N5

Strength of Materials and Structures

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Module 8: Structural frameworks

Module 1 Stress, strain and testing of materials

What is covered?

This module focuses on direct and indirect stresses and discusses the three types of stress. The module also addresses the concept of strain and shows how to draw a stress/strain graph and use the graph to obtain information about a material. Finally, the module discusses how to calculate Young's modulus of elasticity for a material.

Subject Outcomes

After studying this module, you should be able to:

Unit 1

- Name the three different types of stresses
- Calculate direct and shear stresses
- Know what strain is and how to calculate it
- Know what the module of elasticity is and how to calculate it
- Calculate the change in length, final length and percentage change in length of a bar
- Draw a stress/strain graph and use it to obtain information about a material
- Draw a force/extension graph or a stress/strain graph and use it to calculate Young's modulus for a material.

Unit 2

- Calculate stresses for different materials connected in parallel including a pipe with a threaded bar and nut
- Calculate stresses when different materials are connected in series
- Calculate the change in length of each material
- Calculate the final length of the compound bar.

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Unit 1: Stress and strain and tensile testing of materials

LEARNING OUTCOMES

- Name the three different types of stresses
- Calculate direct and shear stresses
- Know what strain is and how to calculate it
- Know what the module of elasticity is and how to calculate it
- Calculate the change in length, final length and percentage change in length of a bar
- Draw a stress/strain graph and use it to obtain information about a material
- Draw a force/extension graph or a stress/strain graph and use it to calculate Young's modulus for a material.

Introduction

This unit shows how to calculate direct and shear stresses and the change in length, final length, and percentage change in length of the bar resulting from stress. The unit then discusses the modulus of elasticity and shows how to calculate it. Next, the unit shows how to draw a stress/strain graph and use it to obtain information about a material. Finally, the unit covers how to draw a force/extension graph or stress/strain graph and how to use it to calculate Young's modulus for a material.

1. Stress

Keyword

Stresses internal forces that are set up in a material when the material is subjected to the action of an external force

Stresses are internal forces that are set up in a material when the material is subjected to the action of an external force.

Stresses can be broadly categorised as direct stress (tensile stress and compressive stress) and shear stress. Figures 1.1, 1.2 and 1.3 illustrate these different categories of stresses.

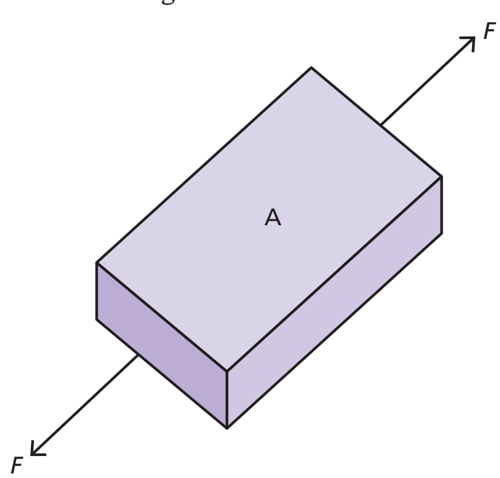

Figure 1.1 A is in tension

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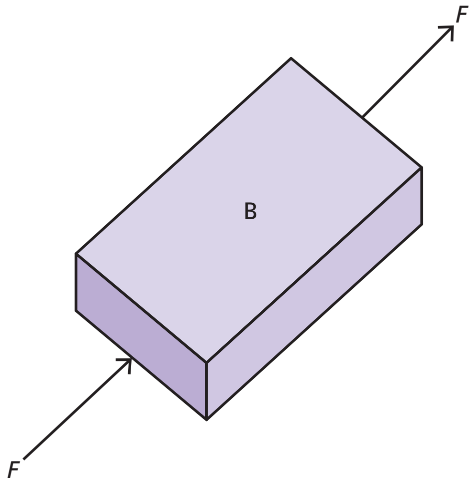

Figure 1.2 B is in compression

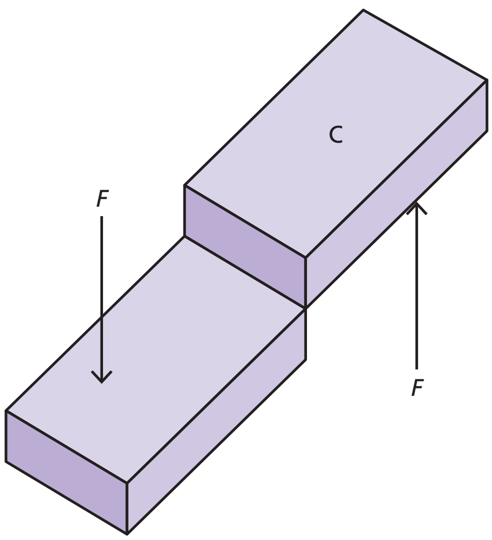

Figure 1.3 C is in shear

1.1 Direct stress

Direct stresses (tensile stress and compressive stress) change the length of a material. Tensile stresses, as shown in Figure 1.1, are produced by forces which try to pull the material apart. **Tensile stress** is a force that stretches a material by pulling on the object.

