N6 Mechanotechnics

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Module
1

Clutches

What is covered?

This module will focus on the function of clutches, as well as the theories applicable to plate and cone clutches. The advanced calculations from basic principles of plate, cone and centrifugal clutches will be covered as well.

Learning Outcomes

After studying this module, you should be able to:

- draw and label a multi-plate clutch.
- explain the uniform wear theory and uniform pressure theory.
- calculate the force required to press the plates together, the number of contact pairs required in a plate clutch in order to apply the required pressure, the combined speed between the engine and shafts at the moment that no slip occurs, the total time of slip, the total time it takes the shaft to reach a certain speed.
- calculate the torque and retardation torque of the engine or shaft and combined speed.
- define conical clutches and explain their applications.
- calculate the torque, frictional force and axial force to transmit given torque, the total axial force required to engage the clutch, the moment of inertia, angular velocity, angular acceleration, time, maximum and minimum pressure.
- describe and explain the application of centrifugal clutches.
- calculate the centrifugal force on spring force and frictional force.
- calculate torque, angular velocity, mass and power.

Unit 1:

LEARNING OUTCOMES

Introduction

Keyword

stress a physical quantity that expresses the internal forces that the nearest particles of a continuous material exert on each other over a specific area of the material

The significant function of a friction clutch is to connect a rotary gear to a stationery one. This must be done in such a way that it smoothly overcomes inertia in order to accelerate the coupled gears to the necessary speed. More so, this must be done in such a way that it causes the minimum **stress** on the parts of mechanism. A clutch allows two coaxial shafts to engage or disengage while at rest or in relative motion. Friction clutches may be of plate, cone or centrifugal type. In each type slippage will occur until the two shafts rotate at the same speed. This feature allows gradual engagement of the driven shaft and also minimises the torque demand from the driving shaft. The input torque from the driving shaft is transmitted by friction between the two contact surfaces and the maximum torque transferable from the driving to the driven shaft is limited by the maximum frictional force between the contact surfaces. In plate and cone clutches, the normal pressure distribution across the contact surfaces can either be according to uniform wear theory or uniform pressure theory.

Figure 1.1: Single-plate clutch

1.1. Uniform wear theory

Assuming that the fit between the contact surfaces is perfect, the normal **intensity** of pressure will be equal at all points on the contact surfaces of a clutch. This theory is relevant to worn clutches, because wear is uniform over the contact area from the inner radius to the outer radius. The rate of wear between the contact surfaces depends on the intensity of the pressure as well as the velocity of the rubbing between the faces. Wear is therefore relative to the product of the normal pressure and the rubbing velocity. This implies that the maximum pressure occurs on the inner radius ($P \times r = c$), which means that the effective radius is equal to the arithmetic mean radius. Refer to Figure 1.2 for the illustration of uniform wear theory.

Keyword

intensity the feature of being very resilient, strong, concentrated, as well as difficult

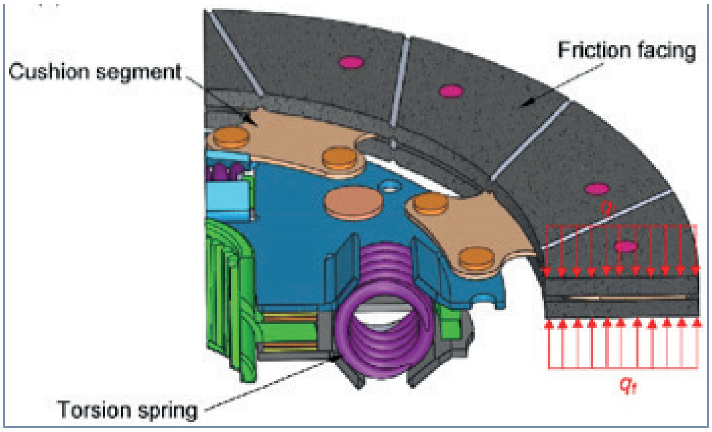


Figure 1.2: Uniform wear theory

1.2 Uniform pressure theory

This theory is based on the assumption that the pressure is distributed uniformly over the frictional surface. The total normal force between the surfaces is the product of the pressure and the contact area. This type of theory provides higher friction torque compared to the uniform wear theory. If the contact area is divided into a number of concentric rings of equal width, the outmost ring will have the largest area, the ring areas decreasing towards the centre. The normal force per will also decrease from the outer ring inwards. The effective radius at which the total normal force can be considered as concentrated is therefore closer to the outer radius than the arithmetic mean radius. Refer to Figure 1.3 for the uniform pressure theory illustration.

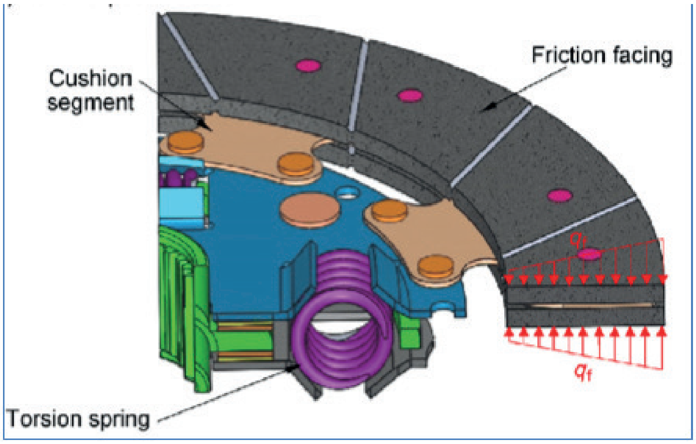
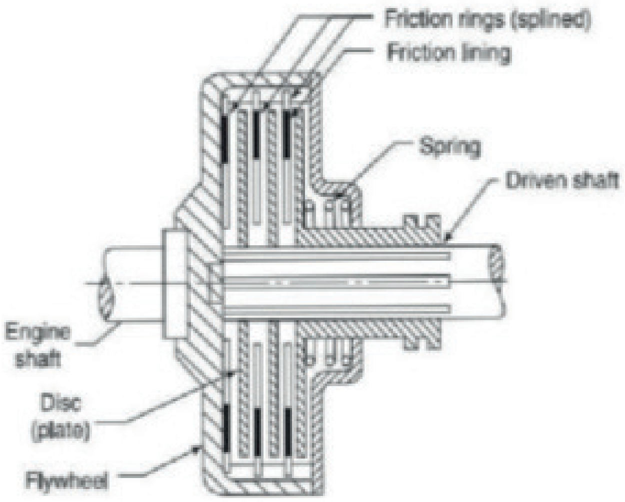


Figure 1.3: Uniform pressure theory

1.3 Plate clutches

A multi-plate clutch is shown in Figure 1.4 below. Clutches are available in a single-plate form as shown in Figure 1.1. The plates are subjected to an axial force when the clutch is engaged. Therefore, a torque is transmitted by friction at the sides of each plate. In a single-plate clutch, both sides of the plate are effective and two contact surfaces are considered. However, in a multi-plate clutch all the surface pairs contribute to the torque in transmission.



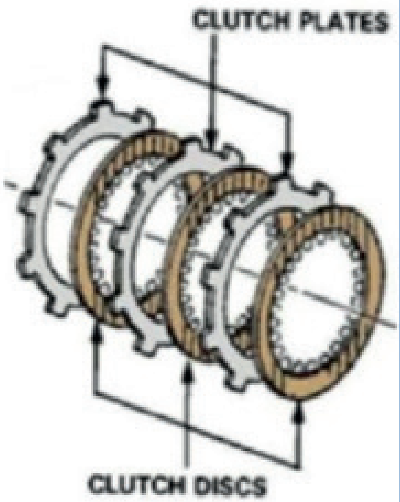


Figure 1.4: Multi-plate clutch

So, for a single-plate clutch there are two contact surfaces, and the normal force is the same as the spring force, as well as the same as the axial force ($F_n = F_s = F_A$). All the formulae applicable to single-plate clutches also apply to the multi-plate clutches as well. It is only the number of contact surfaces that differs, where n = sum of rings and plates − 1.

