

Nigeria

Primary Maths

Grade 6

Teacher's guide

Pearson Education Limited

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England
and Associated Companies throughout the world

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How to use this course

The *New General Mathematics* Primary 6 Pupil's Book (PB) consists of 33 units. Each unit starts with a list of objectives, or commonly known as performance objectives (as listed in NERDC, 2013) that will be covered in each unit.

In addition, the exercises in the PB have been carefully developed to ensure integration of the performance objectives from the curriculum, and a steady progression of skills throughout the year.

It is important that you follow the order of the units, especially for related sub-topics, as units build on the knowledge and skills acquired in preceding units.

The units follow a 'teach and practise' approach:

- New concepts are explained and given context in their meaning.
- Worked-through examples show pupils how to approach problem solving.
- Exercises allow pupils to practise on their own.
- Revision exercises round off each unit as a mixed exercise covering all the problems addressed in the unit.

Summative assessment activities are provided at the end of every term in the form of Term assessments, along with a term project. These assessments test pupils on all the knowledge and skills they have gained in each term, and the projects enable the pupils to apply the work they have learnt in practice.

Additional features include:

- Key words: Key terminology is highlighted for the pupils. Definitions are given in the PB and in the Teacher's Guide (TG).
- Puzzles: Additional problems, usual in a real-life context to help grow an appreciation of mathematics in everyday life.
- Challenges: extension problems for stronger pupils to attempt. These exercises generally

extend the scope of content covered in each unit.

- Teaching notes: advice and ideas for teachers in dealing with the content on each page.

Features of the Teacher's Guide

This *New General Mathematics* Primary 6 TG is lesson-based. The units of the PB are organised into a series of lessons. Units include most of the following features:

- The performance objectives from the curriculum that are covered in the unit
- A list of suggested resources you will need
- Definitions for the key words in the PB, as well as some additional key words and their descriptions
- Frequently asked questions relating to teaching the unit's content (not always applicable)
- Common errors pupils make (not always applicable)
- An evaluation guide showing the key learning milestones.

Each lesson includes the following:

- Preparation for the lesson (all the suggested resources) – remember, these can be tailor-made to suit the requirements of your classroom situation.
- A starter activity which helps you focus on the topic, or revise previous required knowledge.
- Lesson focus which suggests how you should teach the lesson, and the main strategies you can incorporate.
- Answers to all exercises, puzzles and challenges in the PB and Workbook (WB)
- Assessment guidance on how to effectively assess pupils in each lesson
- Extension activities (not always applicable)
- Suggestions for homework activities, where necessary.

Note: The lesson-based guidelines are *suggestions* only. You, as the teacher, will need to assess how much your pupils are able to cover in each lesson.

Features of the Workbook

The *New General Mathematics* Primary 6 WB provides a worksheet for every unit in the PB. Pupils use these worksheets to practise the specific mathematical skills and concepts covered in each unit. It forms as a consolidation of the pupils' understanding and is a useful resource for homework assignments.

Pupils can record their answers and calculations in the spaces provided on each of the worksheets.

The answers to these worksheets are all provided in the TG.

Methodology

Mathematics teaching and learning goes beyond reaching the correct answer. Many mathematical problems have a range of possible answers. Pupils need to understand that Mathematics is a *tool* for solving problems in the real world; not just about giving the correct answers.

The Mathematics classroom must therefore provide an environment in which problem-solving is seen as integral to the teaching programme, and where learning activities are designed to provide pupils with opportunities to think.

Working mathematically involves:

- questioning
- applying strategies
- communicating
- reasoning
- reflecting.

Pupils will require some, or all of the above processes, to make sense of any mathematical concept.

Problem-solving strategies include:

- trial and improvement
- acting it out
- making a model
- drawing a diagram or picture

- looking for patterns
 - working backwards (inverse operations)
 - using tables and data
 - making a list.

Primary level 6 focuses primarily on strengthening the first five strategies listed above, and then builds on the other strategies.

Alongside developing these problem-solving strategies, it is important for pupils to gain specific mathematical knowledge as tools for problem-solving. At Primary level 6, these tools include:

- counting in millions and billions
- writing and reading numbers up to one million
- working with place value
- giving the value and place value for a digit in a decimal fraction
- finding the LCM of 2-digit whole numbers
- finding the HCF of 2-digit whole numbers
- working with demography (reading, writing and comparing populations in small and big cities, and population of HIV prevalence in different countries; using counting in thousands and millions in population studies)
- ordering fractions; expressing decimals as fractions and vice versa
- working with ratios and proportions
- working with ratio sizes of families and resources
- working with ratio of two populations
- working with ratio of prevalence of HIV/AIDS between two sexes and two states
- working with percentage (expressing one number as a percentage of another; calculating numbers expressed as a percentage of another; expressing one population as a percentage of another population; calculate percentage increases; applying profit and loss percent to business)
- adding any set of numbers
- working with multiplication (multiplying 3-digit by 3-digit numbers; multiplying decimal by decimal; multiplying fraction by fraction; calculating the squares of numbers up to 500; calculating the square roots of perfect squares)

- working with dividing (dividing whole numbers and decimals by 2-digit and 3-digit numbers)
- writing numbers in index form
- using basic operations in the right order
- working with open sentences and word problems
- working with money (using a calculator to convert from one currency to another)
- working with length (using the Pythagoras rule to find the unknown length of a right-angled triangle)
- working with weight (expressing the same weight in different units: grams, kilograms and tonnes)
- working with time (telling time in seconds and minutes; reading timetables of journeys especially trains and aeroplanes; using time to plan daily activities)
- working with perimeter
- working with area (calculating the areas of figures which can be divided into rectangles, and/or triangles; calculating land areas in hectares)
- working with volume (calculating volumes of prisms, cylinders and spheres)
- working with capacity (using standard units)
- working with speed (calculating average speed)
- estimating time to complete athletics races
- identifying basic properties of plane figures such as a rectangle and square
- working with angles (measuring angles in degrees and in a plane)
- working with height and distances (measuring heights of human beings, buildings, trees and distances)
- working with scale (drawing plans according to a scale; applying and using scale drawings in converting lengths and distances of objects in the environment to scale)
- interpreting and using pictograms and bar graphs
- finding the mode of data
- calculating the mean of given data.

Curriculum Matching Chart

NERDC Topic	Performance Objective	Pupil's Book Unit	PB Pages	WB Pages
Theme 1: Number and numeration		Sub-theme: Whole numbers		
1. Whole numbers	1. Count in millions and billions	Unit 1 Counting in millions and billions	9	5
	2. Write and read up to one million	Unit 2 Writing and reading numbers up to one billion	17	6
	3. Solve problems involving quantitative reasoning			
	4. Give the place value and the value of a digit in a given whole number	Unit 1 Counting in millions and billions	9	5
	5. Identify numbers in place value			
	6. Solve problems on quantitative reasoning with value and place value			
	7. Give the value and place value for a digit in a decimal fraction			
	8. Solve quantitative aptitude problems related to place value			
	9. Find LCM of 2-digit whole numbers	Unit 3 Highest Common Factor and Lowest Common Multiple	22	7
	10. Find HCF of 2-digit whole numbers			
	11. Solve quantitative aptitude problems on LCM and HCF			
2. Demography	1. Read, write and compare populations of small and big cities	Unit 4 Demography	28	9
	2. Read and compare populations of HIV prevalence in different countries			
	3. Use counting in thousands and millions in population studies			
Theme 1: Number and numeration		Sub-theme: Fractions		
1. Fractions	1. Order fractions	Unit 5 Ordering and comparing fractions	34	11
	2. Solve problems on quantitative reasoning			
	3. Express decimals as fractions and vice versa	Unit 6 Converting between decimals and fractions	40	12
2. Ratios and proportions	1. Solve problems on ratio	Unit 7 Ratio and proportion	46	14
	2. Apply ratio to everyday life			
	3. Solve quantitative reasoning problems involving ratio			
	4. Solve problems in direct proportion			
	5. Solve problems on quantitative reasoning involving direct proportion			
	6. Solve problems on inverse proportions			
	7. Identify some daily life activities that are inversely related			
	8. Solve problems on quantitative reasoning in inverse proportions			

NERDC Topic	Performance Objective	Pupil's Book Unit	PB Pages	WB Pages
Theme 2: Basic operations		Sub-theme: Derived operations		
1. Indices (power)	1. Write numbers in index form	Unit 16 Indices	118	26
	2. Solve problems involving powers (indices)			
	3. Solve problems on quantitative reasoning involving indices			
2. Ratio and percentage	1. Solve problems on ratios	Unit 17 Ratio and percentage	124	28
	2. Express a number as a percentage of another number			
	3. Solve problems on profit and loss percent			
	4. Apply the profit and loss percent to business			
	5. Solve problems on quantitative aptitude on ratio, percentage, profit and loss			
Theme 3: Algebraic processes		Sub-theme: Open sentences		
1. Open sentences	1. Solve problems expressed as open sentences	Unit 18 Open sentences	132	30
	2. Interpret words in open sentences and solve them			
	3. Solve related problems on quantitative aptitude			
Theme 4: Mensuration and geometry		Sub-theme: Primary measures		
1. Money (a) Population and economic consequences	1. Solve problems on taxes and rates	Unit 19 Money	138	32
	2. Solve problems involving buying and selling of shares and dividends			
	3. Solve problems on quantitative aptitude based on taxes and rate as well as selling of share and dividends			
1. Money (b) Conversion of currencies	1. Use a calculator to convert from one currency to another			
	2. Solve problems on quantitative aptitude based on converting from one currency to another			
2. Length	1. Use Pythagoras' rule to find the unknown length of a right angled triangle	Unit 20 Length	148	36
	2. Solve many quantitative aptitude problems involving Pythagoras' rule			
3. Weight	1. Express the same weight in different units: gram, kilogram, tonne	Unit 21 Weight	153	37
	2. Solve word problems on weights			
4. Time	1. Tell time in seconds and minutes	Unit 22 Time	157	39
	2. Solve quantitative aptitude problems on time			
	3. Read timetable of journeys especially by trains and aero planes			
	4. Use time to plan daily activities			