Compressive stress, as shown in Figure 1.2, is produced by forces trying to compress the material (compressive forces). Compressive stress makes components shorter. This change in length is measured by a highly sensitive measuring instrument called an extensometer. The direct stress is represented by the Greek letter sigma σ.

Keywords

Tensile stress a force that stretches a material by pulling on the object

Compressive stress a force trying to compress the material

We calculate direct stress as follows:

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

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or,

$$\sigma = \frac{F}{A}$$

where,

σ is the stress in pascals (Pa)

F is the force in Newtons (N)

A is the cross-sectional area in square metres (m^2).

Worked example 1.1 **Calculate tensile stress**

A steel bar supports a load of 30 kN. The bar has a diameter of 20 mm. Calculate the tensile stress in megapascals.

Solution

Given: F = 30 kN; d = 20 mm

$$\text{Area} = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0{,}02^2}{4}$$

$$= 0{,}000314159 \text{ m}^2$$

$$= 314{,}159 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F}{A}$$

$$= \frac{30\ 000}{0{,}000314159}$$

$$= 95\ 493\ 046{,}51 \text{ Pa}$$

$$= 95{,}493 \text{ MPa}$$

Worked example 1.2 **Calculate the diameter of a steel rod**

Calculate the diameter of a steel rod that must carry a tensile load of 120 kN without exceeding a stress of 100 MPa.

Solution

Given: F = 120 kN; σ = 100 MPa

$$A = \frac{F}{\sigma}$$

$$= \frac{120 \times 10^3}{100 \times 10^6}$$

$$= \frac{120\ 000}{100\ 000\ 000}$$

$$= 0{,}0012 \text{ m}^2$$

$$d = \sqrt{\frac{4 \times 0{,}0012}{\pi}}$$

$$= 0{,}039088195$$

$$= 39{,}09 \text{ mm}$$

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1.2 Shear stress

Shear stresses, as illustrated in Figure 1.4, are produced by equal and opposite parallel forces not in line. **Shear stress** is stress that cuts and changes the shape of an object or body.

Keyword
Shear stress stress that cuts and changes the shape of an object or body

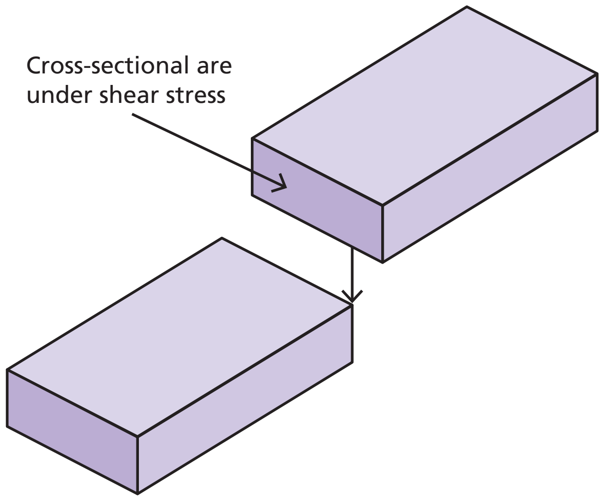

Figure 1.4 A component being cut by a shear force

Shear stress is represented by the Greek letter tau and the symbol is τ. We can calculate shear stress as follows:

$$\tau = \frac{F}{A}$$

where,

τ is the shear stress in pascals (Pa)

F is the shear force in newtons (N)

A is the cross-sectional area in square metres (m^2).

Worked example 1.3 — **Calculate shear stress**

Calculate the shear stress in the bolt in Figure 1.5. The bolt has a diameter of 15 mm and a force of 30 kN is applied on the bolted plate.

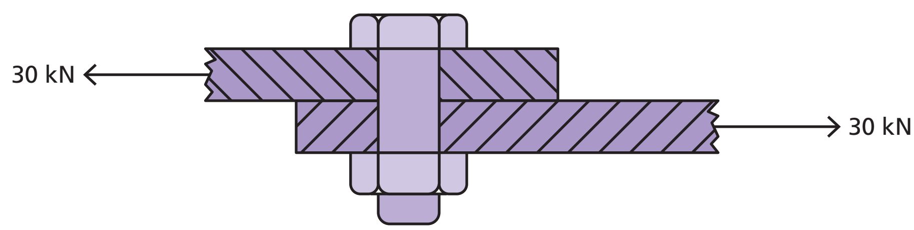

Figure 1.5 A bolt in a single shear

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Solution

Given: d = 15 mm = 0,015 m; F = 30 kN

$$A = \frac{\pi d^2}{4}$$

$$= \frac{\pi \times 0{,}15^2}{4}$$

$$= 176{,}715 \times 10^{-6}\ \text{m}^2$$

$$= 0{,}000176714\ \text{m}^2$$

$$\tau = \frac{F}{A}$$

$$= \frac{30 \times 10^2}{0{,}000176714}$$

$$= 169\ 765\ 836{,}3\ \text{Pa}$$

$$= 169{,}766\ \text{MPa}$$

Worked example 1.4 — **Calculate the shear stress in a blanking operation**

A blanking operation requires a force of 50 kN to punch a hole with a diameter of 30 mm in a steel plate with a thickness of 6 mm. Calculate the shear stress in the plate.