1.3.1 Calculations for plate clutches

Before we look at some solved examples, let us look at the formulae and symbols we need to do calculations for plate clutches.

Formulae and symbols

(i) Uniform wear theory

$$R_f = \frac{R + r}{2}$$

Where:
R_f = effective friction radius in metres (m)
R = outside radius of a friction disc in metres (m)
r = inside radius of a friction disc in metres (m)

(ii) Uniform pressure theory

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right)$$

Where:
R_m = effective friction radius in metres (m)
R = outside radius of a friction disc in metres (m)
r = inside radius of a friction disc in metres (m)

(iii) Final speed in revolutions per second (r/s)

$$N = \frac{\alpha_{shaft} \times N_{engine}}{\alpha_{engine} + \alpha_{shaft}}$$

Where:

N = final speed in revolutions per second (r/s)

α_{shaft} = angular acceleration of the driven shaft in radians per square second (rad/s 2)

α_{engine} = angular acceleration of the engine shaft in radians per square second (rad/s ^{2})

N_{engine} = engine shaft speed in revolutions per second (r/s)

(iv) Axial force (F_A)

$F_A = p_{mean} \times \pi(R^2 - r^2)$ for mean pressure

$F_A = 2\pi r(R - r) \times p_{max}$ for maximum pressure

$F_A = 2\pi R(R - r) \times p_{min}$ for minimum pressure

(v) Friction torque

$$T = \mu \times F_A \times R_e \times n$$

Where:

μ = coefficient of friction

F_A = axial force applied to engage the clutch in newtons (N)

R_e = friction radius or effective radius, depending on the wear applicable, in metres (m)

n = number of contact surfaces

(vi) Time

$$t = \frac{\omega}{\alpha}$$

Note: When wear occur on a plate clutch, there will be an increase in spring length, resulting in a reduction in spring force when assuming equal wear on all surfaces.

(vii) Wear and reduction in spring force

ΔF_s = total wear × stiffness × number of springs

Where:

total wear = n × wear per surface

(vii) Power

$$P = \frac{2\pi NT}{60}$$

Where:

P = power in watts (W) or kilowatts (kW)

T = friction torque in newton metres (N.m)

1.3.2 Solved examples

Having looked at the different theories applicable to plate clutches and the formulae, calculations related to the terms described are explained in the examples that follow.

Note: The main principle in answering questions is to apply the 'info given' against the 'data required' methodology.

Example 1.1

A single-plate clutch with both sides effective has an inside diameter of 280 mm and an outside diameter of 380 mm. The pressure plate is limited to 273 kPa, and the coefficient of friction for the material is 0,35.

Calculate the following:

1. The axial force available if the clutch is new
2. The maximum torque that this clutch can transmit
3. The axial force available if the clutch is worn, assuming the same pressure limits
4. The maximum torque that the worn clutch can transmit.

Solutions

Data given:

d = 280 mm, so r = 0,14 m D = 380 mm, so R = 0,19 m

P = 273 kPa μ = 0,35

1. Calculating F_A for the new clutch:

$$F_A = p_{mean} \times \pi(R^2 - r^2)$$
$$= 273 \times 10^3 \times \pi(0,19^2 - 0,14^2)$$
$$= 14\ 151{,}304 \text{ N}$$

So: $F_A = F_n = 14{,}151$ kN

2. Calculating the maximum torque for the new clutch:

For the new clutch, consider R_m.

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3}\left(\frac{0,19^3 - 0,14^3}{0,19^2 - 0,14^2}\right) = 0,166 \text{ m}$$
$$T = \mu \times F_A \times R_m \times n$$
$$= 0{,}35 \times 14\ 151{,}304 \times 0,166 \times 2$$
$$= 1\ 644{,}382 \text{ N}$$

3. Calculating F_A for the worn clutch:

$$F_A = 2\pi \times r(R - r) \times p_{max}$$
$$= 2\pi \times 0,14 \times (0,19 - 0,14) \times 273 \times 10^3$$
$$= 12\ 007{,}167 \text{ N}$$

So: $F_A = 12{,}007$ kN

4. Calculating the maximum torque for the worn clutch:

For the worn clutch, consider R_f.

$$R_f = \left(\frac{R + r}{2}\right) = \left(\frac{0,19 + 0,14}{2}\right) = 0,165 \text{ m}$$
$$T = \mu \times F_A \times R_f \times n$$
$$= 0{,}35 \times 12\ 007{,}167 \times 0,165 \times 2$$
$$= 1\ 386{,}828 \text{ N.m}$$

Example 1.2

A single-plate clutch, with both sides effective, is required to transmit 130 kW at 20 r/s. The outside diameter of the contact surface is 380 mm and the coefficient of friction is 0,4. The uniform pressure is 173 kPa.

Calculate the inner diameter of the friction surface, assuming uniform pressure theory.

Solution

Data given:

D = 380 mm, so R = 0,19 m P = 130 kW

P = 173 kPa μ = 0,4 N = 20 r/s

Calculating the axial force:

$$P = 2\pi NT$$

$$T = \frac{P}{2\pi N} = \frac{130 \times 10^3}{2\pi \times 20} = 1\ 034{,}507 \text{ N.m}$$

$$F_A = p_{mean} \times \pi(R^2 - r^2)$$

$$= 173 \times 10^3 \times \pi(0{,}19^2 - r^2)$$

For uniform pressure theory, consider R_m.

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3}\left(\frac{0{,}19^3 - r^3}{0{,}19^2 - r^2}\right)$$

Calculate the inner diameter (d):

$$T = \mu \times F_A \times R_m \times n$$

$$1\ 034{,}507 = 0{,}4 \times 173 \times 10^3 \times \pi(0{,}19^2 - r^2) \times \frac{2}{3}\left(\frac{0{,}19^3 - r^3}{0{,}19^2 - r^2}\right) \times 2$$

$$\frac{1\ 034{,}507 \times 3}{0{,}4 \times 173 \times 10^3 \times \pi \times 2 \times 2} = 0{,}19^3 - r^3$$

$$0{,}00357 = 0{,}00686 - r^3$$

$$r = \sqrt[3]{0{,}00686 - 0{,}00357}$$

$$= 0{,}148 \text{ m}$$

$\therefore d = 2 \times 148 \text{ mm} = 296 \text{ mm}$

Example 1.3

A single-plate clutch, with both sides effective, has an inside diameter of 220 mm and an outside diameter of 420 mm. The axial force available is 15 kN, and the coefficient of friction for the material is 0,45.

Calculate the following:

1. The power that a new clutch can transmit at 1 450 r/min
2. The reduction in spring force if each friction surface of the clutch plate has worn away by 4 mm. There are 6 springs, and each spring has a stiffness of 55 N/mm
3. The power that a worn clutch can transmit.

Solutions

Data given:

$d = 220$ mm, so $r = 0{,}11$ m $D = 420$ mm, so $R = 0{,}21$ m

$F_A = 15$ kN $\mu = 0{,}45$

1. Calculating the power that a new clutch can transmit:
   For a new clutch, consider R_m.

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3}\left(\frac{0{,}21^3 - 0{,}11^3}{0{,}21^2 - 0{,}11^2}\right) = 0{,}165 \text{ m}$$

$$T = \mu \times F_A \times R_m \times n$$

$$= 0{,}45 \times 15\ 000 \times 0{,}165 \times 2$$

$$= 2\ 227{,}5 \text{ N.m}$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 1\ 450 \times 2\ 227{,}5}{60}$$

$$= 338\ 231.719 \text{ W}$$

$\therefore P = 338{,}232$ kW

2. Calculating the reduction in the spring force (F_s):
ΔF_s = total wear × stiffness × number of springs
So, total wear = n × wear per surface
total wear = 2 × 4 mm
= 8 mm
$\therefore \Delta F_s = 8 \times 55 \times 6 = 2\ 640$ N

3. Calculating the power for the worn clutch, considering the reduction in force:
For a worn clutch, consider R_f.
$R_f = \left(\frac{R+r}{2}\right) = \left(\frac{0,21+0,11}{2}\right) = 0.16$ m
$F_n = F_A - \Delta F_s$
$\therefore F_n = 15\ 000 - 2\ 640 = 12\ 360$ N
Note: When wear is considered, $F_A \neq F_n$, so F_n represents F_A.
$T = \mu \times F_n \times R_f \times n$
$= 0,45 \times 12\ 360 \times 0,16 \times 2$
$= 1\ 779,84$ N.m
$P = \frac{2\pi NT}{60}$
$= \frac{2\pi \times 1\ 450 \times 1\ 779,84}{60}$
$= 270\ 257.393$ W
= 270,257 kW

Example 1.4

A single-plate clutch, with both sides effective, is required to transmit a torque of 175 N.m when new. The coefficient of friction is 0,35, and the pressure is limited to 73 kPa. The inner diameter is 65% of the outer diameter.

