N4 Engineering Science

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Module 1 Kinematics

What is covered?

Kinematics is a branch of physics that falls under classical mechanics and deals with the study of the motion of a system of bodies or objects without considering forces and their effects on the movement or motion of the bodies. The module covers how to solve problems analytically to practical situations where two objects or bodies move horizontally at constant velocity in different directions. The module also shows how to determine the resultant velocity, shortest distance, time intersection, overtaking and actual velocity. The module gives calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same horizontal plane. Finally, the module covers how to calculate the velocity of projection, angle of projection, height and velocity at any part of the projectile path.

Subject Outcomes

After studying this module, you should be able to:

Unit 1

- Solve problems dealing with constant linear motion analytically (Pythagoras or the sine and cosine rules)
- Determine the relative velocity, shortest distance, time to intercept and actual velocity.

Unit 2

- Calculate the resulting velocity and direction of a maximum of two vectors
- Calculate the time taken to reach a certain destination.

Unit 3

- Do calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same horizontal plane
- Calculate the maximum height reached by an object as well as the time of flight and range
- Calculate the height and velocity at any part of the projectile path
- Calculate the velocity of projection
- Calculate the angle of projection.

Unit 1: Constant linear motion and relative velocity

LEARNING OUTCOMES

- Solve problems dealing with constant linear motion analytically (Pythagoras or the sine and cosine rules).
- Determine the relative velocity, shortest distance, time to intercept and actual velocity.

Introduction

This unit deals with calculations of constant linear motion analytically using Pythagoras or the sine or cosine rules. The unit also shows how to determine relative velocity along parallel and non-parallel lines using the analytical method.

1. Speed and velocity

It is important to understand the difference between speed and velocity. **Speed** is the time rate at which an object moves along a path, while **velocity** is the rate and direction of an object's movement. For example, 100 km/hr describes the speed at which a car is travelling, while 100 km/hr west describes the velocity at which it is travelling.

2. Relative velocity

Keywords

Speed: the time rate at which an object is moving along a path, e.g. 60 km/hr

Velocity: the rate and direction of an object's movement, e.g. 60 km/hr west

Relative velocity: the velocity of an object relative to something else; the difference between two velocities

Relative velocity is defined as the velocity of an object relative to something else. It is also referred to as the difference between two velocities, for example, the velocity of an object minus the velocity of a frame of reference.

Let us consider a variable V_{DE}. This would mean the velocity of object D with respect to object E. D is the object in focus and E is the frame of reference.

To calculate relative velocity V_{DE}, we use the following formula:

$V_{DE} = V_D - V_E$, where V_{DE} is the velocity of object D with respect to E.

We calculate relative velocity R_V as follows:

$R_V = V_{object} - V_{FR}$, where FR is the reference frame.

For example, a car is travelling at 100 km/hr towards the east. If the car passes a boy standing still on the side of the road, it would appear to the boy that the car is moving at 100 km/hr.

Worked example 1.1 **Calculate the relative velocity of two objects moving in the same direction at different speeds**

Two cars, D and E, are moving in the same direction at different speeds. Car D is travelling at 70 km/hr in an easterly direction with respect to the earth and car E is moving at 80 km/hr in the same direction.

1. Calculate the relative velocity of car E with respect to car D.
2. Calculate the relative velocity of car D with respect to car E.

Solution

1. Figure 1.1 shows how to calculate the relative velocity of car E with respect to car D.

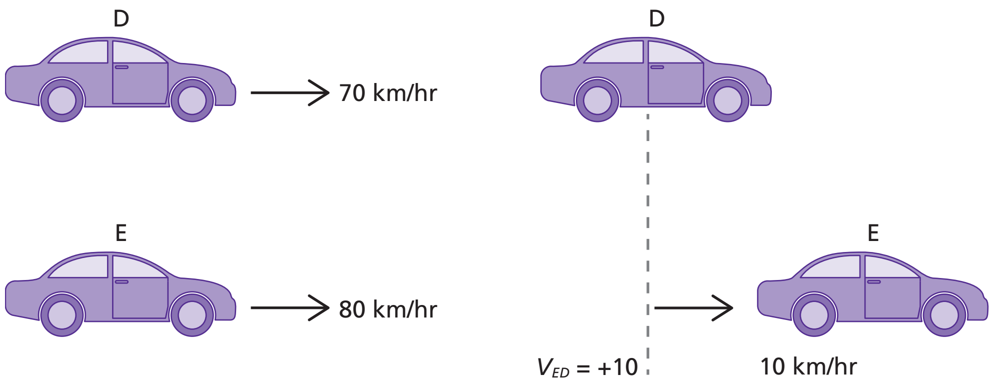

Figure 1.1 How to calculate the relative velocity of car E with respect to car D

Referring to Figure 1.1, relative velocity is calculated as follows:

$V_{ED} = V_E - V_D = 80 - 70 = 10$ km/hr

The relative velocity of car E with respect to car D is 10 km/hr.

$V_{ED} = +10$ km/hr

The diagram shows that car E is moving 10 km faster than car D since the two cars are moving in the same direction. So in one hour, car D will travel 70 km and car E will travel 80 km. Therefore, after an hour Car E will be 10 km ahead of car D. The driver of car D will see that driver in car E is moving +10 km away towards the right (in the diagram). Thus the relative velocity of E with respect to D is positive because car E is moving in the same direction as car D, towards the right.

$V_{ED} = +10$.

2. Figure 1.2 shows how to calculate the relative velocity of car D with respect to car E.

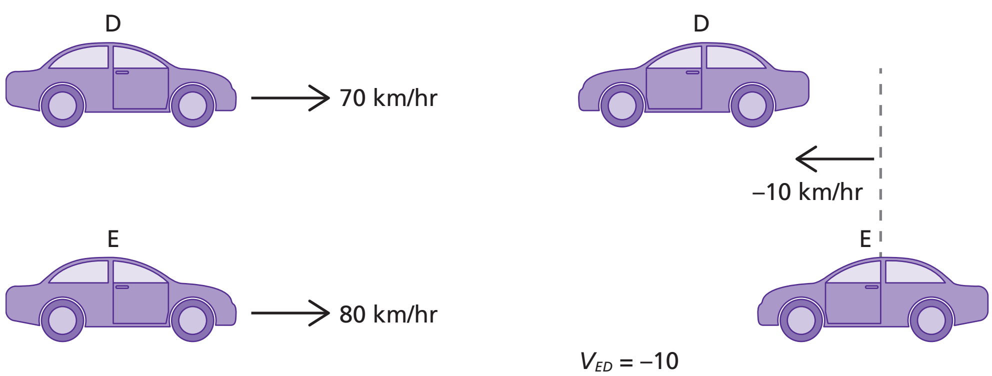

Figure 1.2 How to calculate the relative velocity of car D with respect to car E

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We can calculate the relative velocity of car D with respect to car E as follows:

$$V_{DE} = V_D - V_E$$
$$= 70 - 80$$
$$= -10 \text{ km/hr.}$$

This means that car D is moving 10 km/hr slower than car E.
To the driver of car E, it will appear that car D is moving to the left (in the diagram) with respect to E. Initially, the two cars were at the same location but now it appears that car D is moving towards the left or westwards. In one hour, car D will move 10 km towards the left with respect to car E and the displacement is –10 km. With respect to car E, car D will appear to be moving left and will, therefore, have a negative velocity.

Referring to the solution for both 1 and 2 of the example, we can see that relative velocity can be negative or positive.

Worked example 1.2 **Calculate the relative velocity of two objects moving in the same direction at the same speed**

The two cars D and E in the previous worked example are now both travelling side by side at 120 km/hr in a westerly direction on a double lane.

1. Calculate the relative velocity of car E with respect to car D.
2. Calculate the relative velocity of car D with respect to car E.

Solution

1. Figure 1.3 shows how to calculate the relative velocity car E with respect to car D with both travelling in the same direction at the same speed.

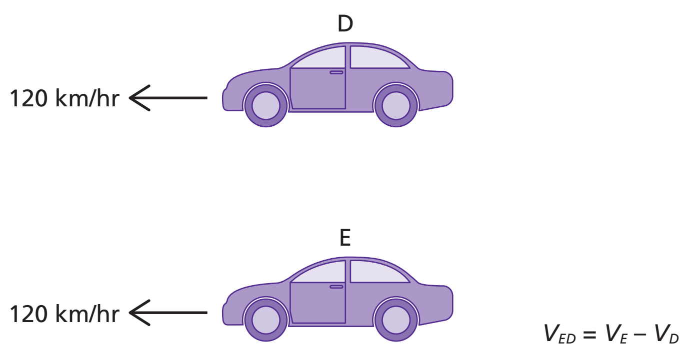

Figure 1.3 How to calculate the relative velocity car E with respect to car D

In Figure 1.3 above,

$$V_{ED} = V_E - V_D$$
$$= 120 - 120$$
$$= 0 \text{ km/hr}$$

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2. Figure 1.4 shows how to calculate the relative velocity of car D with respect to car E with both travelling in the same direction at the same speed.