Solution

Given: F = 50 kN; d = 30 mm = 0,03 m; thickness (t) = 6 mm = 0,006 m

$$\tau = \frac{F}{A}$$

$$= \frac{50 \times 10^3}{\pi \times 0{,}03 \times 0{,}006}$$

$$= 88\ 419\ 412{,}83\ \text{Pa}$$

$$= 88{,}42\ \text{MPa}$$

2. Strain

When a load is applied to a material such as steel bar and there is a change in shape (for example, when if the length of the steel bar increases or decreases or if the bar becomes deformed), this change in the shape of the steel bar as a result of the applied force is called **strain**. Strain refers to deformation due to an external force, for example, the application of a load or by change in temperature.

Direct strain is defined as a change in shape or size or deformation of a material because of stress.

- Strain can be produced in two ways:
- by the application of a load
- by a change in temperature unaccompanied by a load or stress.

Keywords

Strain deformation due to an external force, for example, the application of a load or by change in temperature

Direct strain a change in shape or size or deformation of a material because of stress

The formula for calculating strain is:

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

or,

$$\varepsilon = \frac{x}{L}$$

where,

ε is the strain (no units or unitless)

x is the change in length

L is the original length.

Strain is simply a ratio of two lengths and, therefore, has no units – it is unitless.

Figure 1.6 shows a bar (A) whose length has increased after being subjected to tensile strain.

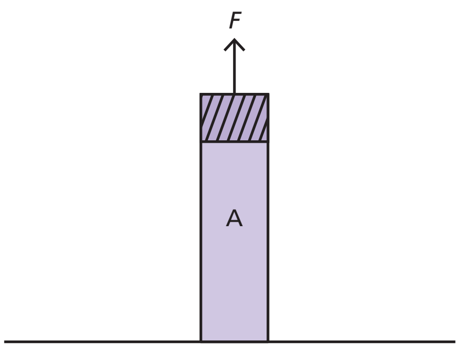

Figure 1.6 A bar whose length has increased after being subjected to tensile strain

Figure 1.7 shows a bar (B) whose length has decreased after being subjected to compressive strain.

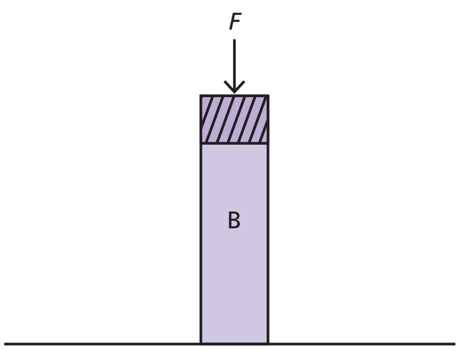

Figure 1.7 A bar whose length has decreased after being subjected to compressive strain

Worked example 1.5 Calculate the strain

A round metal rod has a diameter of 50 mm and a length of 750 mm. The rod hangs vertically and supports a load of 50 kN, which hangs from it. Calculate the strain of the rod if the new length of the rod is 750,165 mm due to this load.

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Solution

Given: L = 750 mm; x = 750,165 – 750 = 0,165 mm

$$\varepsilon = \frac{x}{L}$$

$$= \frac{0,165}{750}$$

$$= 0,00022$$

$$= 220 \times 10^{-6} \text{ (no units for strain)}$$

3. Shear strain

Keyword
Shear strain the angular displacement caused by shear stress

Shear strain is described as the angular displacement caused by shear stress.

We calculate shear strain by using the formula:

$$\gamma = \frac{\Delta x}{l} \text{ in radians}$$

where,

l is the original length of the steel bar or rod

γ (gamma) is the shear strain in radians

Δx is the distance moved by component

$l + \Delta x$ is the length of the bar or rod after loading.

Figure 1.8 shows a shear force applied to the steel bar.

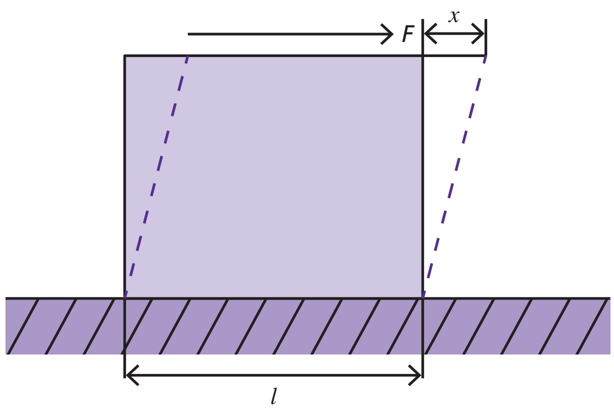

Figure 1.8 A shear force applied to steel bar

4. Modulus of elasticity

The modulus of elasticity refers to the measurement of the elasticity of an object. This section discusses elasticity as well as two related issues, namely, Hooke's law and Young's modulus of elasticity.

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4.1 Elasticity

Elasticity is the physical property of a material to return to its original shape after stress that had caused deformation is removed.

If the strain due to loading in a material disappears when the load is removed, then the material is said to be perfectly elastic. Also, a material is said to be perfectly elastic when the strain at a given load when unloading is equal to the strain at the same load during loading.

Resilience is a measure of the ability of a material to absorb energy while deforming elastically.

