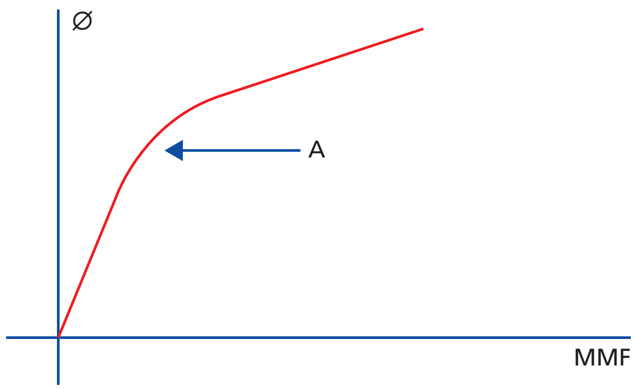

Figure 1.13 Magnetic characterisation curve of a DC motor

As a class, discuss and answer the following questions:

1. What is point A on the curve called.
2. Why is the curve of the magnetic characteristic of a DC motor given in Figure 1.13 non-linear?
3. Derive at least two basic DC motor formulas that can be derived from the curve.

5. Speed, current, EMF, and torque of a DC motor

The performance characteristics of a DC motor can be determined using the appropriate formulae; appropriate in the sense that one needs to determine whether how the motor is connected.

The following equations can be used to describe a DC motor:

$V_a = R_a \cdot i_a + L_a\frac{di_a}{dt} + e_a$ [refers to the armature circuit shown in Figure 1.14 below]

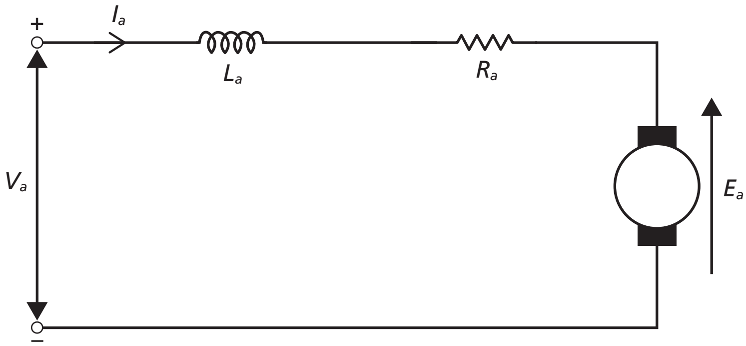

Figure 1.14 An armature circuit

$e_a = k_a \cdot \Phi \cdot \omega$ [back EMF]

$T = k_T \cdot \Phi \cdot i_a = \frac{E_a \cdot i_a}{\omega}$ [rotor torque also known as the electromagnetic torque]

$T = J\frac{d\omega}{dt} + B\omega + T_L$ [rotor torque equated to the mechanical system]

For the above:

ω is the speed in radian/s
Φ is the flux in webers
$k_a = k_T$ are numerically equal constants if SI units are used
J is the moment of inertia of the mechanical system
B is the coefficient of friction in the mechanical system
T_L is the load torque.

The analysis of the motors are done when they are in the steady-state phase; we assume that all transients have died down. The motor is, therefore, running at a certain speed and at a certain load. During this phase, the variables that are dependent on a rate of change are zero, therefore, $L_a\frac{di_a}{dt}$ and $J\frac{d\omega}{dt} = 0$, and the equations reduce to the familiar ones that were used in earlier analyses.

As an example, the DC motor shown in Figure 1.15 is connected in shunt mode.

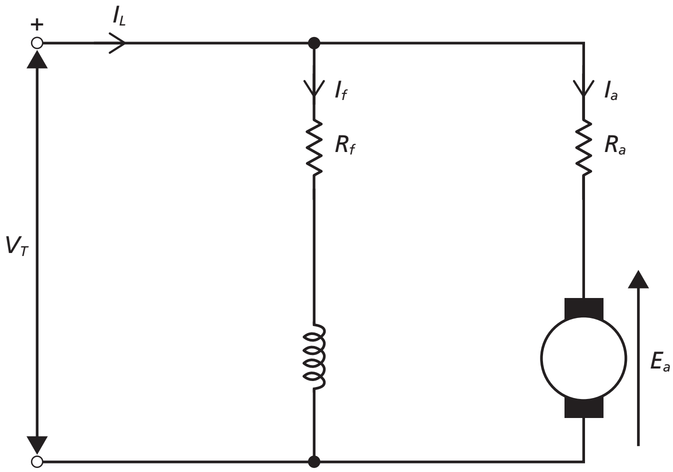

Figure 1.15 A DC motor connected in shunt mode

In this case, the motor is running at rated load with the following conditions:

$V_T = 250$ V

$R_f = R_s + R_w = 23\ \Omega$

$R_a = 0{,}3\ \Omega$

$k_a = 300$ V.s/rad

$\Phi = 15$ mWb

$I_L = 30$ A.