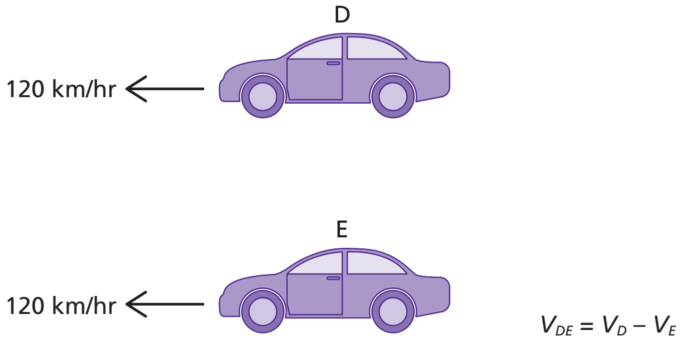

Figure 1.4 How to calculate the relative velocity car D with respect to car E

In Figure 1.4:

$V_{ED} = V_E - V_D$

$= 120 - 120$

$= 0$ km/hr

By referring to the solution for Worked example 1.2, we can see that the relative velocity of two cars D and E is 0 km/hr. This means the two cars are not moving relative to each other, hence they will continue to move next to each other for the entire journey.

Worked example 1.3 Calculate the relative velocity of two objects moving in different directions at different speeds

Two cars are travelling in opposite directions. Car D is travelling at 80 km/hr towards the east and car E is travelling at 70 km/hr in a westerly direction.

1. What is the relative velocity of car D with respect to car E?
2. What is the relative velocity of car E with respect to car D?

Solution

Figure 1.5 shows two cars travelling in opposite directions at different speeds.

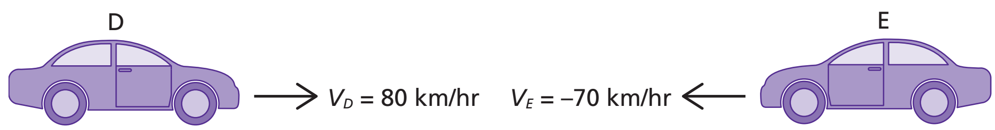

Figure 1.5 Two cars travelling in opposite directions at different speeds

In Figure 1.5, the relative velocity of car D with respect to earth is 80 km/hr, while that of car E is –70 km/hr.

1. We must establish if the velocity is positive or negative. In this case, the velocity of V_{DE} is found to be positive.

$V_{DE} = V_D - V_E$

$= -80 - (-70)$

$= 80 + 70$

$= 150$ km/hr

2. In the second case, the velocity will be negative.

$V_{ED} = V_E - V_D$

$= -70 - (+80)$

$= -70 - 80$

$= -150$ km/hr

Worked example 1.4 **Calculate the relative velocity of two objects moving in different directions and 200 km apart**

The two cars D and E in Worked example 1.3 are now travelling in opposite directions and are 200 km apart. Car D is travelling at 80 km/hr towards the east and car E is travelling at 70 km/hr in a westerly direction. What will be the distance between the two cars after one hour?

Solution

Figure 1.6 shows two cars travelling in opposite directions and 200 km apart.

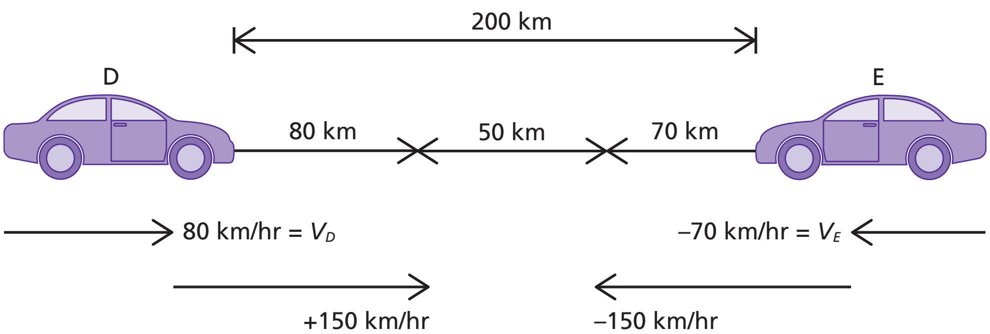

Figure 1.6 **How to calculate the relative velocity of two cars travelling in opposite directions when the distance between the two cars is known**

Referring to Figure 1.6 above, it can be seen that in one hour car D will travel 80 km to the east and car E will travel 70 km towards the west. Every hour the velocity between the two cars will decrease by 150 km/hr.

The distance between the two cars after one hour is calculated as follows:

$S = 200$ km $- (80 + 70)$

$= 50$ km

The distance between the two will decrease from 200 km to 50 km in one hour. The distance will decrease by 150 km, so the velocity will decrease by 150 km/hr.

Worked example 1.5 **Calculate the relative velocity of a stone thrown by the driver of a car towards another car travelling in an opposite direction at a different speed**

Car D is travelling at 50 km/hr towards the east. Car F is travelling at 70 km/hr towards the west approaching car D. The driver in car D throws a stone E that moves at 20 km/hr east with respect to car F.

1. Calculate the relative velocity of the stone E with respect to car F.
2. Calculate the relative velocity of the stone E with respect to car F if the two cars D and F are 400 km apart.

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Solution

1. Figure 1.7 shows he two cars travelling in opposite directions. A stone is thrown by one of the drivers towards the opposite car.

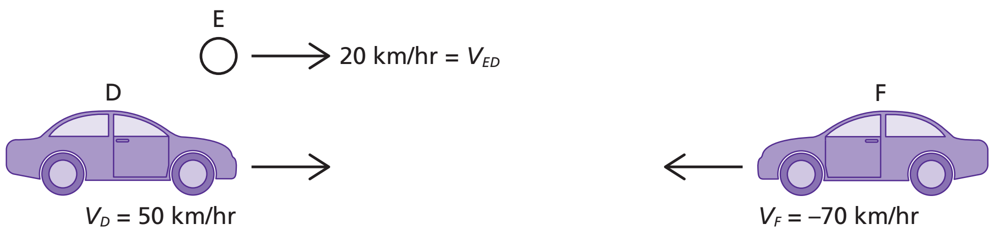

Figure 1.7 **How to calculate the relative velocity of a stone E with respect to car F**

Given: $V_D = 50$ km/hr; $V_F = -70$ km/hr; $V_{ED} = 20$ km/hr

$V_{EF} = V_E - V_F$

$V_{ED} = V_E - V_D$

$20 = V_E - 50$

$20 + 50 = V_E$

$70 = V_E$

Therefore,

$V_{EF} = V_E - V_F$

$= 70 - (-70)$

$= 70 + 70$

$= 140$ km/hr

2. Figure 1.8 Shows two cars travelling in opposite directions. One of the drivers throws a stone towards the opposite car. The two cars are now 400 km apart.

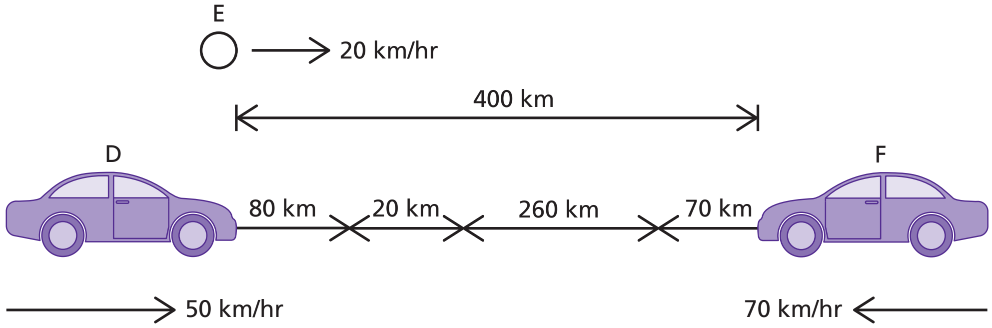

Figure 1.8 **How to calculate the relative velocity of stone E with respect to car F if the two cars D and F are 400 km apart**

In Figure 1.8,

$V_d = 140$ km

Therefore,

$V_{EF} = 140$ km/hr

3. Relative velocity along non-parallel lines

We can use the analytical method, which uses axes of a coordinate system, to solve problems concerning relative velocity along non-parallel lines. We use Pythagoras' theorem and the sine or cosine rules to calculate the relative velocity vector.

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Worked example 1.6	Calculate relative velocity along non-parallel lines (1)

Two ships A and B are 20 km apart with ship B due north of ship A. Ship A is travelling at 10 km/hr in the direction 060° and ship B is travelling at 8 km/hr the direction 135°.

1. Calculate the velocity of ship A relative to B.
2. Calculate the time to the nearest minute taken for ship A to be exactly east of B.
3. Calculate the nearest distance between the two ships.