4.2 Hooke's law

Hooke's law states that within the elastic range, the strain produced is directly proportional to the stress producing it. This law applies to all kinds of stresses.

Stress is resistance per unit area of an external applied force on an object or body. It refers to deformation force per unit area.

$$\text{Stress} = \text{constant} \times \text{strain}$$

The ratio of the stress to strain is equal to a constant if a material obeys Hooke's law. The constant is referred to as the modulus of elasticity or **Young's modulus** and is usually represented by E.

4.3 Young's modulus of elasticity

Young modulus of elasticity (E) simply refers to the stiffness of a material. In other words, how easy it is to bend or stretch material.

Low modulus of elasticity

A low modulus of elasticity is obtained when a material is easy to stretch as a result of a small stress with a large strain material.

High modulus of elasticity

A high modulus of elasticity is obtained when a material is not easy to stretch as a result of a large stress with small strain material.

Table 1.1 shows values of the elasticity constant E.

Table 1.1 Values of the elasticity constant E

Material	Modulus of elasticity (N/mm^2)
Steel	207×10^3
Copper	83×10^3
Brass	69×10^3
Wood	$9{,}6 \times 10^3$

Keywords

Elasticity the physical property of a material to return to its original shape after stress that had caused deformation is removed

Resilience a measure of the ability of a material to absorb energy while deforming elastically

Hooke's law states that within the elastic range, the strain produced is directly proportional to the stress producing it

Stress resistance per unit area of an external applied force on an object or body; deformation force per unit area

Young modulus of elasticity refers to the stiffness of a material; how easy it is to bend or stretch material

We calculate Young's modulus of elasticity as follows:
Stress is directly proportional to strain

$\sigma \propto \varepsilon$

But $\sigma = \varepsilon$ x constant

$\sigma = \varepsilon \times E$

$E = \frac{\sigma}{\varepsilon}$

where,

E is Young's modulus of elasticity, usually in gigapascals (GPa)

σ is stress in pascals (Pa)

ε is strain (unitless).

Since $\varepsilon = \frac{x}{L}$ and $\sigma = \frac{F}{A}$, we can use Young modulus of elasticity to calculate the change in length of a bar as follows:

$x = \frac{\sigma \times L}{E}$

or,

$x = \frac{F \times L}{A \times E}$

Worked example 1.6 — Calculate Young's modulus of elasticity

A wire is 4 m long and is 2 mm in diameter. When stretched by a weight of 10 kg, the length of the wire increases by 0,5 mm.

1. Calculate the stress brought about by the wire.
2. Calculate the strain in the wire.
3. Calculate Young's modulus of elasticity of the material of the wire. Take g as 9,81 m/s^2.

Solution

1. Stress brought about the wire

   Given: Initial length of wire = 4 mm; diameter of wire = 2 mm; g = 9,81 m/s^2

   Radius $r = \frac{d}{2}$

   $= \frac{2}{2}$

   = 1 mm

   = 1 × 10^{-3} m

   = 0,001 m

   Increase in length x = 0,5 mm

   = 0,5 × 10^{-3}

   = 0,0005 m

   $A = \frac{\pi d^2}{4}$

   $= \frac{\pi \times (2 \times 10^{-3})^2}{4}$

   = 0,000003141 m^2

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Stress $= \frac{F}{A}$

$= \frac{mg}{\frac{\pi d^2}{4}}$

or,

$\frac{mg}{\pi r^2} = \frac{8 \times 9{,}81}{0{,}000003141}$

$= \frac{24\ 985\ 673{,}35 \text{ N}}{\text{m}^2}$

$= 2{,}5 \times 10^7 \text{ N/m}^2$

2. Strain in the wire

Strain $= \frac{x}{L}$

$= \frac{0{,}5 \times 10^{-3}}{4}$

$= 0{,}000125$

$= 1{,}25 \times 10^{-4}$

3. Young's modulus of elasticity of the wire

$\gamma = \frac{\text{Stress}}{\text{Strain}}$

$= \frac{2{,}5 \times 10^7}{1{,}25 \times 10^{-4}}$

$= 2 \times 10^{11} \text{ N/m}^2$

Worked example 1.7 Calculate the change in length of a copper tube

A tensile load of 400 kN acts on a 150 mm long copper tube. The tube has an outside diameter of 70 mm and a 10 mm wall. The elastic modulus of copper is 300 GPa. Calculate the change in length of the tube.

Solution

Given: Outside diameter = 70 mm = 0,07 m; inside diameter = 70 – 20 = 50 mm = 0,05 m

$A = \frac{\pi}{4}(0{,}07^2 - 0{,}05^2)$

$= 0{,}001884955 \text{ m}^2$

$= 1{,}885 \times 10^{-3} \text{ m}^2$

$x = \frac{F \times L}{A \times E}$

$= \frac{400 \times 10^3 \times 0{,}15}{0{,}001884955 \times 300 \times 10^9}$

$= \frac{60\ 000}{565\ 486\ 500}$

$= 0{,}000106103 \text{ m}$

$= 106{,}103 \times 10^{-6} \text{ m}$

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Worked example 1.8 Calculate the diameter and the change in length of a brass tube

A 3 m hollow brass tube has a diameter ratio of 0,6:1. A tensile load of 90 kN acts on the tube and the maximum stress that it can withstand 100 MPa.