The armature current can be determined using KCL as:

$$
\begin{aligned}
I_a &= I_L - I_f \\
&= 30 - \frac{250\text{ V}}{23} \\
&= 30 - 10{,}87 \\
&= 19{,}13\text{ A}
\end{aligned}
$$

The back EMF (armature voltage) can be determined as:

$$
\begin{aligned}
E_a &= V_T - I_a \cdot R_a \\
&= 250\ V - (19{,}13 \times 0{,}3) \\
&= 244{,}26\text{ V}
\end{aligned}
$$

The speed can be determined as:

$$
\begin{aligned}
\omega &= \frac{E_a}{k_a \times \Phi} \\
&= \frac{244{,}26\text{ V}}{300 \times 15\text{ mWb}} \\
&= 54{,}28\text{ rad/s}
\end{aligned}
$$

Converting to rpm:

$$
\begin{aligned}
N &= \frac{\omega \cdot 60}{2\pi} \\
&= \frac{54{,}28 \times 60}{2\pi} \\
&= 518{,}33\text{ m}^{-1}
\end{aligned}
$$

The rotor torque is calculated as:

$$
\begin{aligned}
T &= \frac{E_a \cdot I_a}{\omega} \\
&= \frac{244{,}26 \times 19{,}13}{54{,}28} \\
&= 86{,}09\text{ N.m}
\end{aligned}
$$

If the mechanical system has a frictional coefficient of B = 20 × 10-3 N.m/rad/sec, then the torque at the output of the mechanical system is:

$$
\begin{aligned}
T_L &= T - B \cdot \omega \\
&= 86{,}09 - (20 \times 10^{-3} \cdot 54{,}28) \\
&= 85\text{ N.m}
\end{aligned}
$$

Now, consider the motor connected in series as shown in Figure 1.16.

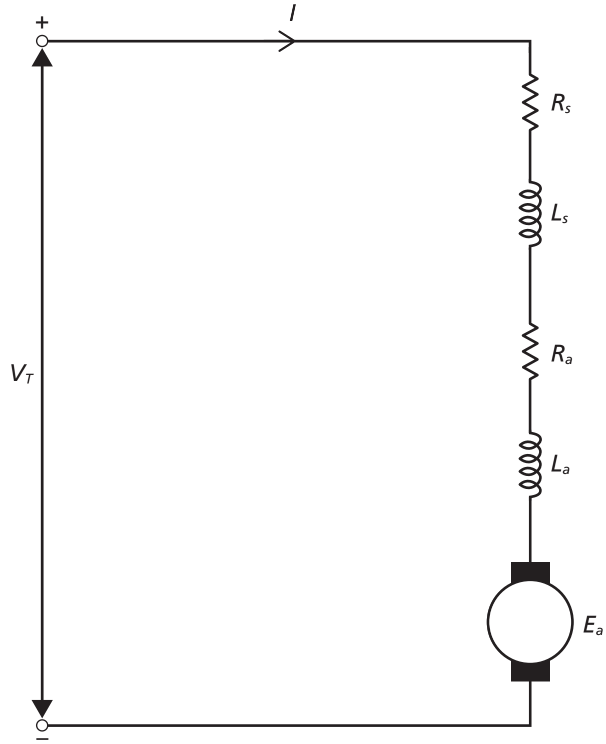

Figure 1.16 A DC motor connected in series

The motor is rated at 120 W and has a rated speed of 2 000 rpm. The efficiency is 80%. The armature winding resistance is 0,5 Ω, the field resistance winding is 0,7 Ω and the series control resistance is 30 Ω.

Converting rpm to rad/s:

$$\omega = \frac{2\ 000 \times 2\pi}{60}$$
$$= 209{,}44 \text{ rad/s}$$

The input power is determined by:

$$P_{in} = \frac{P_{out}}{\eta}$$
$$= \frac{120\ W}{0{,}8}$$
$$= 150\ W$$

The power losses are therefore:

$$P_{losses} = P_{in} - P_{out}$$
$$= 150 - 120$$
$$= 30 \text{ W}$$

Neglecting the losses in the mechanical system, and assuming only winding and resistive losses:

$$P_{losses} = I_a^2 \times (R_s + R_a + R_f)$$

Therefore,

$$\begin{aligned} I_a &= \sqrt{\frac{P_{losses}}{(R_s + R_a + R_f)}} \\ &= \sqrt{\frac{30\ \mathrm{W}}{(30 + 0{,}5 + 0{,}7)}} \\ &= 0{,}98\ \mathrm{A} \end{aligned}$$

The supply voltage is determined as:

$$\begin{aligned} V_T &= \frac{150\ \mathrm{W}}{0{,}98\ \mathrm{A}} \\ &= 153{,}06\ \mathrm{V} \end{aligned}$$

The back EMF is given by:

$$\begin{aligned} E_a &= V_T - I_a(R_s + R_a + R_f) \\ &= 153{,}06 - (0{,}98 \times 31{,}2) \\ &= 122{,}49\ \mathrm{V} \end{aligned}$$

The rotor torque is given by:

$$\begin{aligned} T &= \frac{P}{\omega} \\ &= \frac{120\ \mathrm{W}}{209{,}44} \\ &= 0{,}573\ \mathrm{N.m} \end{aligned}$$

6. Interpret various conditions

This section demonstrates how to interpret the certain conditions, namely, when the ohmic voltage drops are negligible, when the current drawn from the mains is to remain unchanged, and when there are sudden changes in flux.