Solution

1. Figure 1.9 shows the vector diagrams for Worked example 1.6, answer 1

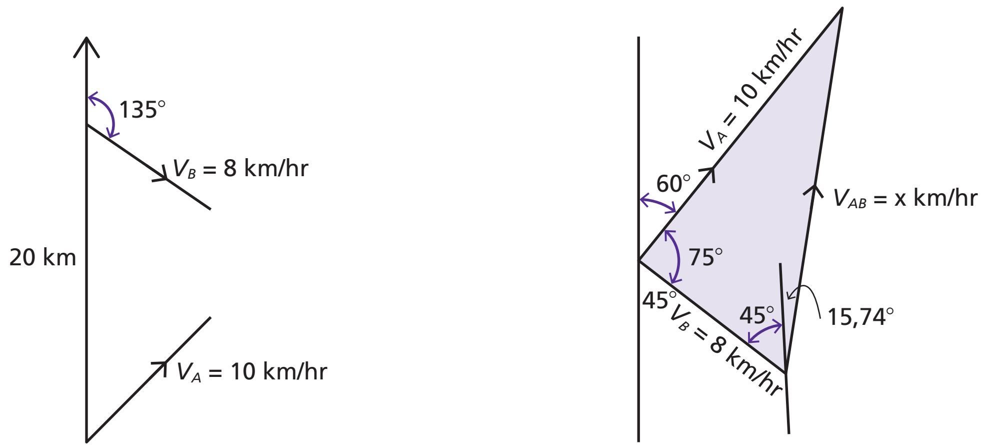

Figure 1.9 Vector diagrams for Worked example 1.6, answer 1

From the diagram:

$x^2 = 8^2 + 10^2 - 2(8)(10)\cos 75°$

$x = 11{,}072$ km/hr

$\frac{10}{\sin\alpha} = \frac{11{,}072}{\sin 75}$

$\alpha = 60{,}74°$

Required bearing:

= 60,74 – 45

= 0,15,74°

2. Figure 1.10 shows the vector diagrams for Worked example 1.6, answer 2.

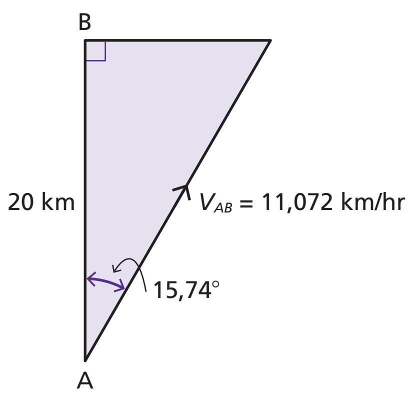

Figure 1.10 Vector diagram for Worked example 1.6, answer 2

From the diagram:

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$\cos 15{,}74 = \frac{20}{y}$

$y = 20{,}779$ km

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$= \frac{20{,}779}{11{,}072}$

$= 1{,}877$ hrs

$= 113$ mins

3. Figure 1.11 shows the vector diagram for Worked example 1.6, answer 3.

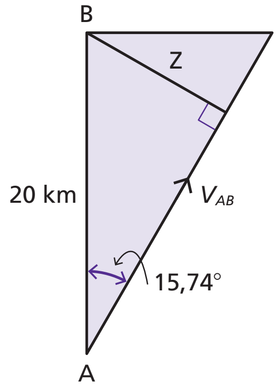

Figure 1.11 Vector diagram for Worked example 1.6, answer 3

From the diagram:

$\sin 15{,}74° = \frac{z}{20}$

$20 \sin 15.74° = z$

$5{,}425 \text{ km} = z$

Worked example 1.7 Calculate relative velocity along non-parallel lines (2)

A light aircraft is 70 nm northeast of Cape Town International Airport and it flies north for seven hours at an airspeed of 100 km/hr. Determine its position (displacement) with reference to Cape Town International Airport in magnitude and direction.

Solution

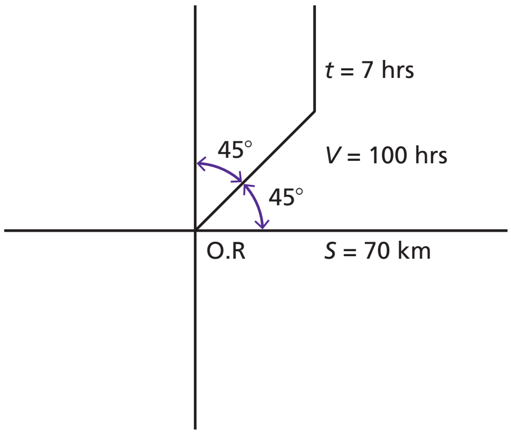

Figure 1.12 Vector diagram 1 for Worked example 1.7

From the diagram:

$S_2 = V \times t$

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$= 7 \times \frac{100}{\text{hour}}$

$= \frac{700 \text{ km}}{\text{N}}$

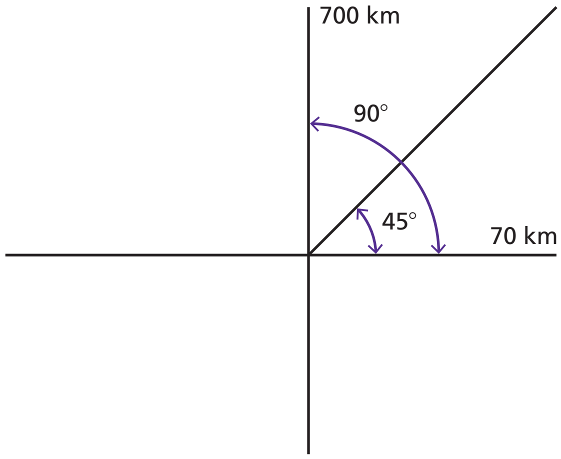

Figure 1.13 Vector diagram 2 for Worked example 1.7

From the diagram:

$V = 700\sin 90° + 70\sin 45°$

$= 749{,}5$ km

$H = 700\cos 90° + 70\cos 45°$

$= 49{,}5$ km

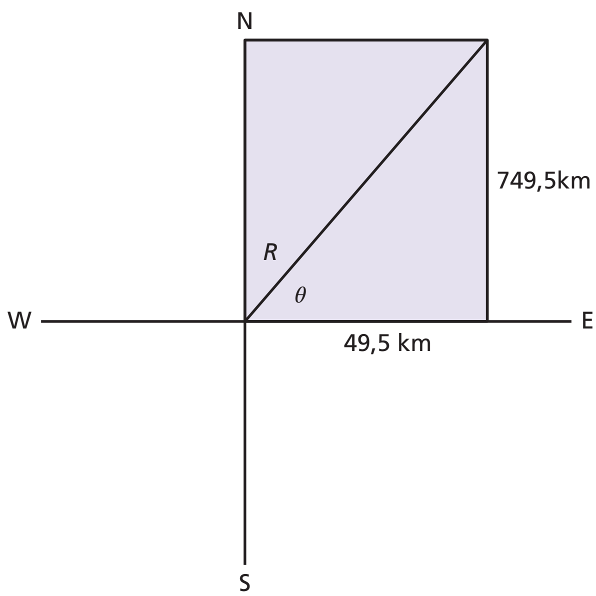

Figure 1.14 Vector diagram 3 for Worked example 1.7

From the diagram:

$R = \sqrt{49{,}5^2 + 749{,}5^2}$

$= \sqrt{564\ 200{,}5}$

$= 751{,}133$ km

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$\Theta = \tan{-1}\ \frac{749,5}{49,5}$
$= 86,22°$

R = 751,133 E86,22°N
or,
N3,78°E

Worked example 1.8 Calculate relative velocity along non-parallel lines (3)

An aircraft A departs Cape Town International Airport in a northerly direction at an airspeed of 250 km/hr. Another Aircraft B departs simultaneously from Stellenbosch aerodrome at an airspeed of 500 km/hr in a direction of 60°. Calculate the velocity of aircraft A relative to the velocity of aircraft B.

Solution

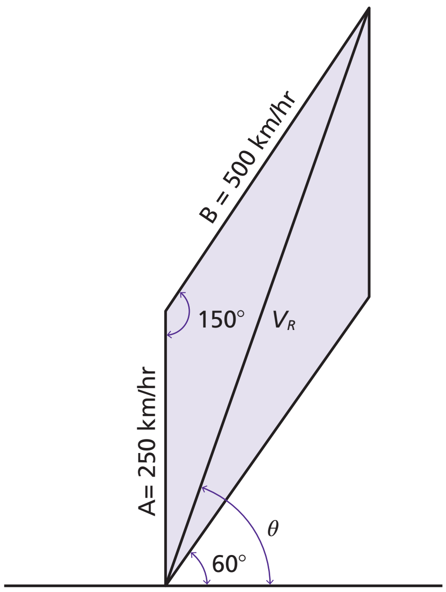

Figure 1.15 Vector diagram 1 for Worked example 1.8

From the diagram:

$V_{r^2} = 250^2 + 500^2 - 2(250)(500)\cos 50°$
$= 312\ 500 - 250\ 000(-0,866025403)$
$= 312\ 500 + 216\ 506,3508$
$= \sqrt{529\ 006,3508}$
$= 727,33$ km/hr

Direction:

$= \frac{\sin\theta}{250}$
$= \frac{\sin 150}{727,33}$
$\theta = \sin^{-1}\left[\frac{\sin 150 \times 250}{727,33}\right]$
$\theta = 9,89°$

Therefore, the velocity of A relative to B is 727,33km/hr W70°S.