1. Calculate the diameter of the tube.
2. Calculate the change in length of the tube if the modulus of elasticity of the tube is 150 GPa.

Solution

Given: F = 90 kN; σ = 100 MPa; d = 0,6D

1. Diameter of the tube

$$A = \frac{\pi(D^2 - d^2)}{4}$$
$$= \frac{\pi}{4}(D^2 - (0,6D)^2)$$
$$= \frac{\pi}{4}(D^2 - 0,36D^2)$$
$$= 0,502654824D^2$$
$$\sigma = \frac{F}{A}$$
$$A = \frac{F}{\sigma}$$
$$0,502654824\ D^2 = \frac{90 \times 10^3}{100 \times 10^6}$$
$$0,502654824\ D^2 = 0,0009$$
$$D^2 = \frac{0,0009}{0,502654824}$$
$$D^2 = 0,001790493$$
$$D = \sqrt{0,001790493}$$
$$D = 0,042314217 \times 1\ 000$$
$$= 42,314 \text{ mm}$$
$$d = 0,6(42,314)$$
$$= 25,389 \text{ mm}$$

2. Change in length of the tube

$$x = \frac{\sigma \times L}{E}$$
$$= \frac{100 \times 10^6 \times 3}{150 \times 10^9}$$
$$= 0,002 \text{ m}$$
$$= 2 \text{ mm}$$

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Worked example 1.9 Calculate the percentage increase in the length of wire when stretched

A wire is stretched by the application of a force of 60 kg per square centimetre. If Young's modulus of elasticity γ is 7×10^{10} N/m² and $g = 9,81$ m/s², what is the percentage increase in the length of the wire?

Solution

Given: $60 \text{ kg/cm}^2 = \frac{60 \times 9,81}{10^{-4}} = 60 \times 9,81 \times 10^4 \text{ N/m}^2$

The percentage increase in length is also referred to as % elongation.

$$\% \text{ elongation} = \% \frac{x}{L}$$

$$\gamma = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\text{Stress}}{\gamma}$$

$$= \frac{60 \times 9,81 \times 10^4}{7 \times 10^{10}}$$

$$= 0,000084085$$

$$= 8,4 \times 10^{-5}$$

$$\% \text{ elongation} = \text{strain} \times 100$$

$$= 8,4 \times 10^{-5} \times 100$$

$$= 0,0084\,\%$$

5. Stress/strain graph

A stress/strain curve or graph shows the relationship between stress and strain of material. A stress/strain graph is also used to ascertain the elastic properties of materials by studying the stress-strain relationships of materials under different loads.

To construct a stress/strain graph, we use data obtained after carrying a mechanical test whereby a load is applied to the material and continuous measurements of stress/strain are made simultaneously.

Figure 1.9 gives an example of a stress/strain graph.

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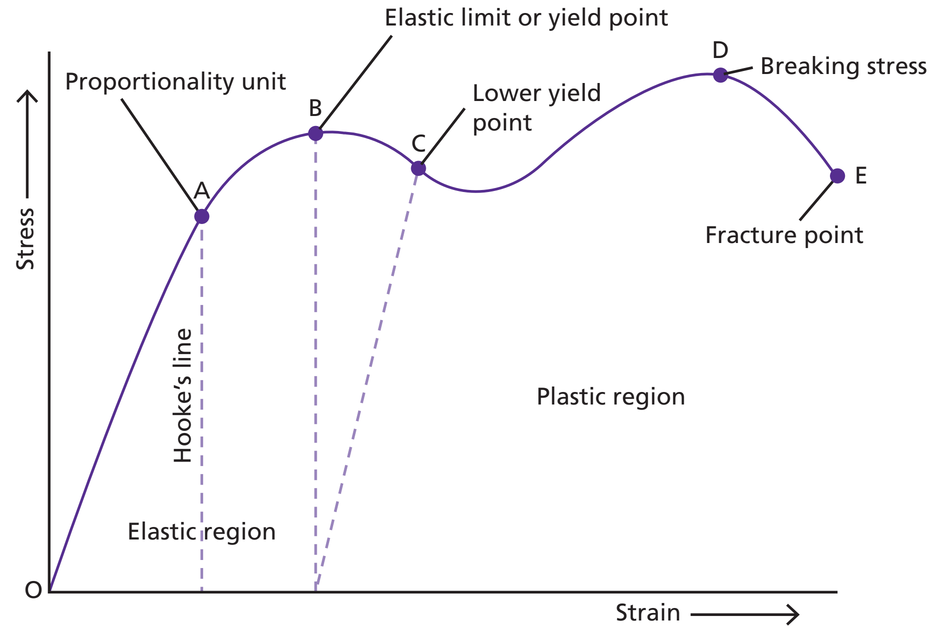

Figure 1.9 A stress/strain graph

A stress/strain graph consists of two main regions, namely, an elastic region and a plastic region.

The elastic region is the region where the material can be deformed, and when released, will return to its original configuration.

The plastic region is the region in which the material deforms permanently.