NERDC Topic	Performance Objective	Pupil's Book Unit	PB Pages	WB Pages
Theme 4: Mensuration and geometry		Sub-theme: Secondary measures		
1. Perimeter	1. Discover that different rectangles with same area have different perimeters	Unit 23 Perimeter	169	40
2. Area	1. Calculate the areas of figure which can be divided into rectangles and or triangles	Unit 24 Area	172	41
	2. Calculate land areas in hectares			
3. Volume	1. Calculate volume of prisms, cylinders and spheres	Unit 25 Volume and capacity	179	43
	2. Solve some quantitative aptitude problems on volume of prism, cylinder and sphere			
4. Capacity	1. To use standard units			
	2. Solve word problems involving litres			
5. Speed	1. Calculate average speed	Unit 26 Speed and athletics time	186	45
	2. Solve quantitative aptitude problems on time and speed			
6. Athletics time	1. Estimate time to complete races			
Theme 4: Mensuration and geometry		Sub-theme: Shapes		
1. Plane figures	1. Identify the basic properties of plane figures such as: rectangle and a square	Unit 27 Plane figures	190	48
2. Angles	1. Measure angles in degrees	Unit 28 Angles	193	51
	2. Measure angles in a plane			
3. Height and distances	1. Measure heights of human beings, buildings, trees and distance	Unit 29 Height and distance	199	53
4. Polygons	1. Solve more problems on two dimensional and three dimensional shapes	Unit 30 Polygons	202	55
5. Scale drawing	1. Draw plans according to a given scale	Unit 31 Scale drawing	212	58
	2. Apply and use scale drawing in converting lengths and distances of objects in his/her environment to any scale			
Theme 5: Everyday statistics		Sub-theme: Data collection and presentation		
1. Population	1. Interpret pictograms and bar graphs	Unit 32 Populations	218	60
	2. Use pictograms and bar graphs in representing population of people or data			
2. Measures of central tendency	1. Find the mode of data	Unit 33 Measures of central tendency	223	62
	2. Calculate the mean of given data			

Objectives

By the end of this unit, each pupil should be able to:

- Count in millions and billions
- Determine place value and value of digits
- Give the value and place value for a digit in a whole number
- Give the value and place value for a digit in a decimal fraction
- Solve problems on quantitative reasoning.

**Suggested resources**

dice; bottle tops; base 10 sheet.

**Frequently asked questions**

Q *Why is place value important?*

A It is place value that allows us to use just ten symbols (the digits 0 to 9) to write numbers of any size, including decimals. Also the relative values of the digits are the basis for all the arithmetical operations involving regrouping.

Q *What use do decimals have in real life?*

A Decimals are met in all areas of measurement. Most money systems use numbers to two decimal places. Athletics and other sporting events use decimals to two and three decimal places to record results in time and distance. There are also many other uses involving mass, length and capacity that require decimals.

Q *What is the difference between decimals, decimal numbers and decimal fractions?*

A A decimal or decimal number is a number with digits to the right of the point. The section to the right of the point is the decimal fraction.

**Evaluation guide**

Pupils to:

1. Count in millions to a specified number.
2. Write up to one million.
3. Solve problems on quantitative reasoning.
4. Give the value and place value of each digit in a given number.

Lesson 1 *Pupil's Book page 9***Preparation**

You will need to have: Pupil's Book.

**Starter activity**

Start with instructions such as:

“Write a five-digit number with a 6 in the hundreds place, a 2 in the units and a 7 in the thousands place.”

“What could you subtract from 3 456 789 to make 3 406 789?”

**Lesson focus**

Discuss the idea of counting in different sequences. The focus has now moved to very large numbers. Remind pupils that skip counting means following a pattern. Here are some examples: Start with 0. Skip count by 9. Read the numbers each time.

Start with 99. Skip count by 99.

Start with 999. Skip count by 999.

Start with 11. Skip count by 11.

Start with 111. Skip count by 111.

Count in thousands starting from 98 000.

Count in ten-thousands, starting from 998 000.

Use a given number as a starting point, count by adding hundreds, or thousands, or ten thousands, or hundred thousands, or millions.

Answers

Challenge page 9

1. Count in 1s
2. Count in 2s, or its multiples.
3. Count in 3s, or its multiples.
4. Count in 2s, or its multiples.
5. Count in 5s, or its multiples.
6. Count in 2s or 3s, or their multiples.
7. Count in 7s.

Exercise 1

1. Check pupils' counting intervals.

Assessment

Pupils should be able to count forwards and backwards in millions starting from any number.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Skip count to millions

Support activity

Interactive learning diagram: Odometer

Homework activity

Find some large numbers – from newspapers, or on the internet. Explain to the class what they stand for.

Lesson 2 *Pupil's Book page 10*

Preparation

You will need to have: Pupil's Book; dice.

Starter activity

Roll a dice three times (or turn over three one-digit cards) to generate the millions, hundred thousands and ten thousands of seven-digit numbers. Use these to complete a set of six random seven-digit numbers (or continue to use the dice or cards to generate the rest of the number). Put these numbers in order.

Lesson focus

There are three key ideas in this lesson:

Place value: As you move to the left in a big number, each digit is worth ten times as much as the digit on its right.

Grouping by thousands: In a big number the digits are grouped into threes to make it easier to read and grasp the meaning of the number. So for a number with at least nine digits:

hundreds, tens and ones of billions
hundreds, tens and ones of millions
hundreds, tens and ones of thousands and
hundreds, tens and ones.

Order of numbers: with different numbers of digits.

Answers

Exercise 2

1. a)–e) Check pupils' counting intervals.
2. a)–e) Check pupils' counting intervals.

Assessment

Pupils should be able to identify numbers in place value.

Solve problems on quantitative reasoning with value and place value.

Assess their performance in Exercise 2, games and activities.

Extension activity

Interactive learning diagram: Skip count to millions

Lesson 3 *Pupil's Book page 11; Workbook page 5*

Preparation

You will need to have: Pupil's Book; Workbook; dice; bottle tops.

Starter activity

Play a game where pupils must try to make the highest number they can using dice or up to 10 cards (billions).

Lesson focus

The way place value works is that each digit is worth 10 of the digit to its right. For example tens are worth 10 times ones (units), and hundreds are worth 10 times tens. The paper abacus shows this clearly.

Count forwards:

The important idea is that the next number after 9 is 10, with the tens digit going up to 1 at the same time as the ones digit drops down to 0.

So when you count from 29 to 30, the tens digit goes up (from 2 to 3) at the same time as the ones digit drops back to 0.

And when you count from 99 to 100, the hundreds digit goes up (0 to 1), and the tens and ones both drop to 0.

Skip count

There are attractive patterns produced when you skip count. For example try counting in 5s. Skip count in 11s (11, 22, 33 ...) and then in 9s (9, 18, 27, 36 ...).

Answers

Puzzle page 12

Dangan lives in Bariga. Biodun lives in Mushin. Ayo lives in Oripanu.

Chukwuma lives in Surulere. Eze lives in Yaba.

Exercise 3

1.

	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units
a)				2	4	3	7	1	5
b)				6	3	5	7	9	5
c)			8	5	6	2	3	7	6
d)		1	9	1	4	3	8	3	5
e)	9	2	0	3	6	1	7	2	5

2. a) 500 b) 5
 c) 50 d) 50 000 000
 e) 500 000 000

Worksheet 1

- a)–c) Check pupils' counting intervals.
- The digit representing the hundred thousand is 2.
 The digit representing the billion is 2.
 The digit representing the million is 6.

Assessment

Pupils should be able to give the place value and the value of a digit in a given whole number.

Assess their performance in Exercise 3, games and activities.

Extension activity

Interactive learning diagram: Skip count to billions

Support activity

Interactive learning diagram: Odometer

Homework activity

Complete WB Worksheet 1.

Lesson 4 *Pupil's Book page 12*

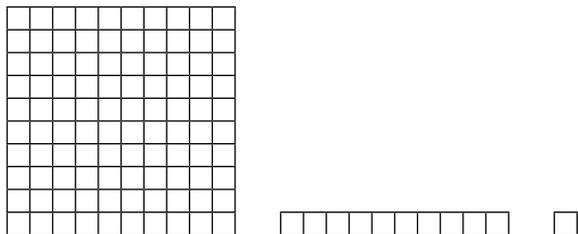
Preparation

You will need to have: Pupil's Book; base 10 sheet; decimal place value squares and strips.

Starter activity

Use decimal place value squares and strips to represent decimal fractions.

The Base 10 sheet includes a large square (for 1), strips for tenths (0.1) and small squares for hundredths (0.01).



Show these to the pupils and explain clearly how they work. Give a set to each group if that helps.

Order of decimals

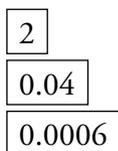
Pupils should use a paper model to put these decimals into order, smallest first.

0.10, 1, 1.1, 0.1, 0.11, 0.01, 1.10, 1.01, 1.11

Lesson focus

The basic idea is that *place value* continues to the right of the decimal point. The way place value works is that each digit is worth 10 of the digit to its right. For example ones (units) are worth 10 times tenths, and tenths are worth 10 times hundredths.

There is a set of place value cards using decimal points that show how the place value of each digit is represented. Use these for pupils who are not sure about place value ideas. Put them together to make many numbers, but you can leave gaps. For example:



becomes 2.0406

Skip counting

Continue these patterns:

0, 0.1, (0.2, 0.3, 0.4, ...)

0, 0.01, (0.02, 0.03, 0.04, ...)

0, 0.5, (1, 1.5, 2, 2.5, 3, ...)

0, 0.4, (0.8, 1.2, 1.6, 2, 2.4, 2.8, 3.2, ...)

0, 0.04, (0.08, 0.12, 0.16, 0.2, 0.24, 0.28, 0.32, ...)

0, 0.09, (0.18, 0.27, 0.36, 0.45, 0.54, ...)

Putting decimals in order

There are two common ways of misunderstanding.

Pupils with the 'longer is larger' misunderstanding simply choose the decimal with the most digits as the larger one – they choose as if they were whole numbers. They ignore place value.

Pupils with the 'shorter is larger' misunderstanding sometimes confuse the number after the point as the denominator of a fraction. They might tell you that 0.4 means one quarter, so therefore 0.8 is one eighth and therefore smaller.

Any pupils with these kinds of misunderstandings need help – NOW.



Answers

Challenge page 13

Check the two numbers that the pupils make. "Close to 50" can mean between 48 and 52.

Exercise 4

1.