Alternative method

Also referring to Figure 1.15

$\sum V_c = A + B\sin 60°$

$= 250 + 500\sin 60°$

$= 683{,}013$ km/hr

$H_c = B\cos 60°$

$\sum = 500\cos 60°$

$= 250$ km/hr

$R^2 = \sum Hc^2 + \sum Vc^2$

$= 250^2 + 683{,}013^2$

$= \sqrt{529\ 006{,}7582}$

$= 727{,}33$ km/hr

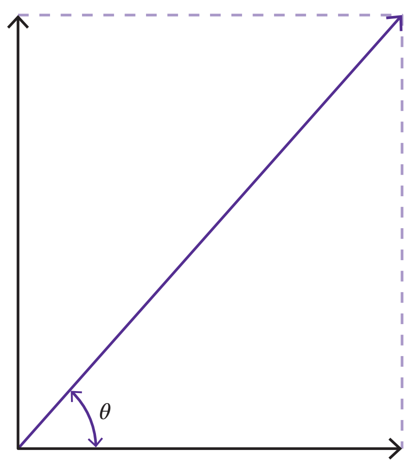

Figure 1.16 Vector diagram 2 for Worked example 1.8

From the diagram:

$\tan\theta = \frac{V_c}{H_c}$

$= \frac{683{,}013}{250}$

$= 2{,}732052$

$\theta = \tan^{-1} 2{,}732052$

$= 70°$

Therefore, the velocity of A relative to B is 727,33 km/hr W70°S.

Activity 1.1 Relative velocity

1. A red car and a blue car are travelling in the same direction due east at speeds of 80 km/hr and 100 km/hr respectively.
   a) Find the velocity of the blue car relative to the red car.
   b) Find the velocity of the red car relative to the blue car.
2. A bus and a car are 20 km apart initially. The bus is travelling at 40 km/hr due east while the car is travelling at 60 km/hr due west.
   a) Find the velocity of the bus relative to the car.
   b) Find the velocity of the car relative to the bus.
   c) Find the time taken for the bus and the car to pass each.
3. A train travels at 40 km/hr due east.

a) What is the relative velocity relative to earth of a man travelling on the train if he walks at 1 5km/hr due east?
b) What is the relative velocity to earth of a man travelling in the train if he walks at 15 km/hr due west?
c) What is the relative velocity to earth of a man travelling on the train if he walks sideways at 15 km/hr?

4 Two cars start moving simultaneously. Vehicle A is travelling at 280 km/hr, W33°N and vehicle B is travelling at 230 km/hr directly east. Calculate the velocity of car B relative to car A.

5. A light aircraft (A) departs from King Shaka International Airport in a northerly direction at an airspeed of 450 km/h. Another light aircraft (B) departs simultaneously from Nelspruit International Airport at an airspeed of 850 km/hr in the direction W60°S. Calculate the velocity of aircraft A relative to the velocity of aircraft B.

6. A light aircraft is 80 km northeast of Lanseria International Airport and flies north for 8 hours at an airspeed of 120 km/hr. Determine its displacement or position with reference to Lanseria International Airport in magnitude and direction.

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Unit 2: Resulting velocity

LEARNING OUTCOMES

- Calculate the resulting velocity and direction of a maximum of two vectors.
- Calculate the time taken to reach a certain destination.

Introduction

This unit shows how to calculate the resulting velocity and direction of a maximum of two vectors and the time taken to reach a certain destination.

1. Resulting velocity

Keywords

Resulting velocity: the vector sum of two or more velocities

Resulting velocity is the vector sum of two or more velocities. Resultant velocity helps us to establish how an external force acting on a moving object can decrease the speed of the moving object. For example, if a cyclist is travelling due east into a very strong wind moving west, the speed of the cyclist will be reduced since the strong headwind will act against the movement of the cyclist's bicycle.

2. How to calculate resultant velocity

Resultant velocity is calculated by simply adding two or more vectors. Depending on the situation you can use the Pythagorean theorem. Depending on the scenario you can calculate the resultant velocity using the formula:

$$V_f - (\text{acceleration})(\text{time})tV_0$$

In N4 Engineering Science, we only calculate the resulting velocity and direction of a maximum of two vectors.

Worked example 1.9 — Calculate resultant velocity (1)

A boat travels at 30 m/s towards the east in calm water. A current flows towards the boat at 10 m/s. Calculate the resultant velocity of the boat.

Solution

Figure 1.17 shows how to calculate the resultant velocity of the boat in Worked example 1.9.

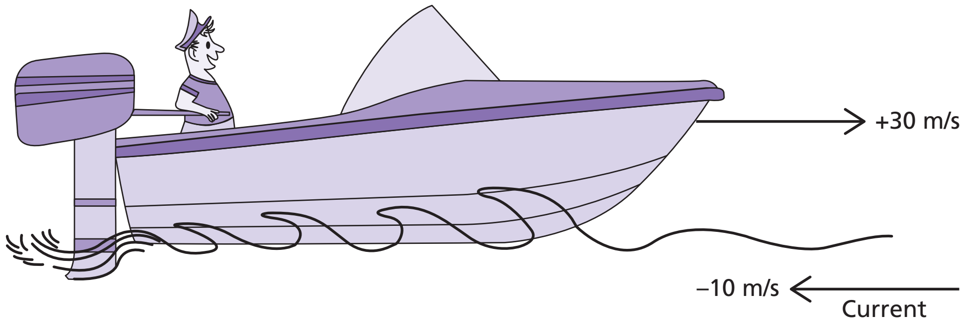

Figure 1.17 How to calculate resultant velocity of the boat in Worked example 1.9

Given: Velocity of the body, V_1 = 30 m/s to the east; velocity of the current V_2 = 10m/s to the west

Therefore, resultant velocity V_R

= $V_1 + V_2$
= 30 m/s + (–10 m/s)
= 10 m/s

Worked example 1.10 **Calculate resultant velocity (2)**

An aircraft takes off from Cape Town International Airport in a direction of N40°W at an airspeed of 400 km/hr. The aircraft is then blown off course by a wind of 150 km/hr from a direction of W30°S. Calculate the resultant velocity of the aeroplane.

Solution

Figure 1.18 shows how to calculate the resultant velocity of the aircraft in Worked example 1.10.

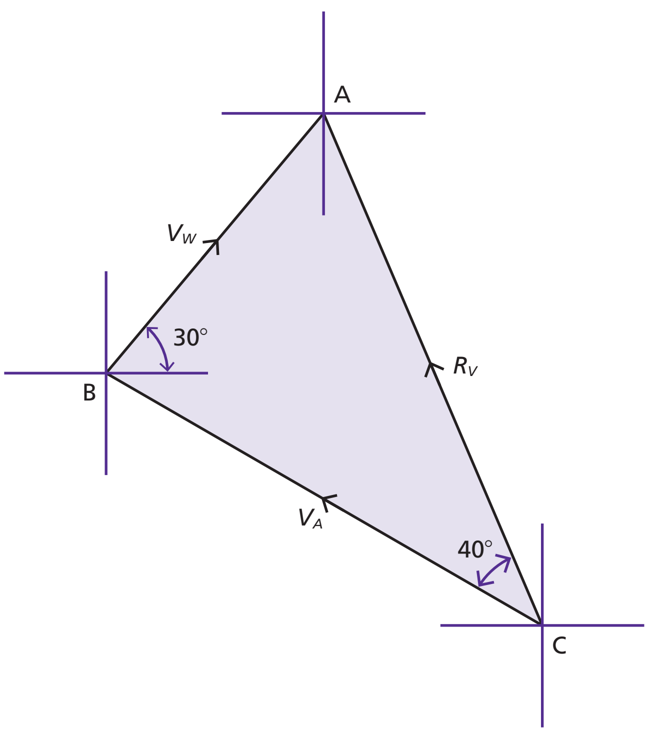

Figure 1.18 How to calculate resultant velocity for Worked example 1.10

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Given: V_a = 400 km/hr N40°W

V_w = 150 km/hr W30°S

R_v = ?

$\beta = (30° + 50°)$

$= 80°$

$b = \sqrt{a^2 + c^2 - 2ac\cos B}$

$= \sqrt{400^2 + 150^2 - 2(400)(150)\cos 80°}$

$= \sqrt{182\ 500 - 20\ 837{,}78132}$

$= \sqrt{161\ 662{,}2187}$

$= 402{,}0724048$ km/hr

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$b\sin C = c\sin B$

$402{,}072\sin C = 150\sin 80°$

$\sin C = \frac{150\sin 80}{402{,}072}$

$C = \sin^{-1}\left(\frac{150\sin 80}{402{,}072}\right)$

$C = \sin^{-1}(0{,}367399776)$

$= 21{,}56°$

$\theta = 40 - 21{,}56$

$= 18{,}44°$

R_V = 402,072 km/hr N18,44°W

Worked example 1.11 — Calculate resultant velocity (3)

A small aircraft flies north at 150 km/h. A 50 km/hr crosswind blows the aircraft off course in a westerly direction. What is the resultant velocity?

Solution

Figure 1.19 shows how to calculate the resultant velocity of the aircraft in Worked example 1.11.