Calculate the following:

1. The inner and outer diameter of the friction surface of the clutch
2. The axial force
3. The power that can be transmitted at the rotational frequency of 1 250 r/min.

Solutions

Data given:
$d = 65\%D$ $T = 175$ N.m $P = 73$ kPa $\mu = 0,35$

1. Calculating the inner and outer diameters of the clutch:
For a new clutch, consider R_m.
$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right)$
Calculate the inner diameter (d):

$$T = \mu \times F_A \times R_m \times n$$

$$175 = 0,35 \times 73 \times 10^3 \times \pi(R^2 - r^2) \times \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) \times 2$$

$$\frac{175 \times 3}{0,35 \times 73 \times 10^3 \times \pi \times 2 \times 2} = R^3 - (0,65)^3 R^3$$

$$0,001635 = R^3 - 0,275 R^3$$

$$0,001635 = 0,725 R^3$$

$$R = \sqrt[3]{\left(\frac{0,001635}{0,725}\right)}$$

$$= 0,131 \text{ m}$$

$\therefore D = 2 \times 131$ mm = 262 mm
$\therefore d = 65\% \times 262$ mm = 170,3 mm

2. Calculating the axial force:

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3}\left(\frac{0,131^3 - 0,0852^3}{0,131^2 - 0,0852^2}\right) = 0,12 \text{ m}$$

$$T = \mu \times F_A \times R_m \times n$$

$$F_A = \frac{T}{\mu \times R_m \times n}$$

$$= \frac{175}{0,35 \times 0,12 \times 2}$$

$$= 2\ 083,333 \text{ N}$$

3. Calculating the power transmitted:

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 1\ 250 \times 175}{60}$$

$$= 22\ 907,446 \text{ W}$$

$$= 22,907 \text{ kW}$$

Example 1.5

The flywheel and engine shaft have a combined moment of inertia of 2,8 kg.m^2 and develop a constant torque of 45 N.m. An engine is coupled via a single-plate clutch to a shaft. Both sides of the plate are effective. The outer and inner diameters are 360 mm and 190 mm respectively. The coefficient of friction is 0,35, and the force applied to the plates is 2,5 kN. The moment of inertia of the driven shaft is 8,5 kg.m^2 and exerts a friction torque of 12 N.m. Assume uniform wear theory.

When the clutch is engaged at an engine speed of 600 r/min and the shaft is active, calculate the following:

1. The combined speed between the engine and the shaft at the moment of no slip
2. The total time of slip
3. The time it takes for the shaft to reach 600 r/min.

Solutions

Data given:

$d = 190$ mm, so $r = 0,095$ m; $D = 360$ mm, so $R = 0,18$ m

$N_{engine} = \frac{600}{60} = 10$ r/s; $F_A = 2,5$ kN; $\mu = 0,35$

$T_\mu = 12$ N.m; $I_{shaft} = 8,5$ kg.m^2; $I_{engine} = 2,8$ kg.m^2

1. Calculating the combined speed between the engine and the shaft at the moment of no slip:

For uniform wear theory, consider R_f.

$$R_f = \left(\frac{R + r}{2}\right) = \left(\frac{0,18 + 0,095}{2}\right) = 0,138 \text{ m}$$

$$T = \mu \times F_A \times R_f \times n$$

$$= 0,35 \times 2,5 \times 10^3 \times 0,138 \times 2$$

$$= 241,5 \text{ N.m}$$

Calculate the **retardation torque** for the engine $\left(T_{R_{engine}}\right)$:

$$T_{R_{engine}} = T_{developed} - T_{constant}$$

$$= 241,5 - 45$$

$$= 196,5 \text{ N.m}$$

Keyword

retardation torque deceleration torque or the required torque to slow down the shaft

Then: $T = I \times \alpha$

$$\alpha_{engine} = \frac{T_{R_{engine}}}{I_{engine}} = \frac{196,5}{2,8} = 70,179 \text{ rad/s}^2$$

Calculate the retardation torque for the shaft $\left(T_{R_{shaft}}\right)$:

$$T_{R_{shaft}} = T_{developed} - T_{friction}$$
$$= 241,5 - 12$$
$$= 229,5 \text{ N.m}$$

Then: $T = I \times \alpha$

$$\alpha_{shaft} = \frac{T_{R_{shaft}}}{I_{shaft}} = \frac{229,5}{8,5} = 27 \text{ rad/s}^2$$

$$\therefore N = \frac{\alpha_{shaft} \times N_{engine}}{\alpha_{engine} + \alpha_{shaft}}$$
$$= \frac{27 \times 10}{70,179 + 27}$$
$$= 2,778 \text{ r/s}$$
$$= 166,703 \text{ r/min}$$

2. Calculating the time of slip:

$$t = \frac{\omega_{shaft}}{\alpha_{shaft}} = \frac{2\pi \times 2,778}{27} = 0,646 \text{ s}$$

3. Calculating the time it takes to reach 600 r/min:
Consider the angular acceleration after slip.

$$T_{after\ slip} = T_{constant} - T_{friction}$$
$$= 45 - 12$$
$$= 33 \text{ N.m}$$

Then: $T = I \times \alpha$

$$\alpha_{after\ slip} = \frac{T_{after\ slip}}{I_{after\ slip}}$$
$$= \frac{33}{8,5 + 2,8}$$
$$= 2,920 \text{ rad/s}^2$$

$$t = \frac{\omega_{after\ slip}}{\alpha_{after\ slip}}$$
$$= 2\pi \times \left(\frac{600 - 166,703}{2,920 \times 60}\right)$$
$$= 15,539 \text{ s}$$

Example 1.6

A multi-plate clutch is fitted with four inner plates and five outer plates. The outside radius of the friction material is 170 mm, with an inside radius of 85 mm.

Calculate the force required to press the plates together in order to transmit 35 kW of power at a rotational frequency of 1 670 r/min. Assume uniform pressure theory and $\mu = 0,4$.

Solution

Data given:

$r = 0,085$ m	$R = 0,17$ m	$N = 1\ 670$ r/min
$P = 35$ kW	$\mu = 0,4$	$n = (4 + 5) - 1 = 8$ contact pairs

Calculating the axial force:

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{\text{P} \times 60}{2\pi \text{N}}$$

$$= \frac{35 \times 10^3 \times 60}{2\pi \times 1\ 670}$$

$$= 200{,}135 \text{ N.m}$$

For uniform pressure theory, consider R_m.

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3}\left(\frac{0{,}17^3 - 0{,}085^3}{0{,}17^2 - 0{,}085^2}\right) = 0{,}132 \text{ mm}$$

$$T = \mu \times F_A \times R_m \times n$$

$$F_A = \frac{T}{\mu \times R_m \times n}$$

$$= \frac{200{,}135}{0{,}4 \times 0{,}132 \times 8}$$

$$= 473{,}804 \text{ N}$$

Example 1.7

A multi-plate clutch transmits 120 kW at 20 r/s. The diameters of the friction plates are 320 mm and 220 mm respectively. Assume a coefficient of friction of 0,35 and that the normal pressure between the contact surfaces is 150 kPa.