	Ten thousands	Thousands	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
a)					4	.	3	7	1	5	
b)				6	3	.	4	7	9	8	
c)			3	4	6	.	1	2	7	8	
d)		2	9	1	4	.	3	8	7	5	6
e)	8	2	0	3	6	.	1	9	7	4	5

2. a) 0.005 b) 0.0005 c) 0.05
 d) 0.00005 e) 0.000005

Exercise 5

1. Pattern: +1 000 000 000

8 576 876 987 9 576 876 987
 10 576 876 987

2. Pattern: +1

34 987 765 234 34 987 765 235
 34 987 765 236

3. *Pattern: +1 000 000 000*
 16 595 043 427 □ 17 595 043 427
4. *Pattern: -1 000 000 000*
 35 765 793 097 □ 34 765 793 097

Number	Hundreds	Tens	Units	.	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths	Ten millionths	Hundred millionths	Billionths
1.023 456 789			1	.	0	2	3	4	5	6	7	8	9
458.987 6	4	5	8	.	9	8	7	6					
74.987 657 432		7	4	.	9	8	7	6	5	7	4	3	2
4.008 097			4	.	0	0	8	0	9	7			
1.000 07			1	.	0	0	0	0	7				

Assessment

Pupils should be able to give the value and place value for a digit in a decimal fraction. Assess their performance in: Exercises 4 and 5, games and activities.

Extension activity

Interactive learning diagrams: Ordering decimals, Reading a scale

Support activity

Interactive learning diagrams: Skip count thousandths, Decimal counting, Estimating decimals

Lesson 5 *Pupil's Book page 15*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.



Answers

Revision exercise page 15

- 111 432 765, 2 111 432 765, 3 111 432 765, 4 111 432 765, 5 111 432 765, 6 111 432 765
 - 452 435 867 341, 453 435 867 341, 454 435 867 341, 455 435 867 341, 456 435 867 341
- a)–c) Check pupils' counting intervals
- a)–e) Check pupils' counting intervals
- a)–e) Check pupils' counting intervals
- 2 000 000, 50 000 000
 - 2 000 000 000, 500 000
 - 20 000 000 000, 5 000 000 000, 500 000 000
 - 2 000, 500 000 000 000
- 700 000, 300 000 000 000
 - 700 000 000, 30 000 000 000
 - 700 000, 30 000 000
 - 700 000 000 000, 300 000 000

7.

	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
a)				2	4	3	7	1	5
b)				6	3	5	7	9	5
c)			8	5	6	2	3	7	6
d)		1	9	1	4	3	8	3	5
e)	9	2	0	3	6	1	7	2	5

- 0.07, seven hundredths, 0.00003, three hundred thousandths
 - 0.000007, seven millionths, 0.3, three tenths
 - 0.007, seven thousandths, 0.000000003, three billionths

Objectives

By the end of this unit, each pupil should be able to:

- Write and read numbers up to one billion in words
- Write and read numbers up to one billion in figures
- Solve problems involving quantitative reasoning.

**Suggested resources**

number expander (millions); number expander (billions)

**Frequently asked questions**

Q *What do pupils need to understand?*

A As for Unit 1, the basic ideas of understanding place value, and the grouping of digits in threes for reading, are important.

Q *What mathematical skills are needed?*

A Reading and writing numbers in numerals and words are essential skills.

**Evaluation guide**

Pupils to:

1. Count in millions to a specified number.
2. Solve problems on quantitative reasoning.
3. Give the value and place value of each digit in a given number.

Lesson 1

Pupil's Book page 17; Workbook page 6

**Preparation**

You will need to have: Pupil's Book; Workbook; number expander (millions).

**Starter activity**

Practise writing numbers up to 100 as words, correcting any spelling as it arises. Then say numbers up to ten thousand and pupils should write down what they have heard. At this point it is important to correct any misunderstandings before

you move on to millions, especially when there are zero place holders in the number, such as 70 005.

**Lesson focus**

Extend correct reading and writing up to one million.

Number expander to millions

Make copies and cut the strips so that each group gets one.

1 million 2 3 4 thousands 5 6 7

You can fold it on the lines. Then you can show just the digits (1,234,567) or the full English name for the number, as spelled out on the strip.

Make sure pupils can give you the value of each digit, and write it correctly.

1,000,000
200,000
30,000
4,000
500
60
7

Use a template of a blank number expander, so that your pupils can write their own digits to show numbers.

**Answers****Challenge page 17**

159923, 343724, 4128835, 21154756,
876898034, 909909909

Exercise 1

1. a)–f) Check pupils' counting intervals
2. a)–e) Check pupils' counting intervals

Worksheet 2

1. Check pupils' counting intervals

Assessment

Pupils should be able to write and read up to one billion.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Powers of 10 (x)

Support activity

Interactive learning diagram: Skip count to billions

Homework activity

Work through WB Worksheet 2, Question 1.

Lesson 2 *Pupil's Book page 18; Workbook page 6*



Preparation

You will need to have: Pupil's Book; Workbook; number expander (billions).



Starter activity

Revise numbers to 10 000. State a number and ask pupils to write it down in words. Include some numbers which have zeroes as place holders, such as 306 or 2 060.



Lesson focus

Ensure that pupils use the correct language for large numbers.

Number expander to billions

Make enough copies of a number expander to use with your class. Cut the strips so that each group gets one. Explain their use.



Answers

Puzzle page 18

245 484

Exercise 2

1. a)–e) Check pupils' counting intervals
2. a)–e) Check pupils' counting intervals

Assessment

Pupils should be able to write and read up to one billion.

Assess their performance in Exercise 2 and activities.

Extension activity

Interactive learning diagram: Roman numerals

Support activity

Interactive learning diagram: Overlay cards

Homework activity

Complete WB Worksheet 2, Question 1.

Lesson 3 *Pupil's Book page 19; Workbook page 6*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Revise numbers to 10 000. State a number and ask pupils to write it down in figures. Include some numbers which have zeroes as place holders, such as 507 or 2 040.



Lesson focus

Ensure that pupils use the correct language for large numbers and understand place value.



Answers

Exercise 3

1. a)–e) Check pupils' counting intervals
2. a)–e) Check pupils' counting intervals

Exercise 4

1. The figures on the right have been moved to match the written numbers.

Ninety billion, one hundred and sixty-three million, forty-five thousand, three hundred and twenty-one	90 163 045 321
Ten billion and ten	10 000 000 010
Eight hundred billion, eight million, four thousand, five hundred and six	800 008 004 506
Four billion, two hundred and fifteen	4 000 000 215
Nine billion nine million, nine thousand, nine hundred and nine	9 009 009 909
One hundred and eleven billion, one hundred and eleven	111 000 000 111

2. a)–e) Check pupils' counting intervals

Worksheet 2

2. Check pupils' counting intervals
- 3.

Four million, one hundred and twenty thousand, nine hundred and six	4 120 906
One billion seven hundred and two thousand and ten	1 000 702 010
Forty million and thirty thousand and forty four	40 030 044
Two hundred thousand and eighty eight	200 088
Sixty six thousand and sixty six	66 066
Two thousand four hundred and fifty nine	2 459

Assessment

Pupils should be able to write and read up to one billion.

Assess their performance in Exercises 3 and 4.

Extension activity

Interactive learning diagram: Roman numerals

Support activity

Interactive learning diagram: Overlay cards

Homework activity

Complete WB Worksheet 2, Questions 2 and 3.

Lesson 4 *Pupil's Book page 21*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.

Answers

Revision exercise page 21

- a)–b) Check pupils' counting intervals
- a)–b) Check pupils' counting intervals
- a)–d) Check pupils' counting intervals
- a) 500 000, 20
b) 500, 2 000 000
c) 5 000 000 000, 20 000 000 000
d) 5 000 000 000, 2 000
- a) 700 000, 30 000 000 000
b) 700 000 000, 30
c) 70 000, 3 000 000 000
d) 7 000 000 000, 3 000 000

Objectives

By the end of this unit, each pupil should be able to:

- Find the factors of a number
- Find HCF of 2-digit whole numbers

- Find multiples of a given number
- Find LCM of 2-digit whole number.

**Suggested resources**

playing cards; counters; dice; checkmath board

**Common errors that pupils make**

Pupils confuse HCF with LCM.

Since a number is a multiple of its factors, this is easily confused. Multiples are the answers to a multiplication. Factors are numbers that divide exactly into a number.

**Evaluation guide**

Pupils to:

1. Find the multiples of given 2-digit numbers.
2. Find the LCM of 2-digit numbers using the factor method and the multiples method.
3. Find the factors and HCF of given whole numbers.
4. Solve quantitative aptitude problems involving LCM and HCF.

Lesson 1

Pupil's Book page 22; Workbook page 7

**Preparation**

You will need to have: Pupil's Book; Workbook; counters; dice.

**Starter activity**

Play 'What's my number?' focusing on questions involving 'multiple' and 'divisibility'.

Here are some examples: "My number is divisible by 6 and is between 20 and 30; my number is a multiple of 4 between 30 and 40; what could it

be?" Continue with questions such as: "My number is divisible by 3 and by 5 and it is less than 20; my number is a multiple of both 4 and 7 and is more than 50."

**Lesson focus****Put counters into rectangles**

Factors multiply to form the product.
The product is a multiple of the factors.
Look at $3 \times 5 = 15$.

The 3 and 5 are factors. The 15 is the product and is also a multiple of 3 and a multiple of 5. (The 15 is also a multiple of 15 and of 1.)

List all the factors of selected numbers.

**Answers****Puzzle page 22**

11 chairs in each row

Exercise 1

1. a) 2; b) 3; c) 2; d) 2; e) 3; f) 1
2. a) 6; b) 36; c) 40; d) 18; e) 56; f) 72

Worksheet 3

1. 20: 1; 2; 4; 5; 10; 20
23: 1; 23
53: 1; 53
64: 1; 2; 4; 8; 16; 32; 64
65: 1; 5; 13; 65
72: 1; 2; 3; 4; 6; 8; 9; 12; 18; 24; 36; 72
80: 1; 2; 4; 5; 8; 10; 16; 20; 40; 80

81: 1; 3; 9; 27; 81
 85: 1; 5; 17; 85
 99: 1; 3; 9; 11; 33; 99

2.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Assessment

Pupils should be able to find the factors of a number.

Assess their performance in Exercise 1 and games.

Extension activity

Interactive learning diagram: Factorgrams

Support activity

Interactive learning diagram: Common factors, Prime sieve

Homework activity

Complete WB Worksheet 3, Questions 1 and 2, page 7.