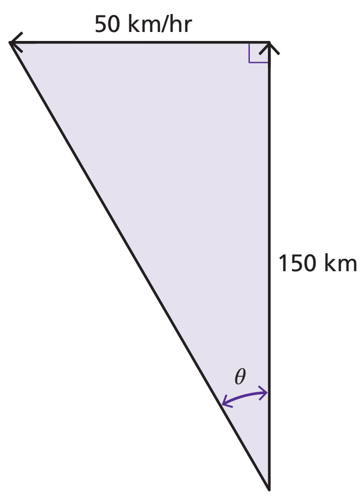

Figure 1.19 How to calculate resultant velocity for Worked example 1.11

Given: Velocity of aircraft, V_P = 150 km/hr; wind velocity, V_W = 50 km/hr

$$R^2_V = Vp^2 + Vw^2$$

$$R_V = \sqrt{150^2 + 50^2}$$

$$= \sqrt{25\ 000}$$

$$= 158{,}114 \text{ km/hr}$$

$$\tan\theta = \frac{O}{A}$$

$$= \frac{50}{150}$$

$$\theta = \tan^{-1}\left(\frac{50}{150}\right)$$

$$= 18°$$

Worked example 1.12 **Calculate resultant velocity (4)**

An aeroplane flies northeast at 400 km/hr and the wind blows to the east at 60km/hr. What is the resultant velocity?

Solution

Figure 1.20 shows how to calculate the resultant velocity of the aircraft in Worked example 1.12.

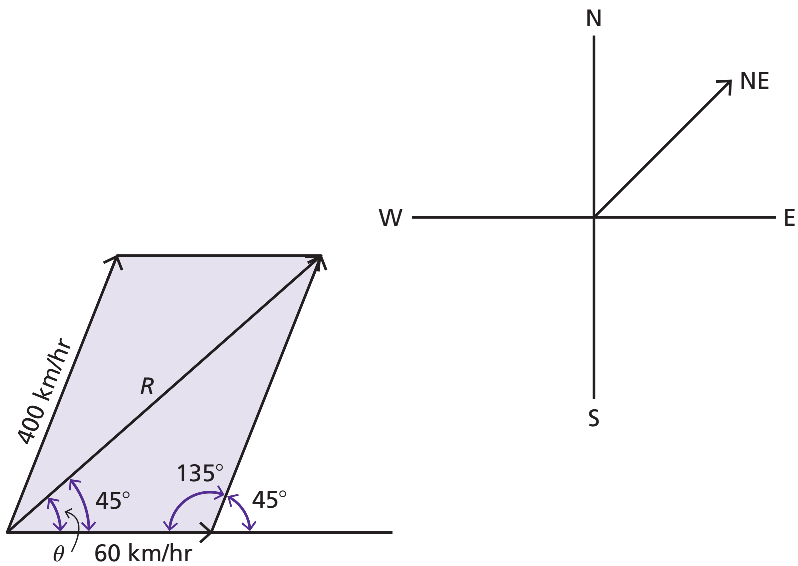

Figure 1.20 How to calculate resultant velocity for Worked example 1.12

Given: Velocity of aircraft, V_P = 400 km/hr; wind velocity, V_W = 60 km/hr

$$R = \sqrt{60^2 + 400^2 - 2(60)(400)\cos 135°}$$

$$= \sqrt{163\ 600 + 33\ 941{,}1255}$$

$$= \sqrt{129\ 658{,}8745}$$

$$= 360{,}082 \text{ km/hr}$$

$$\frac{\sin\theta}{400} = \frac{\sin 135°}{360{,}082}$$

$$\sin\theta 360{,}082 = 400\sin 135°$$

$$\sin\theta = \frac{400\sin 135°}{360{,}082}$$

$$\theta = \sin^{-1}\left[\frac{400\sin 135}{360{,}082}\right]$$

$$= 51{,}77°$$

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Activity 1.2 Resultant velocity

1. Define resultant velocity.
2. A light aircraft with an airspeed of 350 km/hr takes off at Cape Town International Airport in a direction of S33°W. The aircraft is blown off course by a heavy 150 km/hr wind blowing in a direction of N49°W. Calculate the resultant velocity of the aircraft in magnitude and direction.
3. A canoeist is rowing on the Vaal River at 4 m/s in a northerly direction. A wind of 3m/s suddenly starts blowing in a southeasterly direction.
   a) Calculate the resultant velocity of the canoe.
   b) Calculate the displacement of the canoeist after 45 seconds.
4. An aircraft is flying north at 100 km/hr and is being moved off course by a 40 km/hr blowing towards the west. What is the resultant velocity of the aircraft?
5. An aircraft flies northeast at 600 km/hr and a wind blows to the east at 80 km/hr. What is the resultant velocity of the aircraft?

Unit 3: Projectiles

LEARNING OUTCOMES

- Do calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same horizontal plane.
- Calculate the maximum height reached by an object as well as the time of flight and range.
- Calculate the height and velocity at any part of the projectile path.
- Calculate the velocity of projection.
- Calculate the angle of projection.

Introduction

Projectiles are commonly found in warfare and sports and include bullets, round balls and sports equipment, such as a shot putt.

This unit focuses on calculations regarding projectiles. This includes calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same horizontal plane.

The unit also shows how to calculate the maximum height reached by an object, the time of flight and range of these projectiles, the height and velocity at any part of the projectile path, the velocity of projection, and the angle of projection.

1. The motion of a projectile

A **projectile motion** is defined as the motion of an object thrown into the air and subject to only acted upon by gravitational acceleration. The value of gravitational acceleration used is 9,8 m/s^2 and the unit g is used. The object is referred to as the projectile and the path moved by an object is known as the trajectory. Examples of objects are balls or bullets.

Keywords

Projectile motion: the motion of an object thrown into air and subject to only acted upon by gravitational acceleration

For upward movement of an object:

$$\text{Gravitational acceleration } a = -g = -9{,}8\frac{m^2}{s} \text{ (deceleration)}$$

For downward movement:

$$a = g = \frac{9{,}8m^2}{s} \text{ (acceleration).}$$

If an object or a body falls to earth from a height, its downward acceleration is uniform. Similarly, if a body or an object is thrown upwards, its deceleration upwards is uniform and equal to g.

Projectile motion consists of two parts:

- horizontal motion – no acceleration
- vertical motion – constant acceleration due to gravitational acceleration.

The following are examples of projectile motion:

- a ball that has been thrown
- a bullet fired from a gun
- an arrow fired from a bow, or a stone launched with a slingshot
- in sport, a golf ball or shot put
- rockets or missiles.

2. Projectiles launched at an angle

When we throw an object, such as a ball, at an angle, it travels for a distance and then falls. In this case, gravitational force acts on the ball and makes it follow a parabolic path then fall to the ground.

When we fire a bullet from a gun, it travels a distance as a result of a force provided by the force of the propellant used. However, once the bullet leaves the gun, it is affected by gravitational force and will follow a parabolic path. The bullet will fall to the ground when it encounters an opposing force.

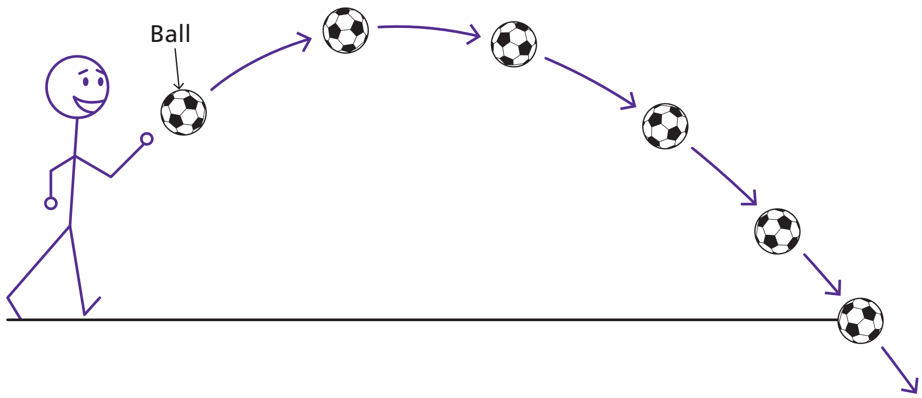

Figure 1.21 Parabolic path of a ball thrown in the air

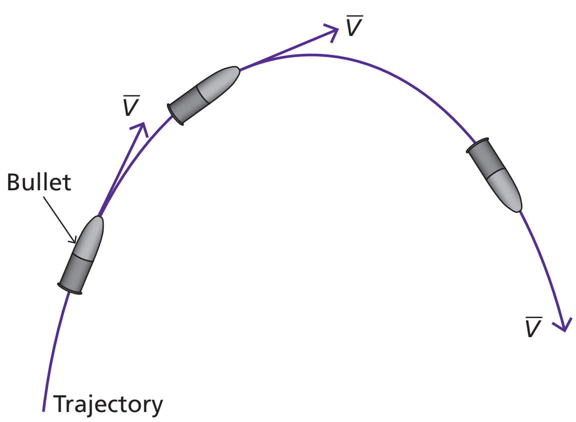

Figure 1.22 Trajectory of a bullet fired from a gun

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Figure 1.23 depicts the horizontal range or distance travelled by a projectile.