Calculate the following:

1. The number of contact pairs required.
2. The actual power that can be transmitted for the same pressure.
3. The maximum pressure on the plates.
4. The minimum pressure on the plates.

Solutions

Data given:

d = 220 mm, so r = 0,11 m — D = 320 mm, so R = 0,16 m

P_{normal} = 150 kPa — μ = 0,35 — N = 20 r/s — P = 120 kW

1. Calculating the number of contact pairs required:

For a worn clutch, consider R_f.

$$R_f = \left(\frac{R + r}{2}\right) = \left(\frac{0{,}16 + 0{,}11}{2}\right) = 0{,}135 \text{ m}$$

$$P = 2\pi NT$$

$$T = \frac{P}{2\pi N}$$

$$= \frac{120 \times 10^3}{2\pi \times 20}$$

$$= 954.93 \text{ N.m}$$

$$F_A = p_{mean} \times \pi(R^2 - r^2)$$

$$= 150 \times 10^3 \times \pi(0{,}16^2 - 0{,}11^2)$$

$$= 6\ 361{,}725 \text{ N}$$

$$T = \mu \times F_A \times R_f \times n$$

$$n = \frac{T}{\mu \times F_A \times R_f}$$

$$= \frac{954{,}93}{0{,}35 \times 6\ 361{,}725 \times 0{,}135}$$

$$= 3{,}177 \text{ pairs}$$

So, $n = 4$ contact pairs.

Note: For the number of contact pairs, always approach the higher value. In the case of this problem, there will be three outer plates and two inner plates.

2. Calculating the power transmitted for the same pressure:

$$T = \mu \times F_A \times R_f \times n$$
$$= 0{,}35 \times 6\ 361{,}725 \times 0{,}135 \times 4$$
$$= 1\ 202{,}366 \text{ N.m}$$
$$\therefore P = 2\pi NT = 2\pi 20 \times 1\ 202{,}366 = 151{,}094 \text{ kW}$$

3. Calculating the maximum pressure on the plates:

$$F_A = 2\pi r(R - r) \times p_{max}$$
$$p_{max} = \frac{F_A}{2\pi r(R - r)}$$
$$= \frac{6\ 361{,}725}{2\pi \times 0{,}11(0{,}16 - 0{,}11)}$$
$$= 184{,}091 \text{ kPa}$$

4. Calculating the minimum pressure on the plates:

$$F_A = 2\pi R(R - r) \times p_{min}$$
$$p_{min} = \frac{F_A}{2\pi R(R - r)}$$
$$= \frac{6\ 361{,}725}{2\pi \times 0{,}16(0{,}16 - 0{,}11)}$$
$$= 126{,}562 \text{ kPa}$$

1.3.3 Assessment

After you have completed the activity below, hand in your work so that the lecturer can evaluate it.

ACTIVITY 1.1

1. A single-plate clutch, with both sides effective, is required to transmit a torque of 185 N.m when new. The coefficient of friction is 0,4, and the pressure is limited to 80 kPa. The inner diameter is 70% of the outer diameter.

   Calculate the folllowing:

   1.1 The inner and outer diameter of the friction surface of the clutch
   1.2 The axial force
   1.3 The power that can be transmitted at the rotational frequency of 1 300 r/min.

2. A single-plate clutch, with both sides effective, is required to transmit 135 kW at 25 r/s. The outside diameter of the contact surface is 400 mm, and the coefficient of friction is 0,35. The uniform pressure is 180 kPa.

   Calculate the inner diameter of the friction surface, assuming uniform pressure theory.

3. A multi-plate clutch transmits 130 kW at 20 r/s. The diameters of the friction plates are 330 mm and 210 mm respectively. Assume a coefficient of friction of 0,35 and that the normal pressure between the contact surfaces is 152 kPa.

   Calculate the following:

   3.1 The number of contact pairs required
   3.2 The actual power sthat can be transmitted for the same pressure
   3.3 The maximum pressure on the plates
   3.4 The minimum pressure on the plates.

4. The flywheel and engine shaft have a combined moment of inertia of 3 kg.m² and develop a constant torque of 48 N.m. An engine is coupled via a single-plate clutch to a shaft. Both sides of the plate are effective. The outer and inner diameters are 380 mm and 200 mm respectively. The coefficient of friction is 0,3, and the force applied to the plates is 2,8 kN. The moment of inertia of the driven shaft is 9 kg.m² and exerts a friction torque of 10 N.m. Assume uniform wear theory.
   When the clutch is engaged at an engine speed of 500 r/min, and the shaft is active, calculate the following:
   4.1 The combined speed between the engine and the shaft at the moment of no slip
   4.2 The total time of slip
   4.3 The time it takes for the shaft to reach 500 r/min.

1.4 Cone clutches

A cone clutch functions similarly to a plate clutch. However, instead of coupling two spinning disks, the cone clutch uses two conical surfaces to transmit torque by friction. They are fitted to precise specialist transmissions, utilised in racing, and in extreme off-road vehicles, even though they are commonly used in power boats. The principal reasoning is because the clutch does not have to be pressed in all the way, so the gears can be changed quicker. Small cone clutches are applied in synchroniser mechanisms in manual transmissions and some limited-slip differentials.

The cone clutch consists of contact surfaces that are in the form of cones. This type of clutch consists of one pair consisting of a male part and a female part of a cone clutch assembly. Refer to Figure 1.5 for a representation of a cone clutch assembly.

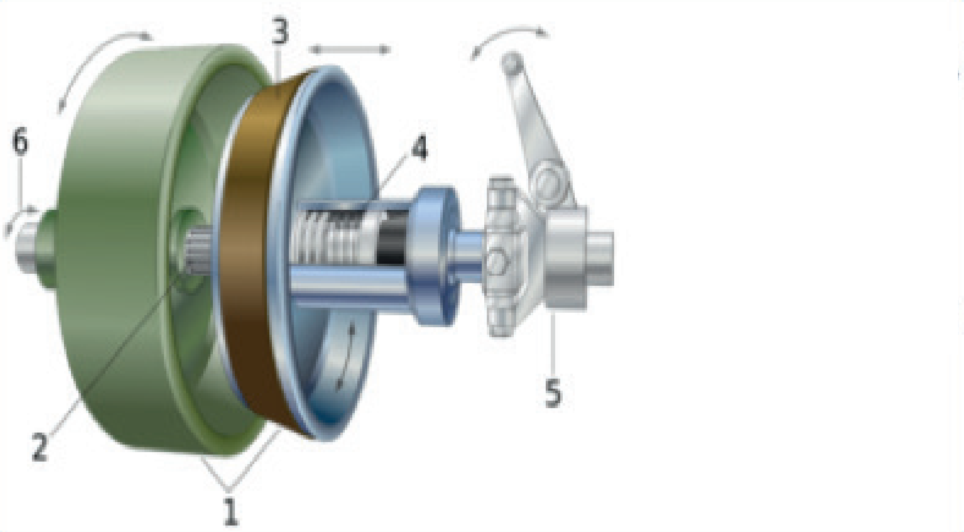


Figure 1.5: Cone clutch

One of the advantages of cone clutches is that the normal force on the contact surfaces is increased due to the involvement of the wedge action. Referring to Figure 1.6 (a) and (b) below, the normal force is $F_n = \frac{F_A}{\sin\alpha}$.

When two cones slide into engagement, there is a friction force opposing the interlocking motion. This force acts laterally to the conical surface and creates a resisting force during engagement, which must be overcome.