A stress/strain graph also consists of the following points, which are located within either the elastic region or plastic region:

- the proportional limit (see point A on the graph)
- the elastic limit (upper yield point) (see point B on the graph)
- the lower yield point (see point C on the graph)
- the breaking (ultimate stress) point (see point D on the graph)
- the fracture point (see point E on the graph).

5.1 The proportional limit

The point OA in the graph represents the proportional limit. The proportional region is the region that obeys Hooke's law. In the region, the stress/strain ratio gives a proportionality constant known as Young's modulus.

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5.2 The elastic limit

The **elastic limit** on a stress/strain curve is the point to which the material returns to its original position when the load acting on the material is completely removed. Beyond the elastic limit, the material will not return to its original position. Plastic deformation will appear on the material.

5.3 The plastic point (yield point)

The **plastic point (yield point)** is the point where the material starts to deform plastically. After the plastic point, permanent deformation will occur. The plastic point is divided into two points, namely, the upper yield point and the lower yield point.

5.4 The ultimate stress point

The ultimate stress point is the point that represents the maximum stress that a material can endure. Beyond this point, complete failure occurs.

5.5 The fracture or breaking point

The fracture or breaking point is the point in a stress/strain curve at which failure of material takes place.

Keywords

Elastic limit (on a stress/strain curve) the point to which the material returns to its original position when the load acting on the material is completely removed

Plastic point (yield point) the point where the material starts to deform plastically

6. Classification of materials

Figures 1.10 and 1.11 show stress/strain graphs and the classification of materials.

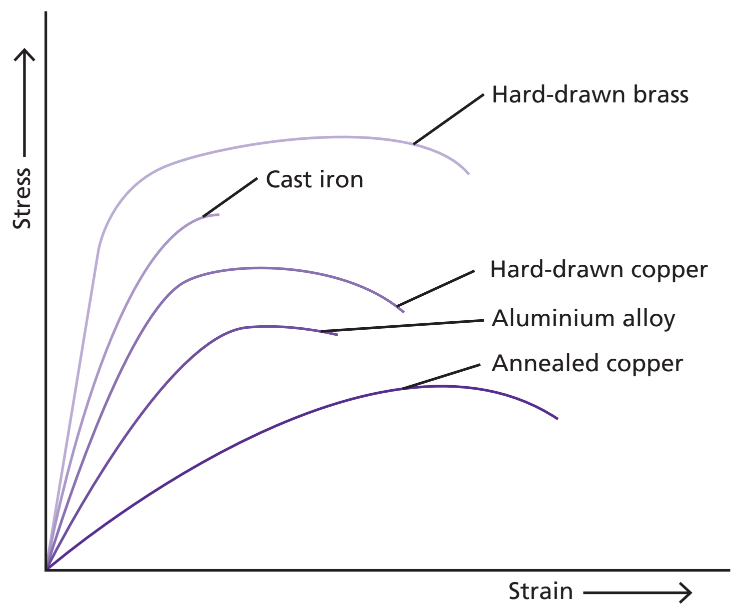

Figure 1.10 A stress/strain graph and the classification of materials

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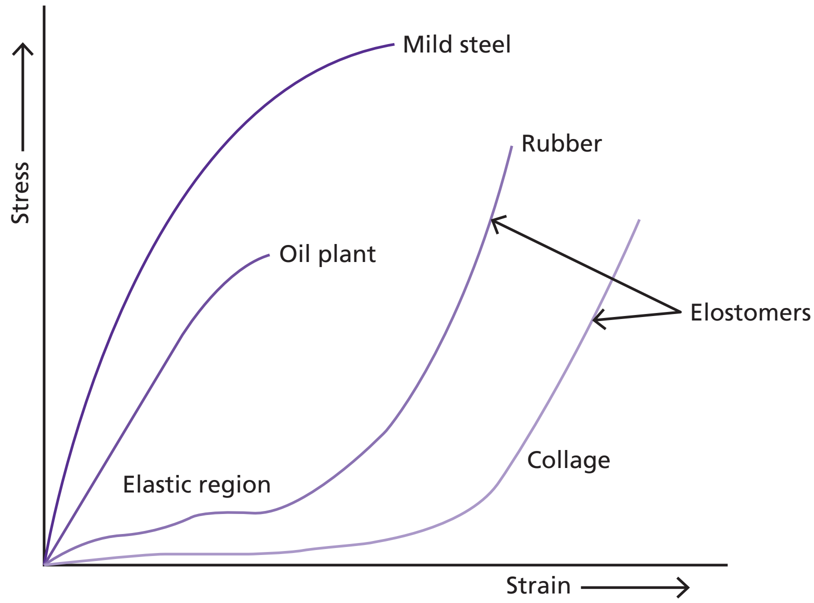

Figure 1.11 A stress/strain graph and classification of materials

Referring to the stress/strain graphs above it can be seen that glass is brittle and strong. Glass can take a lot of stress without deformation. However, glass has a small plastic region.

In the graphs, we can also see that steel has a large elastic region and a small plastic region. As a result, steel cannot be used to make wires – it can break too easily – but it is ideal for building bridges.

7. Mechanical properties of materials

This section discusses two mechanical properties of materials, namely, ductility and brittleness.