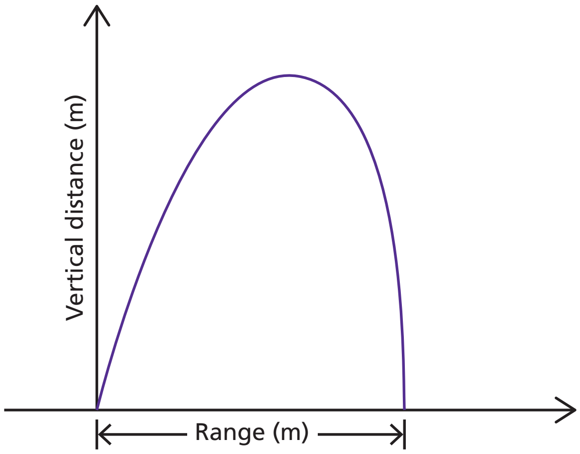

Figure 1.23 The horizontal range or distance travelled by a projectile

The **trajectory** of a projectile follows a curve called a parabola. The horizontal distance travelled by a projectile is known as the **range**

Keywords

Parabola: the trajectory/curve of a launched or thrown projectile

Range: the horizontal distance travelled by a projectile

3. Motion of a ball falling from a cliff

Figure 1.24 shows a ball falling from a cliff. In the diagram, h shows the height of the cliff and dy is the distance the ball drops (vertically).

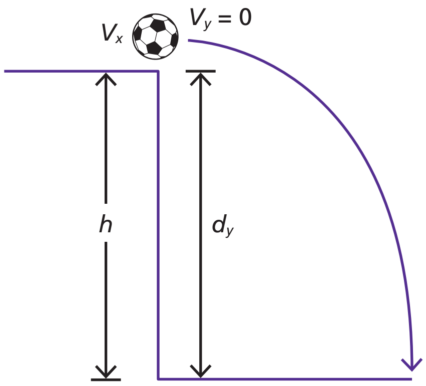

Figure 1.24 Vertical motion of an object (ball) ball falling from a cliff

The formulas that follow are derived from Figure 1.24.

V_0 is initial velocity

$h = \frac{1}{2}at^2$

$d = V_0 t + \frac{1}{2}at^2$

$d_y = V_0 t + \frac{1}{2}ay t^2$

$h = 0 + \frac{1}{2}a t^2$

In Figure 1.25 a ball rolls from a cliff and falls. The drop has a horizontal component. R is the range, which is the horizontal distance between the base of the cliff and where the ball lands.

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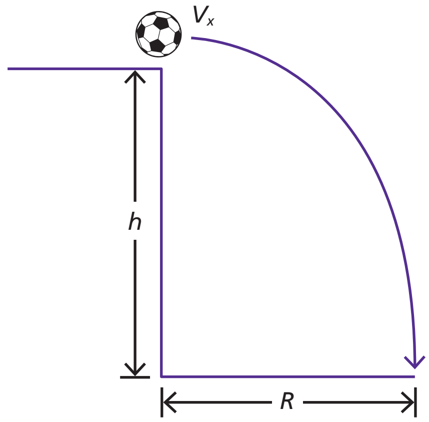

Figure 1.25 Horizontal motion of an object (ball) falling from a cliff

The formulas that follow are derived from Figure 1.25.

$$h = \frac{1}{2}at^2$$
$$R = V_x t$$
$$dx = V_x t$$
$$ax = 0\ ;\ ay = -9{,}8\ \text{m/s}^2$$
$$V_F = V_{YO} + at$$
$$V = \sqrt{Vx^2 + Vy^2}$$
$$Vx = V\cos\theta$$
$$Vy = V\sin\theta$$
$$\sin\theta = \frac{V_y}{V}$$
$$\cos\theta = \frac{V_x}{V}$$
$$\tan\theta = \frac{V_y}{V_x}$$
$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

4. Horizontal displacement of a projectile

Figure 1.26 shows how to calculate the time it takes a ball to move between two points.

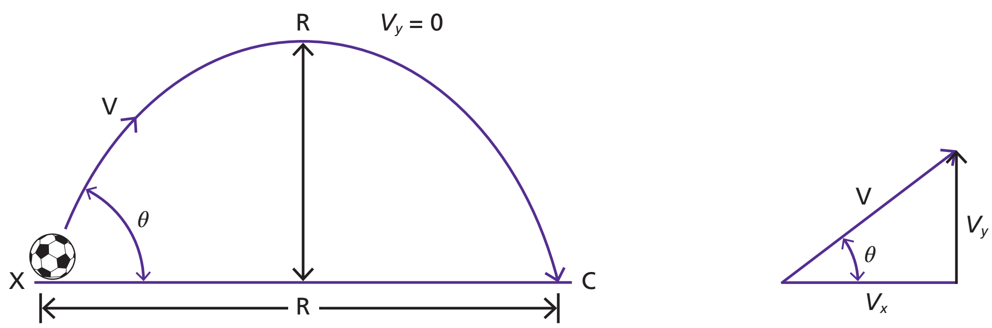

Figure 1.26 How to calculate the time it takes a thrown ball to go from A to B

From Figure 1.26:

$V_Y = V\sin\theta$

$V_X = V\cos\theta$

$V_F = V_0 +$ at, where V_F is the final velocity and V_O is the initial velocity.

The time the takes ball from A to B

$V_F = V_O + at$

$0 = V\sin\theta + gt$

$-V\sin\theta = gt$

$\frac{V\sin\theta}{g} = t$, where t is the time it takes the ball from A to B.

Figure 1.27 shows how to calculate the time it takes a projectile to move from one position to another and the height it reaches.

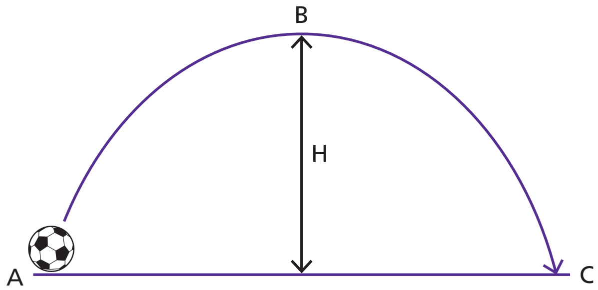

Figure 1.27 How to calculate the time it takes a thrown ball to go from A to C and the height it reaches

From Figure 1.27:

$t = \frac{2V\sin\theta}{g}$, where t is the time it takes the ball to go from A to C.

$V_{yf} = V_{YO^2} + 2_{aydy}$

$VF^2 = VO^2 + 2ad$

$0 = (V\sin\theta)^2 + 2gH$

$-V^2\sin^2\theta = 2gH$

$H = \frac{V^2\sin^2\theta}{2g}$

Figure 1.28 shows how to calculate the range of a projectile.

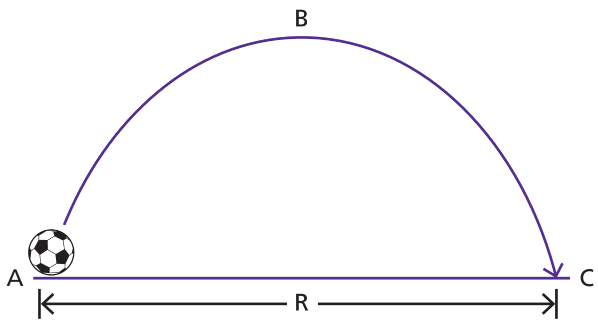

Figure 1.28 How to calculate the range *R* of a thrown ball

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From the diagram,

$R = V_x t$

$R = (V\cos\theta)(t)$

$t = \frac{2V\sin\theta}{g}$ (the time it takes the ball from A to C)

$R = V\sin\theta - \frac{2V\sin\theta}{g}$

$R = V^2\left(\frac{2\sin\theta\cos\theta}{g}\right)$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$R = \frac{V^2\sin(2\theta)}{g}$, where R is the range shown in Figure 1.28.

Figure 1.29 shows how to calculate t using a quadratic equation.

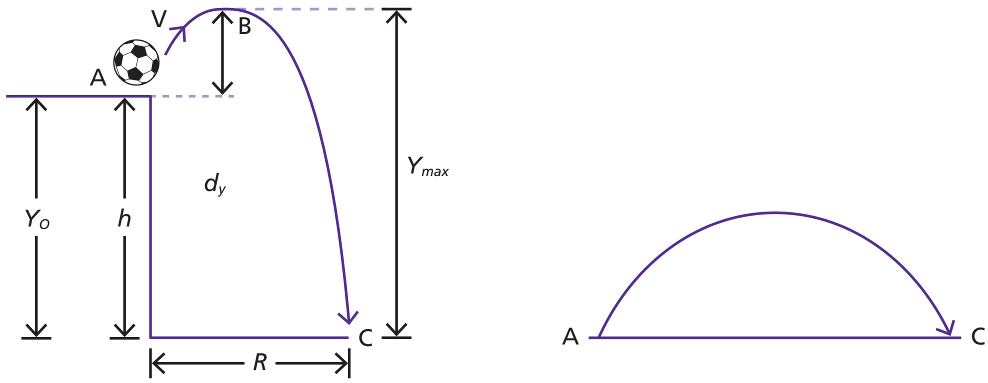

Figure 1.29 How to calculate t using a quadratic equation

From Figure 1.29:

$t\left(\text{A to C}\right) = \frac{2V\sin\theta}{g}$

$d = V_O t + \frac{1}{2}at^2$

$V_F = -Y_O = V_{YO}t + \frac{1}{2}gt^2$

$V_F = Y_O + V_{YO}t + \frac{1}{2}gt^2$

$0 = h + V\sin\theta t + \frac{1}{2}gt^2$

$t = -b \pm \sqrt{\frac{b^2 - 4ac}{2a}}$, where t is the time taken by the ball from A to C.