Lesson 2 *Pupil's Book page 22; Workbook page 7*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Subtract-a-factor

Two players take turns to write a number on a piece of paper. One player chooses a number to start. The next player must find a factor of the number (except the number itself), subtract it and write the new number. Then the first player does the same,

and so on. The winner is the person who has the last turn. To win you must leave the number 1. You cannot subtract 1 from 1 in this game, because that is subtracting the number itself!

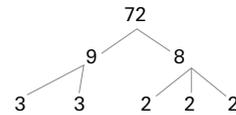


Lesson focus

Revise prime numbers and factors using factor trees.

Factor trees

Factor trees produce the prime factors of any number. To find the prime factors of a number you can use a factor tree method. (It is an upside-down tree!)



This 'tree' shows that the prime factors of 72 are $3 \times 3 \times 2 \times 2 \times 2 = 3^2 \times 2^3$.

The meaning of 3^2 is 3×3 and the meaning of 2^3 is $2 \times 2 \times 2$. *The exponent tells you how many times the factor has been multiplied.*



Answers

Challenge page 23

prime factors of 8 $192 = 2^{13}$

Exercise 2

- a) 5; b) 6; c) 5; d) 8; e) 16; f) 14
- a) 6; b) 24

Worksheet 3

- | | | |
|-------------------|---------------------|---------------------|
| $10 = 25$ | $12 = 2^3$ | $25 = 5^2$ |
| $32 = 2^5$ | $51 = 3 \cdot 17$ | $57 = 3 \cdot 19$ |
| $77 = 7 \cdot 11$ | $88 = 2^3 \cdot 11$ | $99 = 3^2 \cdot 11$ |

Assessment

Pupils should be able to find the HCF of 2-digit whole numbers.

Assess their performance in Exercise 2 and games.

Extension activity

Interactive learning diagrams: Prime factors, HCF

Support activity

Interactive learning diagrams: Factor tree, Prime factor products

Homework activity

Complete Exercise 2.

Complete WB Worksheet 3, Question 3.
page 7.

Lesson 3 *Pupil's Book page 24; Workbook page 8*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Discuss the relationship between factor, multiple and product.

Choose a number (e.g. 24). It is a multiple of what smaller numbers?

(It is a multiple of any of its factors!)



Lesson focus

Patterns in multiples on 100 chart

The diagram shows some shaded multiples of 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Challenge pupils with some thought-provoking ideas.

“Explain why the pattern slopes down to the left.”

“Explain the pattern for the multiples of 2.”

“Explain the pattern for the multiples of 5.”

“Explain the pattern for the multiples of 9.”

“Explain the pattern for the multiples of 11.”

Let the pupils explore similar patterns for other multiples. They will also notice that some numbers are multiples of many numbers.

Patterns in common multiples on 100 chart

This diagram shows multiples of both 3 and 4.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Some numbers appear as multiples of both 3 and 4.
“What multiples are these?”

Predict and check: “What numbers are multiples of both of these numbers?”

- 3 and 5
- 3 and 7
- 4 and 5
- 4 and 7

Predict and check: “What numbers are multiples of both of these numbers?”

- 3 and 6
- 3 and 9
- 4 and 8
- 4 and 12

Predict and check: “What numbers are multiples of both of these numbers?”

- 4 and 6
- 6 and 9
- 6 and 8
- 9 and 12

Pupils should discover that the common multiple is only the product (result of multiplying) when the two numbers have no common factor (that is, there is no number that divides into both of them).

If there is a common factor then to find the common multiple we must divide the product by the common factor.

Find the LCM of two numbers using prime factors

The LCM will include only as many of each prime factor as is needed to include the two numbers as factors.

Relationship between LCM and HCF

For any two numbers: $LCM \times HCF = \text{product}$

Answers

Puzzle page 24

120 seconds, or 2 minutes

Exercise 3

1. a) 72; b) 440; c) 96; d) 210; e) 200; f) 1 980

2. a) 60; b) 144; c) 3 960; d) 198

Challenge page 25

1. 250; 2. 900; 3. 700; 4. 6 000

Exercise 4

Arrow pointing up points to the HCF. Arrow pointing down points to the LCM.

1. 12; 2. 360; 3. 15; 4. 576; 5. 5; 6. 672

Worksheet 3

4. 4 and 5: 20 4 and 8: 8
20 and 30: 60 21 and 90: 630
49 and 63: 441 80 and 90: 720
5. 4 and 8: 4 15 and 25: 5
20 and 28: 4 25 and 90: 5
6 and 63: 3 36 and 72: 36
6. Each daughter gets 3 apples and 4 oranges.
7. The LCM of 6 and 21 is 42.
8. $35 = 15$ stones

Assessment

Pupils should be able to find multiples of a given number.

Pupils should be able to find the LCM of 2-digit whole numbers.

Assess their performance in Exercise 3, games and activities.

Extension activity

Interactive learning diagrams: Multiples, LCM

Support activity

Interactive learning diagrams: Multiple patterns (one number only on 10×10 grid)

Homework activity

Complete WB Worksheet 3 page 8.

Lesson 5 *Pupil's Book page 26*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.

Answers

Revision exercise page 26

1. a) 1, 2, 4; b) 1, 2, 3, 4, 6, 12; c) 5; d) 1, 2, 4
2. a) $40 = 2 \times 2 \times 2 \times 5$
b) $51 = 3 \times 17$
c) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
d) $72 = 2 \times 2 \times 2 \times 3 \times 3$
e) $88 = 2 \times 2 \times 2 \times 11$
f) $90 = 2 \times 3 \times 3 \times 5$
3. a) 20; b) 6; c) 3; d) 5; e) 25; f) 8
4. a) 6, 12, 18, 24, 30
b) 9, 18, 27, 36, 45
c) 10, 20, 30, 40, 50
d) 15, 30, 45, 60, 75
e) 20, 40, 60, 80, 100
f) 25, 50, 75, 100, 125
5. a) 36, 72, 108, 144, 180
b) 30, 60, 90, 120, 150
c) 24, 48, 72, 96, 120
d) 30, 60, 90, 120, 150
e) 100, 200, 300, 400, 500
f) 105, 210, 315, 420, 525
6. a) 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144
b) 16, 32, 48, 64, 80, 96, 112, 128, 144, 160, 176, 192
c) Common multiples: 48, 96, 144
d) LCM = 48
7. $12 = 2 \times 2 \times 2 \times 3$
 $16 = 2 \times 2 \times 2 \times 2$
LCM = $2 \times 2 \times 2 \times 2 \times 3 = 48$
8. a) 240; b) 120; c) 560; d) 576; e) 150; f) 672
9. Ayomipo crosses at: 3 min, 6 min, 9 min, 12 min, 15 min
Dotun crosses at: 5 min, 10 min, 15 min
They will cross the line together at 15 min.

Objectives

By the end of this unit, each pupil should be able to:

- Read, write and compare populations of small and big cities
- Read and compare populations of HIV prevalence in different countries
- Use counting in thousands and millions in population studies.

**Suggested resources**

demography map of Nigeria; video of populations; World Atlas and some published information from WHO on HIV/AIDS; flip charts; newspapers or internet articles that mention population figures

**Common errors that pupils make**

Pupils often misinterpret these questions. They may need help with understanding the questions. Explain any words that are unfamiliar.

**Evaluation guide**

Pupils to:

1. Define population.
2. Read and write the population of big cities in Nigeria.
3. Read and compare population.
4. Read and compare population of HIV prevalence in some given countries.
5. Give populations of countries in Africa.

Lesson 1 *Pupil's Book page 28; Workbook page 9***Preparation**

You will need to have: Pupil's Book; Workbook.

**Starter activity**

Before opening the book on this lesson, ask each pupil to write down what they think a population is. Discuss the different definitions and then turn to page 237 in the PB for the true definition.

**Lesson focus**

The focus of this lesson is to introduce pupils to real-life mathematics. Pupils should see the value of being able to work with large numbers, especially reading and comparing.

Consider Exercise 1 Question 2. Questions a) to c) lead to the answer to d). Extend this idea to calculate the number of pupils in your state. (You will need figures for the number of schools in your area, and the number of areas in your state.) The answer found will be an approximation. Talk about the assumptions that underlie the calculation, for example, it assumes the same number of classes in all schools, or the same number of schools in each area.

Population figures are always approximations and will be based on certain assumptions. Your pupils should be able to reasonably estimate the range of numbers depending on the type of population. For example, the population of your house would be counted in ones, the population of your class may be counted in ones and tens, the population of your school may be counted in hundreds, and so on, until the population of the world which is counted in the millions and billions range.

**Answers****Challenge page 28**

Each answer will be unique. You can look at the average from the houses you sampled and then generalise to the rest of the street. From this you can work out the population of the whole street.

Exercise 1

Answers will vary between students.

Worksheet 4

1. a) 1991; b) 161 253 518
2. a) CAD; b) DPC; c) 14 726 318 503;
d) 21 690 812 723

Assessment

Pupils should be able to define a population. Each pupil should be able to give their definition of a population to the class.

Pupils must be able to accurately record and report simple population figures.

Pupils should understand that population figures are approximations, and that they are made with certain assumptions.

Homework activity

Complete WB Worksheet 4, Questions 1 and 2.

Lesson 2 *Pupil's Book page 29; Workbook page 9*



Preparation

You will need to have: Pupil's Book; Workbook; demography map of Nigeria.



Starter activity

Write the population figures of the main cities in your state on the board. Read the figures together as a class.



Lesson focus

Help pupils to focus on the real-life meaning of the big numbers.

Work with the table on pages 29 and 30 of the PB and a demography map of Nigeria to practise retrieving information from both sources. Pupils record specific population figures and then compare them. For example, ask each pupil to choose a Nigerian state that starts with the first letter of their name. They must then write down

the figure and compare it with a friend to see whose is larger or smaller.



Answers

Challenge page 29

A census is an official count of population in an area and is used to collect data from the population. It is used through collaborating all the data collected from each family to calculate total population.