For a complete engagement, an additional axial force is needed:

$$F_{engage} = F_A\left(1 + \frac{\mu}{\tan\alpha}\right)$$

During disengagement the friction force resists uncoupling of the two cone members. Therefore, once complete coupling has been maintained, there is a smaller axial force required to maintain the engagement. So: $F_{\text{main}} = F_A\left(1 - \frac{\mu}{\tan\alpha}\right)$

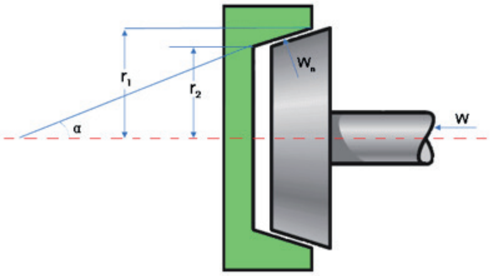


Figure 1.6(a): Cone clutch arrangement

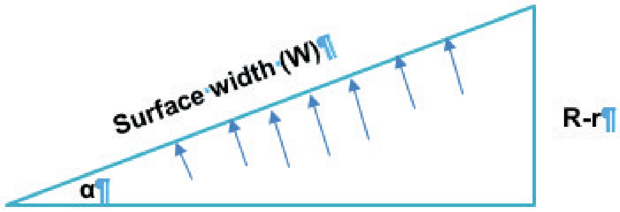


Figure 1.6(b): Cone clutch arrangement

From Figure 1.6(a), $W = F_A$ and $W_n = F_n$.
So, based on the angle α being the semi-cone angle along the surface width, where W_n is applied, $F_n = \frac{F_A}{\sin\alpha}$.

From Figure 1.6(b):
$r_1 = R$ (outside radius of the cone)
$r_2 = r$ (inside radius of the cone)

The surface width (W) is the contact surface where the normal force is applied, so $\sin\alpha = \frac{R - r}{W}$.

1.4.1 Calculations for cone clutches

Before we look at some solved examples, let us look at the formulae and symbols we need to do calculations for cone clutches.

Note that most of the formulae used for cone clutches are the same as those for plate clutches.

Formulae and symbols

(i) Uniform wear theory

$$R_f = \frac{R + r}{2}$$

Where:
R_f = effective friction radius in metres (m)
R = outside radius of a friction disc in metres (m)
r = inside radius of a friction disc in metres (m)

(ii) Uniform pressure theory

$$R_m = \frac{2}{3}\left(\frac{R^3 - r^3}{R^2 - r^2}\right)$$

Where:
R_m = effective friction radius in metres (m)
R = outside radius of a friction disc in metres (m)
r = inside radius of a friction disc in metres (m)

(iii) Maximum and minimum pressure

$$p_{\max} = \frac{F_A}{2\pi r(R - r)}$$

$$p_{\min} = \frac{F_A}{2\pi R(R - r)}$$

(iv) Friction torque

$$T = \frac{\mu \times F_A \times R_e}{\sin\alpha}$$

Where:

μ = coefficient of friction

F_A = axial force applied to engage the clutch in newtons (N)

R_e = friction radius or effective radius, depending on the wear applicable, in metres (m)

α = semi-cone angle in degrees (°)

(v) Engagement force and additional force applicable to cone clutches

$$F_{engage} = F_A\left(1 + \frac{\mu}{\tan\alpha}\right)$$ additional axial force required to complete engagement

$$F_{main} = F_A\left(1 - \frac{\mu}{\tan\alpha}\right)$$ smaller force required to maintain engagement

Where:

F_A = axial force = $F_n \times \sin\alpha$

1.4.2 Solved examples

Having looked at the different theories applicable to plate clutches, cone clutches and the formulae for the latter, calculations related to the terms described are explained in the examples that follow.

Note: the main principle in answering questions is to apply the 'info given' against the 'data required' methodology.

Example 1.8

A conical clutch transmitd 35 kW at 1 400 r/min. The average diameter is 250 mm, with an included cone angle of 26°.

Calculate the engagement force if the coefficient of friction is 0,35, assuming uniform wear theory.

Solution

D_{mean} = 250 mm, so R_f = 0,125 m $\quad N$ = 1 400 r/min

P = 35 kW $\quad \mu$ = 0,35 $\quad \alpha = \frac{26°}{2} = 13°$

Calculating the engagement force:

Find the axial force.

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{35 \times 10^3 \times 60}{2\pi \times 1\ 400}$$

$$= 238{,}732 \text{ N.m}$$

For uniform wear theory, consider R_f.

$R_{mean} = R_f$ = 0,125 m

$$T = \frac{\mu \times F_A \times R_f}{\sin\alpha}$$

$$F_A = \frac{T \times \sin\alpha}{\mu \times R_f}$$

$$= \frac{238{,}732 \times \sin 13°}{0{,}35 \times 0{,}125}$$

$$= 1\ 227{,}497 \text{ N}$$

Find the engagement force.

$$F_{engage} = F_A\left(1 + \frac{\mu}{\tan\alpha}\right)$$
$$= 1\ 227{,}497 \times \left(1 + \frac{0{,}35}{\tan(13°)}\right)$$
$$= 3\ 088{,}403\ \text{N}$$

Example 1.9

A conical clutch transmits 220 N.m. The surface width of the clutch is 72 mm, and it has an included cone angle of 29°. The outside diameter of the clutch is 360 mm, and the coefficient of friction is 0,3.

Assuming uniform wear theory, calculate the following:

1. The inner radius of the clutch
2. The axial force required to engage the clutch
3. The smallest force required to maintain engagement
4. The maximum pressure on the plates.

Solutions

Data given:

$W = 72$ mm, so $D = 360$ mm and $R = 0{,}18$ m

$T = 220$ N.m $\quad \mu = 0{,}3 \quad \alpha = \frac{29°}{2} = 14{,}5°$

1. Calculating the inner radius of the clutch:

$$\sin\alpha = \frac{R - r}{W}$$
$$R - r = W \times \sin\alpha$$
$$0{,}18 - r = 0{,}072 \times \sin(14{,}5°)$$
$$r = 0{,}18 - 0{,}018$$
$$= 0{,}162\ \text{m}$$
$$= 162\ \text{mm}$$
$$\therefore d = 2 \times 162\ \text{mm} = 324\ \text{mm}$$

2. Calculating the axial force required to engage the clutch:

$$R_f = \frac{R + r}{2} = \frac{0{,}18 + 0{,}162}{2} = 0{,}171\ \text{m}$$
$$T = \frac{\mu \times F_A \times R_f}{\sin\alpha}$$
$$F_A = \frac{T \times \sin\alpha}{\mu \times R_f} = \frac{220 \times \sin 14{,}5°}{0{,}3 \times 0{,}171} = 1\ 073{,}754\ \text{N}$$
$$\therefore F_{engage} = F_A\left(1 + \frac{\mu}{\tan\alpha}\right)$$
$$= 1\ 073{,}754 \times \left(1 + \frac{0{,}3}{\tan(14{,}5°)}\right)$$
$$= 2\ 319{,}324\ \text{N}$$

3. Calculating the smallest force required to maintain engagement:

$$F_{main} = F_A\left(1 - \frac{\mu}{\tan\alpha}\right)$$
$$= 1\ 073{,}754 \times \left(1 - \frac{0{,}3}{\tan(14{,}5°)}\right)$$
$$= -171{,}816\ \text{N}$$

Note: If the force required to hold engagement is negative, it simply means that the negative force is the force required to disengage the clutch, which means that the clutch is **self-locking**.

Keyword

self-locking a clutch that automatically aligns itself in place

4. Calculating the maximum pressure on the plates:

$$p_{max} = \frac{F_A}{2\pi r(R - r)}$$
$$= \frac{1\ 073{,}754}{2\pi \times 0{,}162(0{,}18 - 0{,}162)}$$
$$= 58{,}605 \text{ kPa}$$

Example 1.10

The mean diameter of a conical clutch is 280 mm, and the cone has an included angle of 34°. The surface has a width of 68 mm, with a coefficient of friction of 0,325. The pressure is limited to a maximum pressure of 80 kPa, and the clutch rotates at 1 200 r/min.

Calculate the following:

1. The inner and outer diameters of the cone
2. The axial force required to engage the clutch
3. The power that the clutch can transmit.