Also note:

Time taken by the ball from A to B,

$t = \frac{V\sin\theta}{g}$

Time taken by the ball from B to C,

$t = H + Y_O$

$= \frac{1}{2}at^2$

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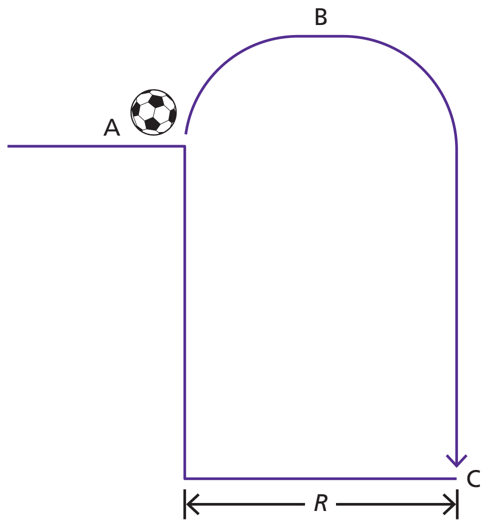

Figure 1.30 Calculate the time it takes the ball from A to B

From the diagram,

$$t = \frac{V\sin\theta}{g}$$

Height (h) is calculated as follows:

$$h = \frac{V^2 \text{Sin}^2\theta}{2g}$$

Time it takes the ball from B to C:

$$h = \frac{1}{2}a\,t^2$$

$$h + H = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2\,Y_{max}}{g}}$$

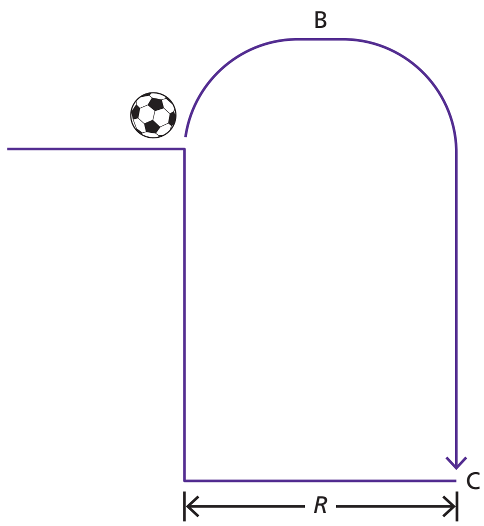

Figure 1.31 Calculate the time it takes a ball from B to C

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From the diagram,

$R = V_X t$

$R = \frac{V^2 \sin(2\theta)}{g}$

$V_X = V\cos\theta$

$R = V\cos\theta t$

V_X is constant

$V_{YF} = V_{YO} + at$

We use $V_{YF} = V\sin\theta + gt$ to calculate the velocity of the ball from A to C.

$V = \sqrt{V_{X^2} + V_{Y^2}}$

$\theta = \tan^{-1}\left(\frac{V_Y}{V_X}\right)$

Worked example 1.13 **Projectile calculations (1)**

A ball rolls horizontally off a cliff at 40 m/s. It takes 20 s for it to hit the ground Calculate the height of the cliff and the horizontal distance travelled by the ball.

Solution

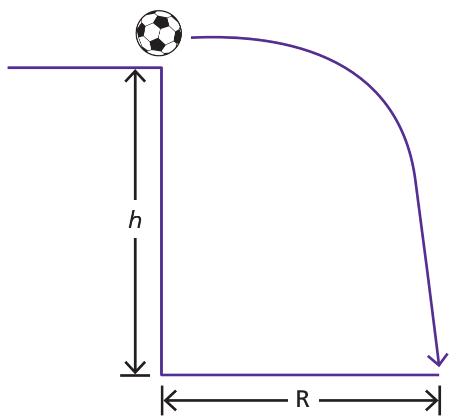

Figure 1.32 Diagram for Worked example 1.13

Given: V_X = 40 m/s; t = 20 s; a = 9.8 m/s^2

Height of cliff

h = 1 _ 2 at 2

= 0,5 × 9.8 × 20 2

= 1 980 m

Horizontal distance travelled by the ball

$R = V_X t$

$= \frac{40 \text{ m}}{s} \times 20\ s$

= 800 m

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Worked example 1.14 Projectile calculations (2)

A ball rolls off a 200m high cliff. Calculate the time it takes for the ball to hit the ground.

Solution

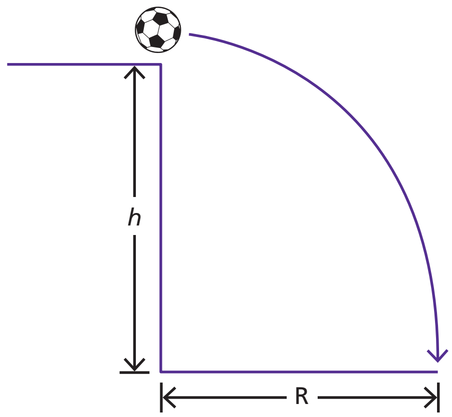

Figure 1.33 Diagram for Worked example 1.14

Given: h = 200 m; a = 9.8 m/s²

$$h = \frac{1}{2}at^2$$

$$400 = 4.9t^2$$

$$81,63265306 = t^2$$

$$\sqrt{81,63265306} = t$$

$$t = 9,04\ s$$

Worked example 1.15 Projectile calculations (2)

A ball is released from rest and drops straight down from a height of 600 m.

1. How long will the ball take to hit the ground?
2. How long will it take the ball to reach the ground if the ball was thrown down with an initial speed of 20 m/s?

Solution

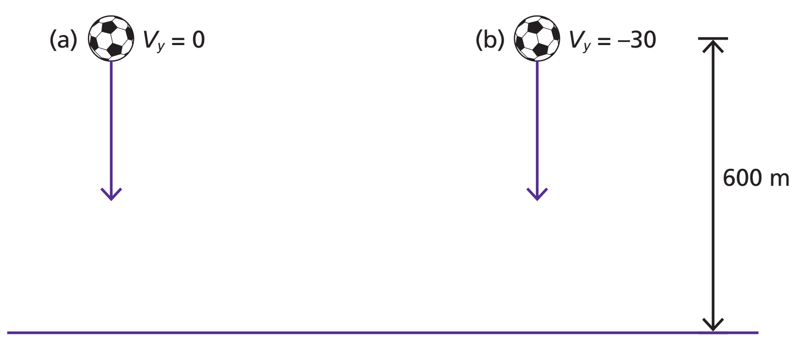

Figure 1.34 Diagram for Worked example 1.15

1. $h = \frac{1}{2}at^2$

$600 = 0,5 \times 9.8 \times t^2$

$600 = 4,9t^2$

$\sqrt{122,4489796} = \sqrt{t^2}$

$11,05s = t$

2. $dy = V_{YO}t + \frac{1}{2}ayt^2$

$-600 = -20\,t + \frac{1}{2}(-9.8)\,t^2$

$-600 = -20\,t + -4,9\,t^2$

$4,9\,t^2 + 20t - 600 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-20 \pm \sqrt{20^2 - 4(4,9)(-60)}}{9.8}$

$t = \frac{-20 \pm \sqrt{400 + 11\,760}}{9,8}$

$= \frac{-20 \pm \sqrt{12\,160}}{9,8}$

$= \frac{-20 \pm 110,272}{9,8}$

$= 9,21$ s

or,

$-13,293$ s

Worked example 1.16 **Projectile calculations (3)**

An object is projected at such an angle that the range (horizontal displacement) is three times the maximum height reached. The initial velocity of the object is 270 m/s. Calculate the angle at which the object is projected.