Exercise 2

1. a) 1 405 201 b) 9 013 534
c) 3 168 101 d) 4 219 244
e) 3 257 298 f) 1 703 358
2. a) 1 434 193 b) 1 630 344
c) 2 710 665 d) 1 230 722
e) 1 761 263 f) 1 492 465
3. a) 2 007 391 b) 1 629 502
c) 1 561 978 d) 1 100 887
e) 2 954 534 f) 1 396 501
4. a) Kano b) Ogun
c) Edo d) Oyo
5. a) Bauchi (4 676 465), Anambra (4 182 032),
Ogun (3 728 098), Edo (3 218 332),
Yobe (2 321 591), Bayelsa (1 703 358)
b) Borno (4 151 193), Ondo (3 441 024),
Kwara (2 371 089), Gombe (2 353 879),
Ebonyi (2 173 501), FCT Abuja
(1 405 201)
c) Jigawa (4 348 649), Delta (4 098 391),
Imo (3 934 899), Sokoto (3 696 999),
Osun (3 423 535), Kebbi (3 238 628),
Ekiti (2 384 212)

Worksheet 4

3. a) 3 276 928 b) 568 272
c) 5 635 086 d) 6 929 062
4. 96 886

Assessment

Pupils should be able to compare populations. Assess their performance in Exercise 2.

Homework activity

Complete WB Worksheet 4, Questions 3 and 4.

Lesson 3 *Pupil's Book page 31; Workbook page 10*

Preparation

You will need to have: Pupil's Book; Workbook; demography map of Nigeria; World Atlas and some published information from WHO on HIV/AIDS.

Starter activity

Have a discussion about HIV/AIDS. Explain that it is a disease of the immune system and is spread through body fluids. Receive questions from the class.

Lesson focus

The questions in Exercise 3 are designed so that pupils will learn to understand and make sense of (interpret) the data in the book. It involves large numbers.

Answers

Challenge page 32

Angola: 11 890 000; Cameroon: 177 000 000;

Ghana: 18 250 000; Nigeria: 138 600 000;

Zambia: 10 252 000; Zimbabwe: 12 211 000

Exercise 3

1. Nigeria; 2. Nigeria; 3. Angola; 4. Angola, Ghana, Cameroon, Zambia, Zimbabwe, Nigeria;
5. Two million, six hundred thousand; 6. 800 000, eight hundred thousand

Worksheet 4

5. a) 550 535 b) 247 893
c) 1 116 066 d) 1 195 465
e) 962 568
6. a) 36 010 874 b) 1 005 578
c) Kano 494 616 d) 144 644 744
e) 148 984 086

Assessment

Pupils should be able to interpret data about people living with HIV/AIDS.

Assess their performance in Exercise 3.

Homework activity

Complete WB Worksheet 4, Questions 5 and 6.

Lesson 4 *Pupil's Book page 33*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.

Answers

Revision exercise page 33

1. 1 150 508; 2. 945 556; 3. Ebonyi
4. a) Three million, two hundred and seventy-eight thousand, four hundred and eighty-seven
b) Two million, eight hundred and nine thousand, eight hundred and forty
c) One million, six hundred and thirty-three thousand and ninety-six
5. a) $3\,000\,000 + 200\,000 + 70\,000 + 8\,000 + 400 + 80 + 7$
b) $1\,000\,000 + 200\,000 + 20\,000 + 500 + 80 + 1$
c) $4\,000\,000 + 300\,000 + 30\,000 + 5\,000 + 500 + 10 + 4$
d) $100\,000\,000 + 40\,000\,000 + 3\,000 + 500 + 40 + 2$

6.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
1	8	6	3	2	7	5

7. Angola, Ghana, Cameroon, Zambia, Zimbabwe, Nigeria
8. $10\,000\,000 + 2\,000\,000 + 800\,000 + 90\,000 + 1\,000$

Objectives

By the end of this unit, each pupil should be able to:

- Order and compare fractions
- Order fractions in ascending and descending order.
- Solve quantitative aptitude problems on fractions.

**Suggested resources**

fraction strips

**Frequently asked questions**

Q *What should the pupils understand?*

A Pupils should have a clear understanding of the concept of a fraction (meaning of denominator and numerator), the equivalence of two fractions, and the relative sizes of fractions.

Q *What mathematical skills do pupils need?*

A Pupils should know various methods for comparing the sizes of two fractions.

**Evaluation guide**

Pupils to:

1. Order given fractions in ascending or descending order.
2. Solve quantitative aptitude problems involving fractions.

Lesson 1 *Pupil's Book page 34; Workbook page 11***Preparation**

You will need to have: Pupil's Book; Workbook; fraction strips.

**Starter activity**

Discuss the order of fractions with the same denominator, and also those with different denominators.

**Lesson focus**

Refer to fraction strips to compare the sizes of two fractions.

Introduce the symbols $<$ (less than) and $>$ (more than). The simple rule is that the smaller number is written on the small side of the symbol.

So $2 < 3$, but $3 > 2$.

**Answers****Challenge page 34**

$$\frac{1}{10395}$$

Puzzle page 35

The mystery fraction is $\frac{5}{15}$

Exercise 1

1. a) $\frac{7}{15} < \frac{5}{6}$ b) $\frac{2}{5} < \frac{4}{7}$ c) $\frac{5}{8} > \frac{7}{12}$
 d) $\frac{2}{3} < \frac{8}{3}$ e) $\frac{3}{10} < \frac{6}{8}$ f) $\frac{2}{5} < \frac{3}{7}$
 g) $\frac{12}{15} < \frac{10}{11}$ h) $\frac{6}{10} > \frac{3}{7}$
2. a) $\frac{1}{12}, \frac{3}{8}, \frac{4}{9}, \frac{2}{3}$ b) $\frac{6}{20}, \frac{5}{12}, \frac{4}{9}, \frac{6}{8}$ c) $\frac{3}{8}, \frac{9}{20}, \frac{3}{5}, \frac{4}{5}$
 d) $\frac{1}{6}, \frac{5}{8}, \frac{2}{3}, \frac{10}{12}$ e) $\frac{2}{6}, \frac{2}{5}, \frac{5}{12}, \frac{7}{10}$ f) $\frac{12}{200}, \frac{8}{15}, \frac{6}{10}, \frac{3}{4}$
3. a) $\frac{3}{4}, \frac{5}{10}, \frac{2}{5}, \frac{1}{9}$ b) $\frac{10}{12}, \frac{2}{3}, \frac{3}{5}$ c) $\frac{8}{7}, \frac{9}{14}, \frac{8}{21}, \frac{2}{7}$
 d) $\frac{6}{9}, \frac{4}{6}, \frac{5}{8}, \frac{7}{12}$ e) $\frac{8}{9}, \frac{10}{12}, \frac{10}{21}, \frac{3}{7}$ f) $\frac{3}{4}, \frac{9}{12}, \frac{4}{6}, \frac{5}{8}, \frac{10}{24}$

Worksheet 5

1. $\frac{1}{5}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3} = \frac{4}{6}, \frac{3}{4}, \frac{5}{6}, \frac{9}{10}$
2. $\frac{1}{2} > \frac{1}{3}, \frac{3}{4} > \frac{2}{5}, \frac{2}{3} > \frac{3}{5}, \frac{5}{8} > \frac{3}{7}, \frac{1}{3} > \frac{1}{6}, \frac{3}{8} > \frac{2}{5}, \frac{9}{10} > \frac{8}{9}, \frac{7}{3} > \frac{5}{8}$

Assessment

Pupils should be able to order fractions.
Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Fraction triangles

Support activity

Interactive learning diagram: Comparing fractions

Homework activity

Complete WB Worksheet 5, Questions 1 and 2.

Lesson 2 *Pupil's Book page 36; Workbook page 11*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Use the examples of fractions from the 'real world' provided by pupils (and supplemented with your own) to show that fractions are used as part of everyday language.



Lesson focus

The fact is that we rarely use fractions in everyday life, and that is one of the reasons they are so difficult for many pupils.

The questions in Exercise 2 are not 'everyday life' but they do provide experience in reading word problems. Discuss what the questions mean, and how we can work out what we must do with the fractions to solve the problems.



Answers

Exercise 2

1. $\frac{1}{5} = \frac{12}{60}$, $\frac{1}{6} = \frac{10}{60}$, $\frac{1}{12} = \frac{5}{60}$.

She spends the most on books.

2. a) $\frac{5}{8}$ left = $\frac{15}{24}$. Danjuma receives .

b) $\frac{3}{8} + \frac{5}{24}$ taken = $\frac{9+5}{24} = \frac{14}{24}$

So there is $\frac{10}{24}$ left for Abiola.

c) Abiola received the biggest share.

d) Uche receives $\frac{3}{8}$ of 72 = $\frac{72 \times 3}{8} = 27$ bags

Danjuma receives $\frac{5}{24}$ of 72 = $\frac{72 \times 5}{24} = 15$ bags

Abiola receives $\frac{10}{24}$ of 72 = $\frac{72 \times 10}{24} = 30$ bags

3. a) $\frac{1}{4}$ of 48 are 12 = $\frac{1 \times 48}{4} = 12$ ten year olds

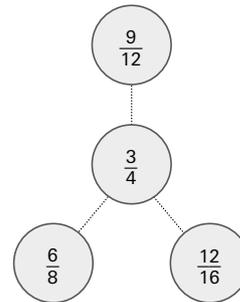
$\frac{1}{3}$ of 48 are 16 = $\frac{1 \times 48}{3} = 16$ eleven year olds

So, $48 - 12 - 16 = 20$ twelve year olds

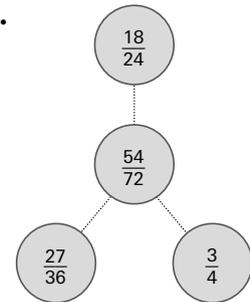
b) $\frac{20}{48} = \frac{5}{12}$ are 12 years old

Exercise 3

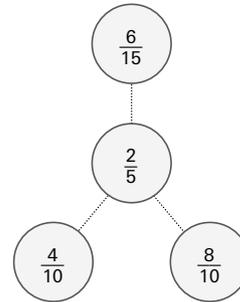
1.



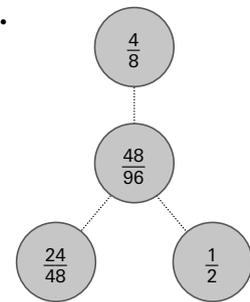
2.



3.



4.



Worksheet 5

3. $\frac{1}{3} < \frac{2}{3}$, $\frac{7}{8} < \frac{8}{9}$, $\frac{1}{10} < \frac{1}{9}$, $\frac{3}{7} < \frac{1}{2}$, $\frac{2}{9} < \frac{3}{10}$, $\frac{1}{5} < \frac{2}{9}$;

$\frac{3}{7} < \frac{2}{3}$, $\frac{3}{7} < \frac{5}{9}$

4. $\frac{1}{4} < \frac{1}{3}$, $\frac{7}{10} < \frac{3}{4}$, $\frac{3}{8} > \frac{2}{7}$, $\frac{5}{8} < \frac{3}{4}$

Assessment

Can the pupil find examples of fractions in use in everyday life?