Solutions

Data given:

$W = 68$ mm, so $D_{mean} = 280$ mm $\quad R_f = 0{,}14$ m $\quad P = 80$ kPa

$\mu = 0{,}325 \quad \alpha = \frac{34°}{2} = 17°$

1. Calculating the inner and outer radius of the clutch: ①②③④

$$\sin\alpha = \frac{R - r}{W}$$
$$R - r = 0{,}068 \times \sin(17°) \quad \ldots①$$
$$R_f = \frac{R + r}{2} = 0{,}14 \text{ m}$$
$$0{,}14 \times 2 = R + r$$
$$R + r = 0{,}28 \quad \ldots②$$

From ②: $R = 0{,}28 - r$

From ①: $R - r = 0{,}0199$

Substitute ② into ①:

$$0{,}28 - r - r = 0{,}0199$$
$$2r = 0{,}28 - 0{,}0199$$
$$r = \frac{0{,}28 - 0{,}0199}{2}$$
$$= 0{,}130 \text{ m}$$

From ②: $R = 0{,}28 - r$

$$R = 0{,}28 - 0{,}13$$
$$= 0{,}15 \text{ m}$$

$\therefore D = 300$ mm and $d = 260$ mm

2. Calculating the axial force to engage the clutch:

$$F_A = 2\pi r(R - r) \times p_{max}$$
$$= 2\pi \times 0{,}13(0{,}15 - 0{,}13) \times 80 \times 10^3$$
$$= 1\ 306{,}903 \text{ N}$$
$$\therefore F_{engage} = F_A\left(1 + \frac{\mu}{\tan\alpha}\right)$$
$$= 1\ 306{,}903 \times \left(1 + \frac{0{,}325}{\tan(17°)}\right)$$
$$= 2\ 696{,}176 \text{ N}$$

3. Calculating the power that the clutch can transmit:

$$T = \frac{\mu \times F_A \times R_f}{\sin\alpha}$$

$$= \frac{0,325 \times 1\ 306,903 \times 0,14}{\sin(17°)}$$

$$= 203.385 \text{ N.m}$$

$$\therefore P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 1\ 200 \times 203,385}{60}$$

$$= 25,558 \text{ kW}$$

Example 1.11

A flywheel with a mass of 25 kg and a radius of gyration of 190 mm is driven by an electric motor through a conical clutch. The operating speed of the motor is 990 r/min and the semi-cone angle of the clutch is 18°. The mean diameter of the clutch lining is 200 mm. The coefficient of friction is 0,3. The axial force applied to the clutch is 300 N.

Assume uniform wear theory, and calculate the following:

1. The torque required to engage the clutch
2. The time it takes for the clutch to reach full speed from rest.

Solutions

Data given:

$m = 25$ kg	$k = 190$ mm, so $D_{mean} = 200$ mm	
$R_f = 0,1$ m	$F_A = 300$ N	$\mu = 0,3$
$\alpha = 18°$	$N = 990$ r/min	

1. Calculating the torque that the clutch can transmit:

$$T = \frac{\mu \times F_A \times R_f}{\sin\alpha}$$

$$= \frac{0,3 \times 300 \times 0,1}{\sin(18°)}$$

$$= 29,125 \text{ N.m}$$

2. Calculating the time it takes for the clutch to reach full speed:

$$I = m \times k^2$$

$$= 25 \times (0.19)^2$$

$$= 0.903 \text{ kg.m}^2$$

Then: $T = I \times \alpha$

$$\alpha = \frac{T}{I} = \frac{29,125}{0,903} = 32,254 \text{ rad/s}^2$$

$$t = \frac{\omega}{\alpha}$$

But: $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 990}{60} = 103,673$ rad/s

$$\therefore t = \frac{103,673}{32,254} = 3,214 \text{ s}$$

1.4.3 Assessment

After you have completed the activity below, hand in your work so that the lecturer can evaluate it.

ACTIVITY 1.2

1. A flywheel with a mass of 28 kg and a radius of gyration of 210 mm is driven by an electric motor through a conical clutch. The operating speed of the motor is 1 100 r/min and the semi-cone angle of the clutch is 19°. The mean diameter of the clutch lining is 220 mm. The coefficient of friction is 0,3. The axial force applied to the clutch is 350 N.

   Assume uniform wear theory, and calculate the following:

   1.1 The torque required to engage the clutch

   1.2 The time it takes for the clutch to reach full speed form rest.

2. The mean diameter of a conical clutch is 300 mm, and the cone has an included angle of 35°. The surface has a width of 70 mm, with a coefficient of friction of 0,35. The pressure is limited to a maximum pressure of 95 kPa, and the clutch rotates at 1 250 r/min.

   Calculate the following:

   2.1 The inner and outer diameters of the cone

   2.2 The axial force required to engage the clutch

   2.3 The power that the clutch can transmit.

3. A conical clutch transmits 250 N.m. The width of the clutch surface is 80 mm, and the included cone angle is 26°. The outside diameter of the clutch is 400 mm and the coefficient of friction is 0,3.

   Assume uniform wear theory, and calculate the following:

   3.1 The inner radius of the clutch

   3.2 The axial force required to engage the clutch

   3.3 The smallest force required to maintain engagement

   3.4 The maximum pressure on the plates.

1.5. Centrifugal clutches

A centrifugal clutch is mainly used in lawn mowers, go-karts, minibikes, as well as chainsaws. Its working principle is to keep the engine running during stalling and disengage loads during starting and idling.

As shown in Figure 1.7, a centrifugal clutch has a driving member with a number of shoes or sliding blocks. These shoes are kept in position by means of a retracting spring that keeps the shoes clear of the rim at low speeds. Doing so allows the motor to gain speed before taking the load. As the speed from the driving shaft increases, the centrifugal force becomes more than the resisting spring force. When this happens, the shoes move outwards and press against the inside of the rim, allowing the transmission of torque.

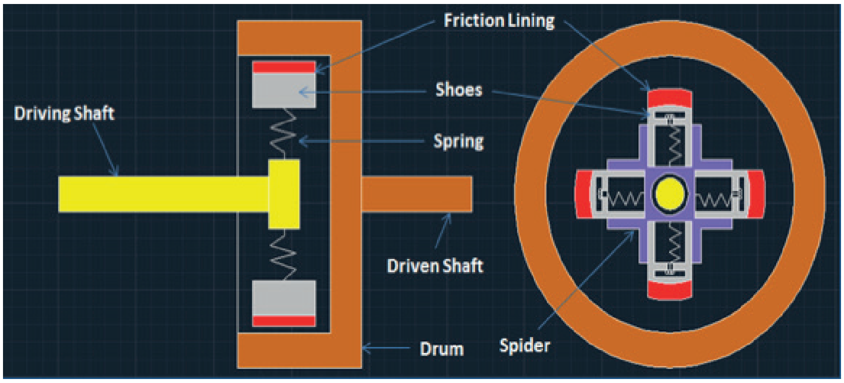


Figure 1.7: Centrifugal clutch

Refer to Figure 1.8 below. When the clutch is stationery, the shoes are held back by an initial spring force. This force also has to be taken into account in calculations. These shoes are usually held in a small clearance between the shoes and the rim during the rest or stationery position. During rotation of the driving shaft, the centrifugal forces increase such that they overcome the resisting spring force to reach engagement. Once engagement is reached, the driving shaft will speed up and cause a normal reaction between the two surfaces.

At the point of engagement, the centrifugal force is equal to the spring force: $F_c = F_s$.

After engagement, the centrifugal force increases, but the spring force remains as it was during engagement, so $F_n = F_c - F_s$.