Solution

S (horizontal) = 3 × S (vertical)

$\frac{U^2\sin\theta}{g} = \frac{3\,U^2\sin\theta}{2g}$

$\frac{U^2\sin\theta \times g}{g \times U^2} = \frac{3\,U^2\sin^2\theta \times g}{g \times U^2}$

$\frac{3\sin\theta \times \sin\theta \times 2}{2 \times \sin2\theta \times 3} = \frac{\sin2\theta \times 2}{\sin2\theta \times 3}$

But,

$\sin2\theta = 2\sin\theta\cos\theta$ (from compound angles)

$\frac{\sin\theta\sin\theta}{2\sin\theta\cos\theta} = \frac{2}{3}$

$\frac{\sin\theta}{\cos\theta} = \frac{2 \times 2}{3}$

$\tan\theta = \frac{4}{3}$

$\theta = 53,13°$

Or,

$S(\text{horizontal}) = 270\cos\alpha(2)t$

$S(\text{vertical}) = 270\sin\alpha t + \frac{1}{2}g\,t^2$

$S(\text{horizontal}) = 3 \times S(\text{vertical})$

$270\cos\alpha(2)t = 3(270\sin\alpha t) + \frac{1}{2}g\,t^2$

$t(\text{maxheight}): V = U + gt$

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$0 = 270\sin\alpha - 9{,}8\,t$
$9{,}8t = 270\sin\alpha$
$t = 27{,}55\sin\alpha$
$270\cos\alpha(2) = 3\left(270\sin\alpha + \frac{1}{2}gt\right)$
$540\cos\alpha = 810\sin\alpha - 14{,}7t$
$540\cos\alpha = 810\sin\alpha - 14{,}7(27{,}55\sin\alpha)$
$540\cos\alpha = 810\sin\alpha - 404{,}985\sin\alpha$
$540\cos\alpha = 405{,}015\sin\alpha$
$\frac{540}{405{,}015} = \frac{\sin\alpha}{\cos\alpha}$
$\frac{540}{405{,}15} = \tan\alpha$
$53{,}129° = \tan\alpha$

Worked example 1.17 — Projectile calculations (4)

A store is thrown at a velocity of 42 m/s at an angle of 26° to the horizontal.

1. Calculate the maximum height that the stone reaches.
2. Calculate the horizontal displacement of the stone.

Solution

From the first principle:

1. S(vertical)
   $= \frac{U^2 \sin^2\theta}{2g}$
   $= \frac{42^2 \sin^2 26°}{2 \times 9{,}8}$
   $= 17{,}295$ m
2. S(horizontal)
   $= \frac{U^2 \sin 2\theta}{g}$
   $= \frac{42^2 \sin 2 \times 26°}{9{,}8}$
   $= 141{,}841$ m

Alternative method:

1. $S(\max) = \frac{V^2 - U^2}{2g}$
   $= \frac{0^2 - 425 \times 26}{2 \times -9{,}8}$
   $= 17{,}295$ m
2. $S(\text{range}) = U \times t$
   $= U\cos\theta \times \frac{2(U\sin\theta)}{g}$
   $= \frac{42\cos 26° \times 2 \times 42\sin 42°}{9.8}$
   $= 141{,}841$ m

Worked example 1.18 — Projectile calculations (5)

A boy throws a cricket ball at an angle of 15° to the horizontal with an initial velocity of 30 m/s. The path of the ball is that of a projectile.

1. Calculate the time the ball takes to reach its maximum height.
2. Calculate the maximum height reached by the ball.
3. Calculate the horizontal displacement of the ball.

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Solution

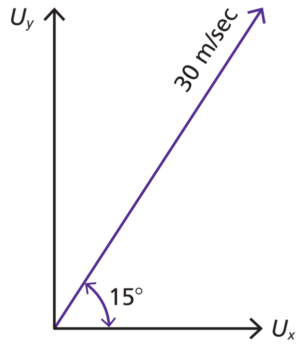

Figure 1.35 Diagram for Worked example 1.18

Given: $U = 30$ ms ; $V = 0\frac{m}{s}$; $a = g = -9{,}8\frac{m^2}{s}$; $U_X = 30\cos15° = 29\frac{m}{s}$;
$U_Y = 30\sin15° = 7{,}76$ m/s

1. $t = \frac{V-U}{a}$
$= \frac{0 - 7{,}76}{-9{,}8}$
$= 0{,}792\ s$

2. $S = Ut + \frac{1}{2}at^2$
$= (7{,}76 \times 0{,}792) + \frac{1}{2}(-9.8(0{,}792^2))$
$= 3{,}072$ m

3. $S = Ut$
$= U_X \times 2t$
$= 29 \times 2 \times 0{,}792$
$= 45{,}936$ m

Worked example 1.19 **Projectile calculations (6)**

A bullet is fired at a muzzle velocity of 125m/s at an angle of 25° to the horizontal.

1. Calculate the horizontal distance of the bullet after 8 seconds.
2. Calculate the maximum height of the bullet.

Solution

1. $U_X = U\cos\theta$
$= 120\cos25°$
$= 108{,}757$ m/s
$\Delta_X = U_X \times 2t$
$= 108{,}757 \times 2 \times 8$
$= 1740{,}111$ m

2. $\Delta_Y = (U_Y \times t) - \frac{1}{2}gt^2$
$= (52{,}83 \times 8) - \left(\frac{1}{2} \times 9{,}8 \times 8^2\right)$
$= (422{,}62) - (313{,}6) = 109{,}02$ m

Activity 1.3 **Projectiles**

1. Define a projectile motion.
2. Define a trajectory.
3. List FOUR examples of projectile motions.
4. An object is projected at such an angle that the range (horizontal displacement) is

three times the maximum height reached. The initial velocity of the object is 280 m/s. Calculate the angle at which the object is projected.

5. A stone is thrown at a velocity of 40 m/s at an angle of 25° to the horizontal. Calculate the following:
   a) The maximum height that the stone reaches.
   b) The horizontal displacement of the stone.
6. A boy throws a cricket ball at an angle of 20° to the horizontal with an initial velocity of 35 m/s. The path of the ball is that of a projectile. Calculate the following:
   a) The time to reach the maximum height.
   b) The maximum height reached by the ball.
   c) The horizontal displacement of the ball.
7. A ball is released from rest and drops straight down from a height of 500 m.
   a) How long will it take to hit the ground?
   b) How long will it take to reach the ground if the ball was thrown down with an initial speed of 25 m/s?
8. A bullet is fired at a muzzle velocity of 130 m/s at an angle of 30° to the horizontal. Calculate the following:
   a) The horizontal distance of the bullet after 6 seconds.
   b) The maximum height of the bullet.

Module summary

- Relative velocity is defined as the velocity of an object relative to something else; it is the difference between two velocities.
- Resulting velocity is the vector sum of two or more velocities.
- Resulting velocity is calculated by simply adding two or more vectors.
- Pythagorean theorem is also used to calculate resulting velocity depending on a situation.
- The formula, $V_f - at + V_0$ is also used to calculate resulting velocity depending on a given scenario.
- A projectile motion is defined as the motion of an object thrown into the air and subject only acted upon by gravitational acceleration.
- The value of gravitational acceleration used in this module is 9.8 m/s^2.
- Projectile motion consists of two parts: horizontal motion and vertical motion.

Module 1 Checklist

Before you attempt to answer the exam type questions, go through the following checklist with a list of learning outcomes taken from the syllabus. Check that you understand all concepts /learning outcomes covered in Module 1 of the N4 Engineering Science syllabus.

Learning content	Learning outcomes	Yes	No
1.1 Relative velocity	■ Solve problems dealing with constant linear motion analytically (Pythagoras or the sine and cosine rules). ■ Determine the relative velocity, shortest distance, time to intercept and actual velocity.		

Learning content	Learning outcomes	Yes	No
1.2 Resulting velocity	■ Calculate the resulting velocity and direction of a maximum of two vectors. ■ Calculate the time to reach a certain destination.		
1.3 Projectiles	■ Do calculations dealing with projectiles that are launched horizontally from a certain vertical height or launched at an angle from the horizontal landing on the same plane. ■ Calculate the maximum height reached by an object as well as the time of flight and range. ■ Calculate the height and velocity at any part of the projectile path. ■ Calculate the velocity of projection. ■ Calculate the angle of projection.		

Exam practice questions

1. A light aircraft is 50 km northeast of Cape Town International Airport and flies north for five hours at an airspeed of 95 km/hr. Determine its position (displacement) with reference to Cape Town International Airport in magnitude and direction. (6)

2. A gold mine shaft has a depth of 230 m. Hoist A descends at 6 km/hr and hoist B ascends at 5 km/hr. Calculate the following:

a) The velocity of the hoist A relative to the velocity of the hoist B in magnitude and direction (2)

b) The velocity of hoist B relative to the velocity of hoist A in magnitude and direction. (2)

3. A stone is thrown at a velocity of 38 m/s at an angle of 20° to the horizontal. Calculate the following:

a) The maximum height that the stone reaches. (3)

b) The horizontal displacement of the stone. (2)

4. An aircraft A departs from Lanseria International Airport in a northerly direction at an airspeed of 280 km/hr. Another aircraft B departs simultaneously from OR Tambo International Airport at an airspeed of 600 km/hr in a direction W60°S.

Calculate the velocity of aircraft A relative to the velocity of aircraft B. (5)

5. A boy throws a cricket ball at an angle of 10° to the horizontal with an initial velocity of 30 m/s. The path of the ball is that of a projectile. Calculate the following:

a) The time to reach the maximum height. (3)

b) The maximum height reached by the ball. (2)

c) The horizontal displacement of the ball. (2)

6. A canoeist can row 20 m/s in calm water. The river flows at 5 m/s and is 100 m wide.

a) In which direction does the canoeist have to row to cross the river at a rectangular angle? (3)

b) How long will it take to reach the opposite side of the river? (2)

7. Two vehicles start moving simultaneously, a blue car at 250 km/hr W33°N and a red car at 210 km/hr directly east. Calculate the velocity of the red car relative to the blue car. (4)

Total: 36 marks

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