Assess their performance in Exercises 2 and 3.

Extension activity

Interactive learning diagram: Fractions of numbers

Support activity

Interactive learning diagram: Fractions of time

Homework activity

Complete WB Worksheet 5, Questions 3 and 4.

Lesson 3 *Pupil's Book page 39*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.



Answers

Revision exercise page 39

1. $\frac{48}{36}, \frac{11}{12}, \frac{7}{8}, \frac{5}{6}, \frac{19}{24}, \frac{3}{4} = \frac{36}{48}, \frac{11}{24}, \frac{1}{3}$

2. $\frac{5}{3}, \frac{6}{7}, \frac{3}{4}, \frac{1}{6}$

3. $\frac{375}{1000}, \frac{27}{72}, \frac{333}{888}, \frac{36}{96}, \frac{153}{408}$

4. $\frac{7}{8} = \frac{42}{48} \quad \frac{5}{6} = \frac{40}{48}$

The $\frac{7}{8}$ advertisement uses more space

5. Math: $\frac{3}{5}$ of 60 min = $\frac{3 \times 60}{5} = 36$ minutes

History: $\frac{5}{6}$ of 60 min = $\frac{5 \times 60}{6} = 50$ minutes

Geography: $\frac{2}{3}$ of 60 min = $\frac{2 \times 60}{3} = 40$ minutes

The order is: math, geography, history

Objectives

By the end of this unit, each pupil should be able to:

- Express decimals as fractions and vice versa.

**Suggested resources**

playing cards; decimal strips; decimal square;
fraction-decimal-percentage sheet

**Common errors that pupils make**

Misconceptions regarding fractions and decimals

There are some pupils who seem to believe that 0.4 and $\frac{1}{4}$ mean the same thing because they both use 4. They often extend this to believing that 0.25 and $\frac{1}{25}$ mean the same thing. Any pupils who reveal this misunderstanding need to be helped immediately. Some personal tuition using hands-on materials can remove these errors quickly.

Pupils often struggle to convert common fractions with a denominator of 8 to decimal fractions.

If you find that this is a problem for your class, make a poster showing the following conversions:

$$\frac{1}{8} = 0.125; \frac{2}{8} = \frac{1}{4} = 0.25; \frac{3}{8} = 0.375; \frac{4}{8} = \frac{1}{2} = 0.5;$$

$$\frac{5}{8} = 0.625; \frac{6}{8} = \frac{3}{4} = 0.75; \frac{7}{8} = 0.875; \frac{8}{8} = 1.$$

**Evaluation guide**

Pupils to:

- Express decimals as fractions and vice versa.

Lesson 1

Pupil's Book page 40; Workbook page 12

**Preparation**

You will need to have: Pupil's Book; Workbook;
decimal square; playing cards.

**Starter activity**

In this unit we are reviewing place value as applied to decimals. Revisit this with this game:

Card game – Decimals in order

Shuffle the pack and turn the cards over. Each player takes six cards. The player must put out the cards to make three numbers.

One must use tenths.

$$0 \cdot \square$$

One must use hundredths.

$$0 \cdot \square\square$$

One must use thousandths.

$$0 \cdot \square\square\square$$

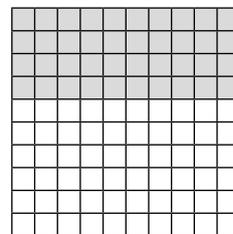
They must be put in order of size – smallest to largest.

To win the point, explain to the other players why the order is correct.

**Lesson focus**

Use a decimal square (10 × 10)

Use it to demonstrate converting 0.4 into a fraction and reduce it to its simplest terms.



Using the diagram, this is 4 tenths, or 2 fifths.

Using equivalent fractions it is: $\frac{40}{100} = \frac{20}{50} = \frac{4}{10} = \frac{2}{5}$

Ask pupils to show you how to do these:

Convert into fractions, reducing to simplest terms.

0.5; 0.2; 0.25; 0.05; 0.15; 0.6; 0.75; 0.8; 0.85; 0.95.

Pupils should draw each one, and show how they get the answer.



Answers

Exercise 1

1. $\frac{9}{10}$; 2. $\frac{27}{100}$; 3. $4\frac{23}{100}$; 4. $9\frac{3}{125}$ (HCF = 8);
 5. $\frac{9}{20}$ (HCF = 5); 6. $3\frac{2}{25}$ (HCF = 4);
 7. $10\frac{1}{200}$ (HCF = 5); 8. $8\frac{307}{500}$ (HCF = 2);
 9. $10\frac{1}{20}$ (HCF = 5); 10. $20\frac{12}{25}$ (HCF = 4);
 11. $14\frac{17}{50}$ (HCF = 2); 12. $\frac{5}{8}$ (HCF = 125);
 13. $11\frac{69}{200}$ (HCF = 5); 14. $15\frac{1}{40}$ (HCF = 25);
 15. $5\frac{7}{8}$ (HCF = 125)

Worksheet 6

1. a) $\frac{6}{10}$ b) $\frac{3}{10}$ c) $\frac{32}{100}$ d) $\frac{2}{10}$
 e) $\frac{6}{100}$ f) $\frac{36}{100}$ g) $\frac{258}{1\ 000}$ h) $\frac{6}{1\ 000}$
 i) $\frac{26}{1\ 000}$ j) $\frac{279}{1\ 000}$
 3. a) $\frac{13}{10}$ b) $\frac{608}{100}$ c) $\frac{888}{100}$ d) $\frac{18\ 007}{1\ 000}$
 e) $\frac{999}{10}$ f) $\frac{2\ 777}{100}$ g) $\frac{35\ 989}{100}$ h) $\frac{761}{10}$
 i) $\frac{58201}{1\ 000}$ j) $\frac{99999}{1\ 000}$

Assessment

Pupils should be able to express decimals as fractions.

Assess their performance in Exercise 1 and games.

Extension activity

Interactive learning diagram: Decimal to fraction

Support activity

Interactive learning diagram: Guitar fractions

Homework activity

Complete WB Worksheet 6 Questions 1 and 3.

Lesson 2

Pupil's Book page 42;

Workbook pages 12, 13



Preparation

You will need to have: Pupil's Book; Workbook; decimal strips; fraction-decimal-percentage sheet.



Starter activity

Ask pupils to find the factors of 10, 100, 1 000 and 10 000. From these they will find the multipliers needed to convert fractions into equivalent fractions with denominators of 10, 100, 1 000 or 10 000.



Lesson focus

Use decimal strips.

Label the tenths and the twentieths: 0.05, 0.1, 0.15, 0.2 etc.

Fold into halves to show $\frac{1}{2} = 0.5$.

Fold into quarters to show $\frac{1}{4} = 0.25$, and $\frac{3}{4} = 0.75$.

Fold into eighths: $\frac{1}{8} = 0.125$, $\frac{3}{8} = 0.375$, etc.

Fold into fifths: $\frac{1}{5} = 0.2$, $\frac{2}{5} = 0.4$, $\frac{3}{5} = 0.6$, $\frac{4}{5} = 0.8$

Continue in this way to see the decimals equivalent to each fraction.

For example $\frac{1}{3}$ is about 0.33.

Use a fraction-decimal-percentage sheet. This shows the same idea with the fractions and the decimals (and percentages) lined up. Pupils may use a ruler to estimate the decimal equal to a given fraction, or the nearest fraction to a given decimal.



Answers

Puzzle page 42

1. $\frac{1}{16}$; 2. $\frac{9}{20}$

Challenge page 44

1. $\frac{5}{32}$; 2. $\frac{3}{32}$; 3. $\frac{3}{5}$; 4. $\frac{3}{20}$

Exercise 2

1. a) 0.75; b) 0.3; c) 2.8; d) 0.92
 2. a) 0.4166...; b) 0.75; c) 0.44...; d) 0.166...;
 e) 0.33...

Worksheet 6

2. a) 0.9 b) 0.5 c) 0.02
d) 0.025 e) 0.001 f) 0.25
g) 0.99 h) 0.277 i) 0.68
j) 0.999
4. a) 32.1 b) 6.02 c) 92.06
d) 123.99 e) 49.049 f) 3.999
g) 27.27 h) 13.002 i) 22.001
j) 3.142
- 5.

$\frac{1}{4}$	$\frac{1}{10}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{3}{8}$	$\frac{9}{10}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{3}{4}$
0.25	0.1	0.667	0.333	0.5	0.2	0.4	0.6	0.375	0.9	0.7	0.8	0.75

Assessment

Pupils should be able to express decimals as fractions and vice versa.

Assess their performance in Exercise 2, the decimal line folding activity and the game.

Extension activity

Interactive learning diagram: Fraction to decimal

Pupils should commit common conversions to memory: halves, quarters, eighths, tenths, twentieths.

Support activity

Interactive learning diagram: Guitar fractions

Homework activity

Complete WB Worksheet 6, Questions 2, 4 and 5.

Lesson 3 *Pupil's Book page 45*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.



Answers

Revision exercise page 45

1. a) 0.75; b) 0.52; c) 0.875; d) 0.525
2. a) $\frac{12}{25}$; b) $2\frac{18}{25}$; c) $\frac{47}{125}$; d) $\frac{78}{125}$
3. a) 0.6875; b) 1.86; c) 4.48; d) 0.66; e) 0.47826...
4. a) $\frac{13}{20}$; b) $\frac{11}{25}$; c) $\frac{4}{25}$; d) $\frac{1}{4}$; e) $\frac{18}{25}$; f) $\frac{7}{8}$
5. a) 0.625; b) 0.7272...; c) 0.55...; d) 0.45; e) 0.66...; f) 0.5833...

Objectives

By the end of this unit, each pupil should be able to:

- Solve problems on ratio
- Relate ratio to everyday life situations
- Solve problems on direct proportion
- Solve problems on inverse proportion
- Relate inverse proportion to everyday life situations
- Solve quantitative reasoning problems involving ratio
- Solve problems on quantitative reasoning involving direct and inverse proportion.

**Frequently asked questions**

Q *What do pupils need to understand?*

A The true meaning of a ratio is a comparison of two quantities using division. It is not just the listing of two numbers with ':' between them. There is a clear link to fractions here, but a fraction compares one part to the whole thing, while a ratio compares one part to another. (Also, because there can be more than one part making up the whole, a ratio can have more than two numbers.)