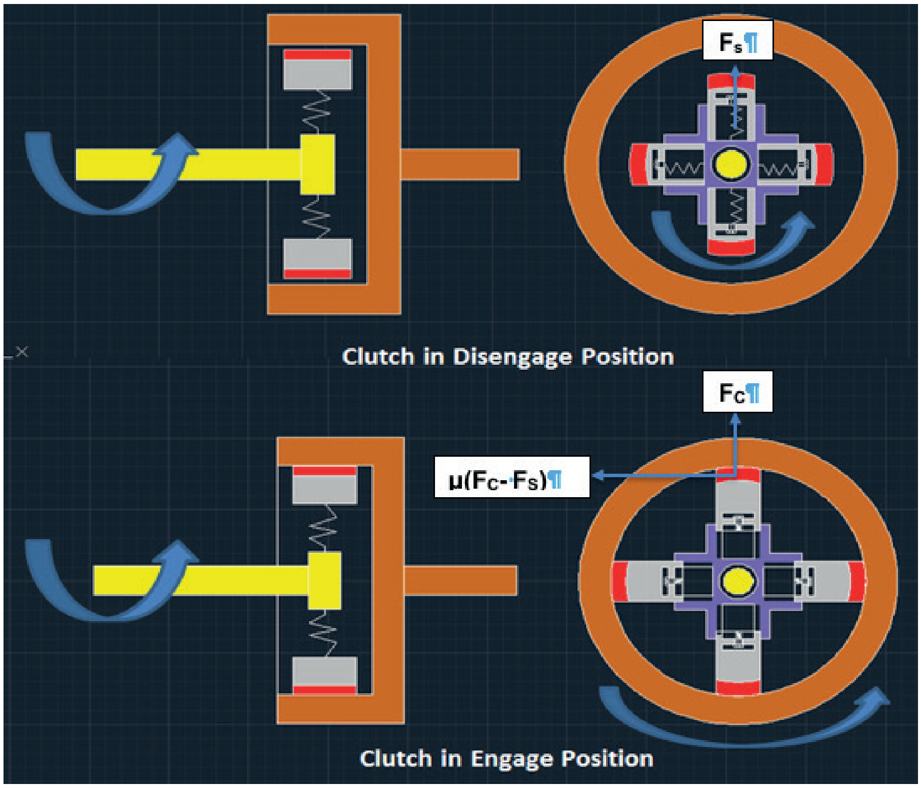


Figure 1.8: Operation principle of a centrifugal clutch

1.5.1 Calculations for centrifugal clutches

Before we look at some solved examples, let us look at the formulae and symbols we need to do calculations for centrifugal clutches.

Note that the centrifugal clutch does not apply the different types of theories applicable to cone and plate clutches.

Formulae and symbols

(i) Centrifugal force

$F_c = m \times \omega^2 \times r$ or $F_c = \frac{mv^2}{r}$

Where:

F_c = centrifugal force of the clutch in newtons (N)

m = mass of a clutch shoe in kilograms (kg)

r = radius of the center of gravity in metres (m)

ω = angular velocity im radians per second (rad/s)

v = linear velocity in metres per second (m/s)

(ii) Spring force

F_s = spring tension × wear × number of shoes

$F_s = m \times (\% \text{ operating speed} \times \omega)^2 \times r$

Where:

F_s = spring force in newtons (N)

% operating speed × ω = engagement speed of the shoes with the drum in radians per second (rad/s)

(iii) Torque

$T = \mu \times (F_c - F_s) \times R \times n$

Where:

μ = coefficient of friction

F_c = centrifugal force in newtons (N)

F_s = spring force in newtons (N)

R = internal radius of the drum in metres (m)

n = number of clutch shoes

(iv) Power

$P = \frac{2\pi NT}{60}$

Where:

P = power in watts (W) or kilowatts (kW)

T = friction torque in newton metres (N.m)

1.5.2 Solved examples

Having looked at the formulae applicable to centrifugal clutches, calculations related to the terms described are explained in the examples that follow.

Note: The main principle in answering questions is to apply the 'info given' against the 'data required' methodology.

Example 1.12

A centrifugal clutch is designed to transmit power of 35 kW at a maximum operating speed of 900 r/min. The clutch starts engagement at 70% of the operating speed. It operates using four shoes, making a friction coefficient of 0,35. The inside diameter of the drum is 330 mm, and the radius of the centre of gravity is 135 mm from the centre of the driving shaft.

Calculate the mass of each shoe.

Solution

Data given:

P = 35 kW	% operation speed = 70 %	$R = \frac{330}{2} = 165$ mm
r = 135 mm	$\mu = 0{,}35$	N = 900 r/min

Calculating the mass of each shoe:

$$F_c = m \times \omega^2 \times r$$

$$= m \times \left(\frac{2 \times \pi \times 900}{60}\right)^2 \times 0{,}135 = 1\ 199{,}157\ m$$

$$F_s = m \times \left(0{,}7 \times \frac{2 \times \pi \times 900}{60}\right)^2 \times 0{,}135 = 587{,}587\ m$$

$$P = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{35 \times 10^3 \times 60}{2\pi \times 900} = 371{,}361 \text{ N.m}$$

$$T = \mu \times (F_c - F_s) \times R \times n$$

$$\frac{T}{\mu \times R \times n} = (F_c - F_s)$$

$$\frac{371{,}361}{0{,}35 \times 0{,}165 \times 4} = m \times (1\ 199{,}157 - 587{,}587)$$

$$m = \frac{1\ 607{,}623}{(1\ 199{,}157 - 587{,}587)} = 2{,}629 \text{ kg}$$

Example 1.13

A centrifugal clutch is designed to transmit power at a maximum operating speed of 930 r/min. When the clutch is stationery, the centre of gravity of all four shoes is held 3 mm clear of the drum by means of retracting springs exerting a spring force of 450 N at this position. The centre of gravity of the shoes if 135 mm from the axis of rotation. The stiffness of each spring is 38 N/mm. Each shoe has a mass of 2,9 kg. The internal diameter of the drum is 320 mm.

Assume the coefficient of friction between the shoes and the drum is 0,35, and calculate the following:

1. The spring force at engagement
2. The engaging speed of the clutch
3. The power that the clutch can transmit.

Solutions

Data given:

$F_s = 450$ N	$r = 135$ mm	$R = \frac{320}{2} = 160$ mm	
$m = 2{,}9$ kg	$\mu = 0{,}35$	$N = 930$ r/min	stiffness = 38 N/mm

1. Calculating the spring force at the point of engagement:
   Note: The springs have a force of 450 N before covering the clearance of 3 mm, so $F_s = F_{s_{initial}}$ + (stiffness × clearance).
   $\therefore F_s = 450 + (38 \times 3) = 564$ N

2. Calculating the engaging speed of the clutch:
   At the point of engagement $F_c = F_s = 564$ N and $r = 135 + 3 = 138$ mm.
   $$\therefore F_c = m \times \omega^2 \times r$$
   $$= m \times \left(\frac{2 \times \pi \times N_{engage}}{60}\right)^2 \times r$$
   $$\therefore \frac{F_c \times 60^2}{m \times 2^2 \times \pi^2 \times r} = N_{engage}^{\ 2}$$
   $$N_{engage} = \sqrt{\frac{564 \times 60^2}{2{,}9 \times 2^2 \times \pi^2 \times 0{,}138}}$$
   $$= 358{,}486 \text{ r/min}$$

3. Calculating the power transmitted:
   $$F_c = m \times \omega^2 \times r$$
   $$= 2{,}9 \times \left(\frac{2 \times \pi \times 930}{60}\right)^2 \times 0{,}138 = 3\ 795{,}773 \text{ N}$$
   $$T = \mu \times (F_c - F_s) \times R \times n$$
   $$= 0{,}35 \times (3\ 795{,}773 - 564) \times 0{,}16 \times 4 = 723{,}917 \text{ N.m}$$
   $$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 930 \times 723{,}917}{60} = 70{,}502 \text{ kW}$$

Example 1.14

A centrifugal clutch is designed to transmit power of 28 kW at a maximum operating speed of 900 r/min. The clutch starts to engage at the speed of 580 r/min. The centre of gravity of four shoes is held 60 mm from the contact surface when it is engaged. The inside diameter of the drum is 360 mm. The stiffness of each spring is 72 kN/m.

Assume a coefficient of friction of 0,32, and calculate the following:

1. The mass of each shoe
2. The spring force applied on the shoes
3. The power that the clutch can transmit if each shoe is worn by 3 mm.