6.1 When ohmic voltage drops are negligible

In DC machine equations, because the inductive reactance ($X = 2\pi fL$) is zero, the frequency is zero. When a DC motor is in steady-state, then the armature current is constant according to $I = \frac{V - E}{R_a}$.

6.2 When the current drawn from the mains is to remain unchanged

Under steady state conditions, the motor torque equals the load torque. As soon as the motor torque is changed (different to the load torque), then the motor either accelerates or decelerates.

6.3 When there are sudden changes in flux

A sudden change in flux in a DC machine alters the back EMF and drastically changes the armature current. So, if the flux is suddenly decreased, the back EMF decreases, because back EMF is directly proportional to magnetic flux and speed $\left(E_a = k\text{Ø}\omega\right)$. The armature current increases significantly $\left(I_a = \frac{V - E_b}{R_a}\right)$. The motor torque increases because it is directly proportional to armature current $\left(T = k\text{Ø}I_a\right)$. The increased motor torque leads to a change in motor speed, according to the type of motors as shown in the graph in Figure 1.17. Keep in mind that at first the speed stays constant due to inertia.

Due to the presence of back EMF, the DC motor becomes a self-regulating machine. Just enough armature current is drawn to develop the required torque for the load. Back EMF ensures the automatic adjustment of armature current according to the load.

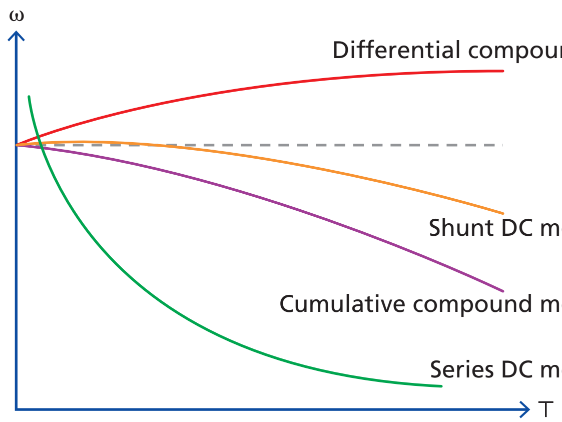

Figure 1.17 Torque-speed characteristics of different DC motors

Worked example 1.2 **Change in armature current**

Calculate by what percentage the armature current would change if the flux were reduced by 5%, the supply voltage is 230 V and the armature resistance is 0.5 Ω. During normal operation E_b=220 V and I_a = 20 A.

Solution

The new armature voltage $E_b = (0{,}95)(220) = 209 \text{ V}$

the new armature current $I_a = \frac{V - E_b}{R_a} = \frac{230 - 209}{0.5} = 42 \text{ A}$

A 5% decrease in flux increased the armature current by 110%.

So, even when the flux is decreased by a small amount, then there is a drastic increase in armature current, which also leads to an increase in torque. This results in an increase of speed. The speed increases up to when the armature voltage and current adjust to a constant power load.

One advantage a DC motor is that it can create a constant torque over a wide speed range. Since torque is proportional to armature current, all we need to do is control the current in order to control the torque. This can be done with a DC drive.

ACTIVITY 1.2 DC motors

Select the most appropriate answer for the following questions:

1. In the region from 0 to the rated speed of a DC motor, the motor will exhibit:
   a) varying torque
   b) constant torque
   c) constant power
   d) varying flux
   e) none of the above.
2. In the region from the rated speed to the maximum speed of a DC motor, the motor will exhibit:
   a) varying torque
   b) constant torque
   c) constant power
   d) constant flux
   e) none of the above.
3. Speed control above the rated speed of a DC motor can be achieved by:
   a) armature voltage control
   b) armature current control
   c) increasing the load
   d) field weakening
   e) none of the above.
4. A resistive starter circuit is employed in a shunt-connected DC motor to:
   a) provide starting current
   b) increase the efficiency of the motor
   c) prevent high starting currents
   d) increase the field flux.
   e) none of the above.
5. A low load or low current in a series-connected DC motor will result in:
   a) an increase in flux
   b) an increase in speed
   c) a decrease in back EMF
   d) a decrease in the terminal voltage
   e) none of the above.