**Evaluation guide**

Pupils to:

1. Solve given problems on ratio, direct proportion and inverse proportion.
2. Solve some quantitative reasoning problems on ratio, direct proportion and inverse proportion.

Lesson 1 *Pupil's Book page 46***Preparation**

You will need to have: Pupil's Book.

**Starter activity**

Ratio is used to compare the amount of one thing with an amount of another.

In everyday life you might think about paint mixing and the way ratio affects the shades of mixes of paint. Think about all the shades of paint made

from mixing a single colour and white. Discuss the possible ratios of colour with white.

If possible, give pupils an opportunity to experiment with making different shades using two colours of paint, using a small container or a spoon to measure the ratios.

**Lesson focus**

Ratios compare 'part to part', not 'part to whole'. Point out the difference: with 1 girl, 2 boys, the girl is $\frac{1}{3}$ of the group, but the number of girls is $\frac{1}{2}$ of the number of boys.

Counters

Use four counters to make three different ratios: 1:3, 2:2 and 3:1. Make and write all the ratios you can with five counters – with six counters – with seven ...etc.

Nigerian coins

Compare the value of the Nigerian coins in pairs:

$$1 \text{ kobo to } 5 \text{ kobo} = 1:5$$

$$1 \text{ kobo to } 10 \text{ kobo} = 1:10$$

$$1 \text{ kobo to } 25 \text{ kobo} = 1:25$$

$$5 \text{ kobo to } 10 \text{ kobo} = 5:10 = 1:2$$

$$5 \text{ kobo to } 25 \text{ kobo} = 5:25 = 1:5$$

$$10 \text{ kobo to } 25 \text{ kobo} = 10:25 = 2:5$$

**Answers****Exercise 1**

1. 1:3; 2. 4:3; 3. 4:5

Exercise 2

1. a) 1:2 b) 5:6
2. 5:3:4; 3. 27:18

Assessment

Pupils should be able to solve problems on ratio.
Assess their performance in Exercises 1 and 2.

Extension activity

Interactive learning diagram: Right-angled triangles

Support activity

Interactive learning diagram: Orange cordial

Lesson 2 *Pupil's Book page 48;
Workbook pages 14 and 15*

Preparation

You will need to have: Pupil's Book; Workbook; counters.

Starter activity

Ask a group of pupils to show the ratio of three boys to two girls (3:2). Ask each of this group to find a friend of the same gender. Work out the ratio of this new group (6:4). Explain that these two ratios are equivalent $3:2 = 6:4$. This is because in both of them there are $1\frac{1}{2}$ times as many boys as girls.

Ask questions based on the same ratio:
"If there were 12 boys, how many girls would there be?"

"If there were six girls, how many boys?"

If it would be helpful, draw a chart on the board.

Number of boys	3	6	9	12
Number of girls	2	4	6	8

Repeat with a different ratio, with more girls this time.

Lesson focus

Counters

Groups of pupils can do the same activity with counters. Ask pupils to start with the simplest ratio e.g. 3:4. Then they add another 3 and 4 to get 6:8, and so on, writing the new ratio each time. Work through the example. By now pupils will have discovered the basic 'rule' for themselves – that you can find equivalents by multiplying each side by the same number.

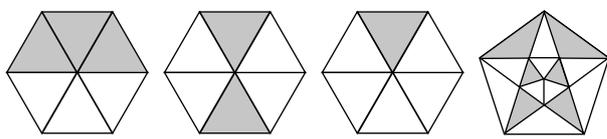
Now work backwards. Start with a ratio, such as 12:8, and ask them to work out the simplest numbers that have the same ratio. They will 'discover' for themselves they can simplify ratios by finding a common factor of both sides and dividing by it, just as they did for fractions. Sometimes they may need to reduce the ratio more than once to find the simplest form, if they do not identify the highest common factor at first.

Answers

Exercise 3

1. *Answers will vary*
- a) 2:6, 3:9 b) 4:10, 20:50
c) 6:14, 9:21 d) 8:18, 12:27
e) 1:3, 2:6
2. a) 2:3 b) 2:5
c) 3:4 d) 3:4
e) 1:3
3. a) 1:2 b) 1:15
c) 2 cm:5 cm d) 50K:₹1
e) 2 min:3 min f) 1 kg:6 kg

Worksheet 7

1. 
- 1:1 1:2 1:5 2:3

2.

Any other combination as long as there are 9, 12 and 15 blocks of the respective colours.

5. *Answers will vary.*

Assessment

Pupils should be able to find equivalent ratios from the simplest, or find the simplest ratio from an equivalent ratio with larger numbers.

Assess their performance in:
Exercise 3, and the counters activity.

Extension activity

Interactive learning diagram: Proportion graph

Support activity

Interactive learning diagram: Graph fractions

Homework activity

Complete WB Worksheet 7, Questions 1, 2 and 5.

Lesson 3 *Pupil's Book page 49*

Preparation

You will need to have: Pupil's Book.

Starter activity

Ask pupils to write down how much milk and sugar they like in their tea or coffee. This may be 1 sugar and 40 ml milk, or 3 sugars and 10 ml milk. Answers will vary. Use this as a starting point to show how to write the quantities of milk and sugar as a ratio. Remind them that the quantities must first be expressed in the same units.

Lesson focus

Use a recipe and use the list of ingredients to work out some ratios, choosing quantities that can be simplified. Explain that the challenge involves equivalent ratios and that they should choose one quantity and work out the other from that.

Answers

Exercise 4

- a) 5 g : 3 g; b) 9 g : 10 g; c) 2 g : 3 g
- a) 2 ml : 3 ml; b) 2 ml : 9 ml; c) 3 ml : 1 ml

Assessment

Assess their performance in Exercise 4.

Extension activity

Interactive learning diagram: Proportion triangles

Support activity

Interactive learning diagram: Sharing by ratio

Lesson 4 *Pupil's Book page 50*

Preparation

You will need to have: Pupil's Book; counters.

Starter activity

Remind pupils about equivalent ratios. For example 2 : 3 is equivalent (equal) to 6 : 9 by multiplying both numbers by 3.

Choose a few incomplete equivalences so that pupils can find the missing numbers:

$$3 : 5 = 6 : ? \quad 3 : 2 = ? : 8 \quad 2 : 5 = 6 : ?$$

(Keep the multiplier to whole numbers at this stage.)

Lesson focus

Counters

Groups of pupils can solve problems with counters. For example, ask them to share 12 counters in the ratio 1 : 3, or 2 : 1, or 5 : 1.

After some working of these kinds of problems with trial and error, they will be ready to tell you the method.

Count how many parts, e.g. 3 : 1 has 4 parts.
Divide the number to be shared by the number of parts to get the size of each part e.g. $20 \div 4 = 5$
Then multiply by the numbers in the ratio, e.g. the ratio 3 : 1 shares 20 as 15 and 5.

Answers

Exercise 5

- 1 part = $\frac{4}{15}$; 5 parts = $\frac{4}{3}$
 - 1 part = 100 g; 3 parts = 300 g
 - 3 parts = 300 kg; 4 parts = 400 kg
 - 2 parts = $\frac{87}{70}$; 5 parts = $\frac{87}{28}$
 - 5 parts = 450 km; 3 parts = 270 km
 - 4 parts + 5 parts = 9 parts
1 part = $2\,700 \text{ ml} \div 9 = 300 \text{ ml}$
4 parts = $4 \times 300 \text{ ml} = 1\,200 \text{ ml}$
5 parts = $5 \times 300 \text{ ml} = 1\,500 \text{ ml}$
- story books = 180 books;
reference books = 300 books
- brown rolls = 144 rolls; white rolls = 216 rolls
- green beads = 1 000 beads;
purple beads = 2 500 beads

Assessment

Pupils should be able to find new quantities by using a given ratio.

Assess their performance in Exercise 5 and the counter activity.

Extension activity

Interactive learning diagram: Proportion

Support activity

Interactive learning diagram: Proportions

Lesson 5 *Pupil's Book page 51*

Preparation

You will need to have: Pupil's Book; playing cards.

Starter activity

Card game – Changing using ratios

Use the pack of cards for the numbers 1 to 6 and 10 only. (These are factors of 60.) Shuffle the pack. Each player imagines they have ₦60. Each player gets two cards. They make a ratio (larger to smaller) to increase the ₦60, and work out the answer. Now take the answer and swap the numbers in the ratio, so that the ratio now decreases the amount. The answer should be ₦60.

Lesson focus

Work through the examples on page 51. If you feel that your pupils are struggling with this concept, try using more concrete examples. You could bring some basic ingredients into class to show how to change the ratio in order to change the quantities required.

Answers

Challenge page 51

1. 6 kg cement; 3 kg gravel; 2. 12 kg cement; 6 kg gravel; 3. 45 kg concrete

Challenge page 52

- a) 260 g sugar; 300 g butter; b) 720 g flour; 432 g sugar; 480 g butter
- 6 000 ml red paint

Exercise 6

- a) 100 ml; b) 600 ml
- a) lime juice = 180 ml; soda = 720 ml
b) soda = 2 880 ml; lime juice = 720 ml

Exercise 7

- 3 hours 20 min; 2. 25 students; 3. 8 boys;
- 14 mints; 35 fruit

Assessment

Pupils should be able to change quantities using a given ratio.

Assess their performance in Exercises 6 and 7.

Extension activity

Interactive learning diagram: Proportion problems, Spirograph

Support activity

Interactive learning diagram: Proportion cordial

Lesson 6

Pupil's Book page 53;

Workbook pages 14 and 15



Preparation

You will need to have: Pupil's Book; Workbook; counters.



Starter activity

Show the class some pictures of items with price tags, for example, a mango with a price of ₦15. Ask pupils how much they would expect to pay for a given number of each item.



Lesson focus

Direct proportion is the situation when two quantities increase in the same ratio. When one is doubled, the other is doubled as well. In fact we have used this situation several times already in this unit.

Ask 1 pupil to come to the front and give that pupil 3 counters, telling the class what you have done. The ratio of counters:pupils is 3 : 1.

Ask three more pupils to come to the front, and give each one three counters. There are now 12 counters in the hands of 4 pupils. The ratio is 12 : 4 which is still 3 : 1. We multiplied the number of pupils by 4, and the number of counters had to be multiplied by 4 as well ($3 \times 4 = 12$).



Answers

Exercise 8

1. ₦300
2. a) ₦120; b) ₦1 800
3. 270 kg

Exercise 9

1. 50 yellow flowers; 2. potatoes = 20 rows;
3. 6 people = 375 g butter

Worksheet 7

3.