Solutions

Data given:

$P = 35$ kW $\quad N_{engage} = 580$ r/min $\quad R = \frac{360}{2} = 180$ mm

$r = 60$ mm from the rim end $\quad \mu = 0,32 \quad N = 900$ r/min

$\therefore r = 180 - 60 = 120$ mm

1. Calculating the mass of each shoe:

$$F_c = m \times \omega^2 \times r$$

$$= m \times \left(\frac{2 \times \pi \times 900}{60}\right)^2 \times 0,120 = 1\ 065,917\ m$$

$$F_s = m \times \left(\frac{2 \times \pi \times 580}{60}\right)^2 \times 0,120 = 442,685\ m$$

$$P = \frac{2\pi NT}{60}$$

$$\therefore T = \frac{P \times 60}{2\pi N} = \frac{28 \times 10^3 \times 60}{2\pi \times 900} = 297,089 \text{ N.m}$$

$$T = \mu \times (F_c - F_s) \times R \times n$$

$$\therefore \frac{\text{T}}{\mu \times \text{R} \times \text{n}} = (\text{F}_c - \text{F}_s)$$

$$\frac{297,089}{0,32 \times 0,18 \times 4} = m \times (1\ 065,917 - 442,685)$$

$$m = \frac{1\ 289,449}{(1\ 065,917 - 442,685)}$$

$$= 2,069 \text{ kg}$$

3. Calculating the power if there is wear of 3 mm on each shoe:

Note: The spring force will increase due to wear and will change the normal force and torque.

However, the centrifugal force will remain unchanged, because the speed remains the same.

$$\therefore \Delta F_s = \text{stiffness} \times \text{wear}$$

$$= 72 \times 10^3 \times 3 \times 10^{-3} = 216 \text{ N}$$

$$\therefore F_{s_{new}} = F_{s_{initial}} + \Delta F_s$$

$$= 915,915 + 216 = 1\ 131,915 \text{ N}$$

$$F_c = m \times \omega^2 \times r$$

$$= 2,069 \times \left(\frac{2 \times \pi \times 900}{60}\right)^2 \times 0,120 = 2\ 205,382 \text{ N}$$

$$T = \mu \times (F_c - F_s) \times R \times n$$

$$= 0,32 \times (2\ 205,382 - 1\ 131,915) \times 0,18 \times 4 = 247,327 \text{ N.m}$$

$$\therefore P = \frac{2\pi NT}{60} = \frac{2\pi \times 900 \times 247,327}{60} = 23,310 \text{ kW}$$

1.5.3 Assessment

After you have completed the activity below, hand in your work so that the lecturer can evaluate it.

ACTIVITY 1.3

1. A centrifugal clutch is designed to transmit power of 30 kW at a maximum operating speed of 920 r/min. The clutch begins to engage at the speed of 600 r/min. The centre of gravity of four shoes is held 60 mm from the contact surface when it is engaged. The inside diameter of the drum is 360 mm. The stiffness of each spring is 72 kN/m.

   Assume a coefficient of friction of 0,32, and calculate the following:

   1.1 The mass of each shoe
   1.2 The spring force applied on the shoes
   1.3 The power that the clutch can transmit if each shoe is worn by 3 mm.
2. A centrifugal clutch is designed to transmit power of 32 kW at a maximum operating speed of 880 r/min. The clutch starts to engage at 65% of the operating speed. The clutch operates using four shoes, making a friction coefficient of 0,38. The inside diameter of the drum is 333 mm and the radius of the centre of gravity is 133 mm from the centre of the driving shaft.

   Calculate the mass of each shoe.
3. A centrifugal clutch is designed to transmit power at a maximum operating speed of 950 r/min. When the clutch is stationery, the centre of gravity of all four shoes is held 3,5 mm clear of the drum by means of retracting springs exerting a spring force of 465 N at this position. The centre of gravity of the shoes is 132 mm from the axis of rotation. The stiffness of each spring is 40 N/mm. Each shoe has a mass of 3 kg. The internal diameter of the drum is 360 mm.

   Assume the coefficient of friction between the shoes and the drum is 0,3, and calculate the following:

   3.1 The spring force at engagement
   3.2 The engaging speed of the clutch
   3.3 The power that the clutch can transmit.

Guidelines for students

Remember the following main guidelines when doing assessment exercises:

- Identify the problem and the formulae you need to use.
- Memorise the formulae for future use in similar exercises.
- Convert all similar values to standard units, for example, change measurements in millimetres to metres.
- Do calculations in short phases.

Above all, work systematically, using these guidelines to help you achieve your objectives.

Module summary

This module covered friction clutches, including single-plate and multi-plate clutches, conical clutches and centrifugal clutches. In this module you have learnt the following:

- The significant function of a friction clutch is to connect a rotary gear to a stationery one.
- A clutch allows two coaxial shafts to engage or disengage while at rest or in relative motion.
- Friction clutches may be of plate, cone or centrifugal type.
- In plate clutches, the plates are subjected to an axial force when the clutch is engaged, with a torque being transmitted by friction to the sides of each plate.
- A cone clutch functions similarly to a plate clutch, but instead of coupling two spinning discs, uses two conical surfaces to transmit torque by friction.
- The working principle of a centrifugal clutch is to keep the engine running during stalling and disengage loads during starting and idling by using a driving member with a number of shoes or sliding blocks, kept in position by a retracting spring that keeps the shoes clear of the rim at low speeds.
- In plate and cone clutches, the normal pressure distribution across the contact surfaces can either be according to uniform wear theory or uniform pressure theory.
- According to the uniform wear theory, the normal intensity of pressure is equal at all points on the contact surfaces of a clutch, assuming that the fit between the contact surfaces is perfect.
- According to the uniform pressure theory, the total normal force between the surfaces is the product of the pressure and the contact area, assuming that the pressure is distributed uniformly over the frictional surface.

Exam questions

1. An electric motor drives a machine by means of a single-plate friction clutch that transmits 142 N.m during engagement. The rotor of the motor has a mass of 22,8 kg and a radius of gyration of 75 mm. The machine has an equivalent mass of 63,5 kg and a radius of gyration of 140 mm. The motor speed is 1 500 r/min, and the machine is at rest at the clutch engagement. The clutch plate has a mean diameter of 95 mm and a coefficient of friction of 0,3.

Assume uniform wear, and calculate the following:

1.1 The power the clutch can transmit at 1 500 r/min (2)

1.2 The axial force required to transmit the 142 N.m (3)

1.3 The combined speed after engagement (5)

1.4 The time for which slippage occur (4)

1.5 The loss of energy during the engagement period. (3)

2. An electric motor that rotates at 1 200 r/min is connected by means of a conical clutch to a flywheel that is at rest. The mass of the flywheel is 30 kg and the radius of gyration is 190 mm. The clutch cone has a semi-angle of 16°, and the coefficient of friction is 0,3, with a mean diameter of 380 mm. The axial force applied to the clutch is 280 N.

Assume uniform wear, and calculate the following:

2.1 The torque required to engage the clutch (2)

2.2 The time it takes for the clutch to reach full speed of 1 200 r/min from rest. (7)

3. A centrifugal clutch transmits 25 kW at 600 r/min, with engagement beginning at 420 r/min. The coefficient of friction between the shoes and the drum is 0,3. The centre of gravity for each of the four shoes is 130 mm from the shaft axis, and the inside diameter of the drum is 300 mm.

Calculate the mass of each shoe. (9)

4. A machine is driven by means of a centrifugal clutch that has four shoes, each with a mass of 2 kg. The stationery tension in each retraction spring is 400 N, while the clearance between the shoes and the drum is 3 mm. When the clutch is stationery, the centre of gravity of each shoe is 35 mm from the contact surface of the drum. The internal diameter of the drum is 270 mm, and the coefficient of friction between the shoes and the drum is 0,28. The spring stiffness is 150 N/mm.

Calculate the following:

4.1 The maximum power that the clutch can transmit at 1 200 r/min (8)

4.2 The maximum power that the clutch can transmit at 1 200 r/min if the diameter of the drum increases to 275 mm because of wear. The spring stiffness does not change. (8)

Total: 51 marks