Figure 1.12 shows ductile and brittle material on a stress/strain graph.

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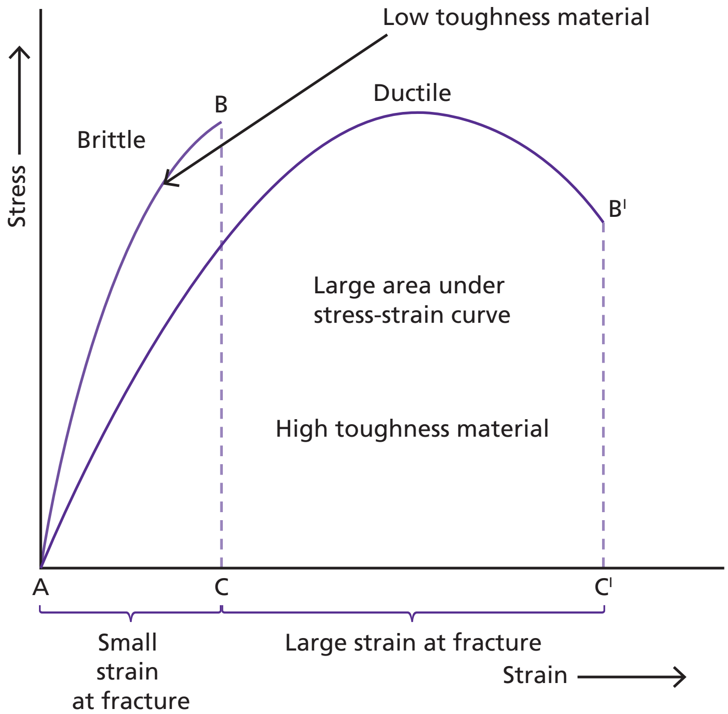

Figure 1.12 Ductile and brittle material on a stress/strain graph

7.1 Ductility

Ductility is a measure of the ability of a material to deform plastically before fracture. These materials fracture at very large strain. Materials such mild steel and gold are very ductile.

7.2 Brittleness

Materials that fracture at very low strain with little or no plastic deformation are **brittle**. A material is brittle when subjected to stress when it fractures with elastic deformation and without plastic deformation. A brittle material is hard but breaks easily.

Brittle materials don't easily deform plastically so the concept of yield strength is irrelevant for brittle materials. Glass, porcelain, and concrete are examples of brittle materials.

Keywords

Ductility a measure of the ability of a material to deform plastically before fracture

Brittle material does not easily deform plastically; materials that fracture at very low strain with little or no plastic deformation

8. Calculation of stresses from a stress/strain curve

If we carry out a tensile test on a piece of material such as steel and obtain stress/strain results, we can draw a stress/strain graph. we can then use the graph to determine factors, such as elastic limit, yield stress, percentage elongation and modulus of elasticity.

These are some of the formulae we could use:

$$\text{Nominal breaking stress} = \frac{\text{Load at breaking point}}{\text{Original cross} - \text{sectional area}}$$

$$\text{Actual breaking stress} = \frac{\text{Load at breaking point}}{\text{Reduced cross} - \text{sectional area at fracture}}$$

$$\text{Yield stress} = \frac{\text{Load at yield point}}{\text{Original cross} - \text{sectional area}}$$

$$\text{Ultimate stress} = \frac{\text{Maximum load}}{\text{Original cross} - \text{sectional area}}$$

$$\text{Working stress} = \frac{\text{Actual axial load}}{\text{Original cross} - \text{sectional area}}$$

$$\text{Percentage reduction is area} = \frac{\text{Original area} - \text{final area}}{\text{original area}}$$

$$\text{Percentage elongation} = \frac{\text{Final length} - \text{original length}}{\text{Original length}}$$

$$\text{Factor of safety (FoS)} = \frac{\text{Ultimate stress}}{\text{Working permissible stress}}$$

Note: FoS has no units

FoS > 1

Worked example 1.10 **Plot a load-extension curve**

The following results were obtained from a tensile test carried out on a mild-steel test piece with a diameter of 40 mm and with 300 mm between the gauge points.

Load (kN)	50	100	150	155	165	170	185	195	200
Extension (mm)	0,08	0,2	0,3	0,35	0,5	0,8	0,85	0,9	0,99

Load (kN)	215	225	230	240	250	260	230
Extension (mm)	1,05	1,5	1,6	1,8	2,4	3,9	4.9

Assume that the change in cross-sectional area is so small that is not considered in the early part of the test.

1. Plot the load-extension curve.
2. Use the curve to determine the following results:
   a) the modulus of elasticity
   b) the yield stress
   c) the ultimate stress
   d) the percentage elongation
   e) the allowable working stress, with a factor of 3 for the yield stress.

Solution

Given: Original area = $\frac{\pi d^2}{4} = \frac{\pi}{4} \times 0{,}04^2 = 0{,}001256637\ m^2 = 125{,}664 \times 10^{-5}\ m^2$

1. Load-extension curve

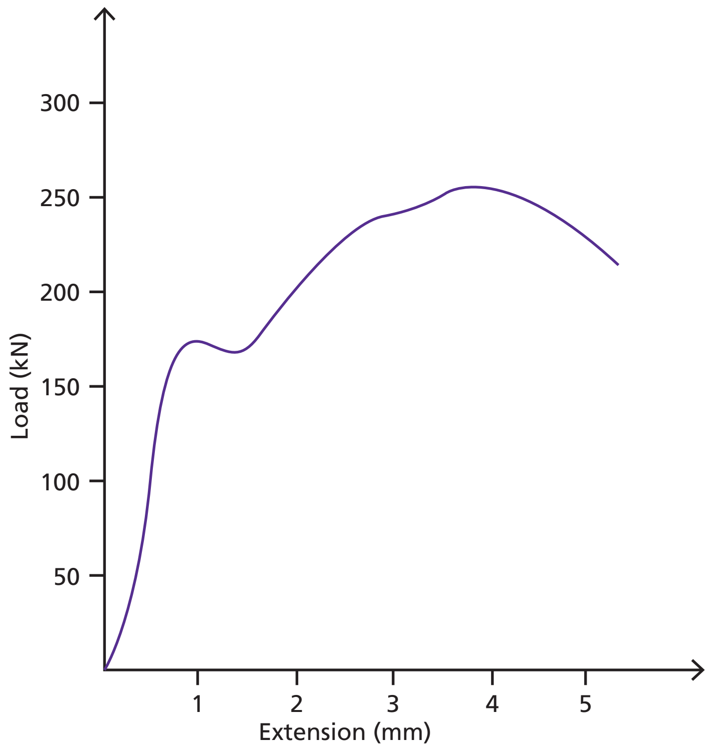

Figure 1.13 Load-extension curve for Worked example 1.10

2. Results obtained from the curve
   a) Modulus of elasticity
   The modulus of elasticity is calculated within the straight-line part of the graph.

$$E = \frac{F \times L\text{ (original)}}{\text{Original area} \times x}$$

$$= \frac{100 \times 10^3 \times 0{,}3}{125{,}664 \times 10^{-5} \times 0{,}2 \times 10^{-3}}$$

$$= \frac{30\ 000}{0{,}000000251}$$

$$= 119{,}522\text{ GPa}$$

   b) Yield stress
   Yield stress is determined from the graph where the length quickly becomes longer, namely, between 155 kN and 165 kN.

$$\sigma\text{ Yield point} = \frac{F\text{ yield}}{A\text{ original}}$$

$$= \frac{155 \times 10^3}{125{,}664 \times 10^{-5}}$$

$$= 123{,}345\text{ MPa}$$

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c) Ultimate stress

Find the ultimate stress by using maximum force:

$$\sigma \text{ ultimate} = \frac{F \text{ ultimate}}{A \text{ original}}$$

$$= \frac{260 \times 10^3}{125{,}664 \times 10^{-5}}$$

$$= 206{,}9 \text{ MPa}$$

d) Percentage elongation

$$\% \text{ elongation} = \frac{x}{L \text{ original}} \times 100\%$$

$$= \frac{4{,}9}{300} \times 100\%$$

$$= 1{,}63\%$$

e) Allowable working stress

$$\sigma \text{ working} = \frac{\sigma \text{ yield point}}{\text{FoS}}$$

$$= \frac{123{,}345}{3}$$

$$= 41{,}115 \text{ MPa}$$

Activity 1.1 SINGLE Stresses

1. Name the THREE types of stress.
2. A steel bar supports a load of 60 kN. The bar has a diameter of 30 mm. Calculate the tensile stress in megapascals.
3. Calculate the diameter of a steel rod supporting a load of 150 kN, without exceeding stress of 120 MPa.
4. A round metal rod has a diameter of 60 mm and a length of 800 mm. The rod hangs vertically and supports a load of 70 kN hanging from it.
   a) Calculate the tensile stress in the rod.
   b) Calculate the strain of the rod if a new length.
5. State Hook's law.
6. A 5 m long wire is 2 mm in diameter. When stretched by a weight of 20 kg, its length is increased by 0,9 mm.
   Calculate:
   a) the stress brought in the wire
   b) the strain in the wire
   c) Young's modulus of a material of the wire. Take g as 9,81m/s^2.
7. A 2 m hollow brass tube has a diameter ratio of 0,6. A tensile load of 100 kN acts on the tube, which can withstand maximum stress of 120 MPa.
   Calculate:
   a) the diameter of the tube
   b) the change in length of the tube if the modulus of elasticity is 170 GPa.
8. Draw a neat sketch of a typical stress/strain graph for mild steel in good proportions. Label all the main points on the graph.
9. What does a stress/strain curve show?

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10. The following tables give details of a tensile test conducted on a small piece with a diameter of 50 mm and with 280 mm between the gauge points.

Load in kN	60	110	150	160	170	180	210
Extension in (mm)	0,09	0,2	0,3	0,5	1,8	0,8	0,85

Load in kN	215	225	230	245	260	220
Extension in (mm)	1,05	1,05	1,5	1,8	3,9	4,8

If the change in cross-sectional area is so small that it is not considered in the early part of the test, plot the load extension graph, and determine the following results.

a) the modulus of elasticity
b) the yield stress
c) the ultimate stress
d) the percentage elongation
e) the allowable working stress with a factor of 2 for the yield stress.

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