Number of oranges	1	2	3	4	5	6	7	8
Selling price	₦12	₦24	₦36	₦48	₦60	₦72	₦84	₦96

7. a) ₦12 320 b) 62.5 litres
c) ₦2 720 d) 175 km

Assessment

Pupils should be able to solve direct proportion problems.

Assess their performance in Exercises 8 and 9.

Extension activity

Interactive learning diagram: Proportion problems, Spirograph

Support activity

Interactive learning diagram: Proportion cordial

Homework activity

Complete WB Worksheet 7, Questions 3 and 7.

Lesson 7

Pupil's Book page 55;

Workbook pages 14 and 15



Preparation

You will need to have: Pupil's Book; Workbook; counters.



Starter activity

Talk about how long it would take to do particular jobs with a given number of workers. For example, if it takes six workers six days to do a job, how long will it take 12 workers? (Answer: three days.)



Lesson focus

Inverse proportion requires a constant product of two numbers. One way to show this is sharing.

Shares of ₦60 (use counters)

Demonstrate with 24 counters.

Number of pupils	1	2	3	4	6
Size of share (₦)	24	12			
Number of pupils	8	12	24		
Size of share (₦)	3	2	1		

This should be enough for pupils to see that they are dividing to get the answers. But always the two numbers in the table multiply to 24.

Give each group 30 counters.

They should work together to complete this table.

Number of pupils	1	2	3	5	6
Size of share (₦)	30				
Number of pupils	10	12	15	30	
Size of share (₦)	3				

In groups pupils should complete this table for sharing ₦840.

Number of pupils	1	2	3	4	6
Size of share (₦)	840	420			
Number of pupils	6	7	8	10	12
Size of share (₦)	3				

Discuss similar situations to those in the book. The more workers there are, the less each one has to do, so the shorter the time the job takes. If you double the workers the time is halved. This is inverse proportion.

Answers

Puzzle page 55

Each answer will be different for each girl. But the working out would be similar:

Work out the ratio of boys to girls in the class then multiply by the number of classes in the school.

Exercise 10

- a) 40 days; b) 20 days; c) 15 days
- a) 12 hours; b) 24 hours; c) 8 hours
- a) 120 km/h; b) 60 km/h

Exercise 11

- 5 days
- a) 800 labourers; b) 80 labourers; c) 20 labourers
- 8 hours

Worksheet 7

4.

Number of builders to build a wall	1	2	3	4	6	10	12
Days	120	60	40	30	20	12	10

6. 9 days

Assessment

Pupils should be able to solve problems on inverse proportions, identify some daily activities that are inversely related, and solve problems on quantitative reasoning in inverse proportions.

Assess their performance in counter activities and Exercises 10 and 11.

Support activity

Interactive learning diagram: Inverse proportion

Homework activity

Complete WB Worksheet 7, Questions 4 and 6.

Lesson 8 *Pupil's Book page 57*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observation.

Answers

Revision exercise page 57

- Answers will vary*
a) 2:6; b) 8:10; c) 22:26; d) 10:4
- a) 1:3; b) 1:4; c) 4:7; d) 4:3; e) 1:2;
f) 1:3; g) 3:2; h) 14:5
- a) 1 part = 100 ml; 4 parts = 400 ml
b) 4 parts = 16 kg; 5 parts = 20 kg
c) 5 parts = 450 km; 2 parts = 180 km
d) 3 parts = 438 g; 7 parts = 1 022 g
- 20 years; 5. 64 kg; 6. 70 sheep
- a) 40 words; b) 160 words

Objectives

By the end of this unit, each pupil should be able to:

- Find the ratio of family size and resources
- Correctly use ratio in relating the proportion of resources to family size
- Express two populations in a given ratio
- Express the ratio of prevalence of HIV/AIDS between the sexes in a town or country
- Correctly express the ratios in monitoring the trend of infection of HIV/AIDS in different sexes and among states
- Solve quantitative aptitude problems on the ratio of HIV/AIDS prevalence.

**Suggested resources**

counters

**Common errors that pupils make**

Pupils struggle to find an unknown quantity using a given ratio, where neither part is 1, e.g. 2:3.

In this case the given quantity needs to be divided by its ratio number (e.g. 2), to find the quantity of one part, then multiplied by the other ratio number (e.g. 5) to find the missing quantity. Emphasise the part-to-part relationship of ratios.

**Evaluation guide**

Pupils to:

1. Calculate the ratio of given family size to their earning.
2. Give examples of everyday activities that demand correct application of ratio.
3. Find the ratio of given populations.
4. Find the given ratio of prevalence of HIV/AIDS by gender of a population.
5. Solve quantitative aptitude problems on the ratio of prevalence of HIV/AIDS of a population.

Lesson 1 *Pupil's Book pages 58 and 59;
Workbook page 16*

**Preparation**

You will need to have: Pupil's Book; Workbook.

**Starter activity**

Review equivalent ratios, and the simplification of a ratio by dividing by a common factor.

Bring six girls and two boys to the front of the class. Pupils will quickly see that there are 3 girls for each boy. (3 : 1)

Now bring 8 girls and 4 boys out the front. This time there are 2 girls for each boy. (2 : 1)

In the first group the ratio (3 : 1) is larger than in the second group (2 : 1).

Now bring 5 girls and two boys out. How many girls for each boy this time? The ratio is 2.5 : 1 (or $2\frac{1}{2} : 1$). We can use fractions or decimals in ratios.

**Lesson focus**

Go through the first example in the book, which demonstrates the same idea with income and people.

The second example (page 59) does the same but adds the comparison of the two ratios. Ask the pupils to take the two ratios and make unit ratios from them (Ola $100\ 000 : 5 = 20\ 000 : 1$ and Anene $100\ 000 : 8 = 12\ 500 : 1$. there is more money per person in Ola's family.

Note that we could have realised that Ola's family was better off because they have fewer people to share the same money.

In Exercise 1, Question 5 revisits the idea of sharing a quantity using ratios. This was covered on page 49. In fact there are three numbers in this ratio. This will be a good time to review that topic.

Answers

Exercise 1

- 1:2; 2. 5:7; 3. 8:16
- a) 36 000; b) 15 000
- a) 1:11 000; b) 7:99 000;
c) 1:49 500; d) 1:19 800
- 16 chickens;
- a) 27 eggs; b) 63 eggs
- 16 people

Worksheet 8

- a) 400 g rice, 1.2 kg beans b) 800 g
- 3:1
- 12 000, 16 000 and 20 000
- 4 days, 2 days
- Method 1 is indirect; Method 2 is direct
- Blue 36 000, red 24 000

Assessment

Pupils should be able to find ratio of family size and resources.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Proportion, Proportion cordial

Support activity

Interactive learning diagram: Proportions

Homework activity

Complete WB Worksheet 8.

Lesson 2 *Pupil's Book page 62*

Preparation

You will need to have: Pupil's Book; counters.

Starter activity

Use the picture in the PB on page 62, and then do it for your own classroom. Simplify the ratios if possible.

Lesson focus

In this lesson we are reviewing finding missing numbers in a proportion.

The 'unitary method' is often used.

Here is a simple example.

Use 14 counters. Start with just 4 in the ratio 2:2. We want another set of counters, so that the ratios of numbers in the smaller to the bigger set is 2:5. In the larger pile we want 5 for each 2 in the smaller pile.



What we are doing is finding how many 2s are in the smaller pile, and making that number of 5s. In other words we are dividing by 2, and then multiplying by 5.

Answers

Exercise 2

- a) 25:17; b) 0.68 male
- 6 million people; 3. 3:4; 4. 2:3

Assessment

The pupils should be able to increase or decrease a number in a given ratio.

Assess their performance in Exercise 2.

Lesson 3 *Pupil's Book page 64*

Preparation

You will need to have: Pupil's Book.

Starter activity

Have a discussion about HIV/AIDS. Start with looking at some statistics from the internet showing the prevalence within certain areas. Try to show trends in the data.

Lesson focus

Carefully work through the challenge on page 64. This will give pupils an example of how to calculate the relevant values to identify trends and understand the data.



Answers

Challenge page 64

Country	Population	HIV/AIDS Patients		
		Male	Female	Male:Female Ratio
A	800 000	8 000	12 000	2:3
B	2 000 000	22 000	32 000	11:16
C	1 000 000	23 000	18 000	23:18
D	1 300 000	30 000	20 000	3:2
E	1 010 000	30 000	50 000	3:5

1. 4:5; 2. 5:6; 3. 4:11; 4. 3:8

There is a general increase in the amount of people infected with HIV which influences the nature of the ratios.

Puzzle page 65

Ratio = 5:5005

Exercise 3

- 57:182
- 7:101

Question number	Country	Population	HIV/AIDS Patients		
			Male	Female	Male:Female ratio
3	A	6 220 000	240 000	200 000	6:5
4	B	2 650 000	300 000	350 000	6:7
5	C	2 120 000	40 000	80 000	1:2
6	D	4 300 000	140 000	100 000	7:5
7	E	4 360 000	120 000	240 000	1:2
8	F	6 200 000	140 000	60 000	7:3

Exercise 4

- a) 19:1200; b) 3:100; c) 13:700
- a) 15:26; b) 17:3; c) 69:110
d) Angola, Ghana, Cameroon, Zimbabwe

Country	Children with HIV/AIDS: Population	Decimal rate = children with HIV/AIDS/population
Angola	11 000:12 000 000 = 11:12 000	0.000916
Cameroon	45 000:18 000 000 = 1:400	0.0025
Ghana	17 000:18 400 000 = 17:18 400	0.000923
Zimbabwe	120 000:12 891 000 = 40:4297	0.0093

Assessment

Pupils should be able to find the ratio of two populations.

Assess their performance in Exercises 3 and 4.

Extension activity

Interactive learning diagram: Right-angled triangles

Lesson 5 *Pupil's Book page 67*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 67

- 1: ~~38~~ 000
- ~~4~~: ~~7~~
- a) 5:4; b) 4:5; c) 5:9
- 4:5; 5. 1:4; 6. 7:4

Objectives

By the end of this unit, each pupil should be able to:

- Express one number as a percentage of another number
- Calculate numbers expressed as a percentage of another
- Express one population as a percentage of another population.

**Suggested resources**

metre ruler; base 10 square – like the one on page 68 of the Pupil's Book

**Frequently asked questions**

Q *What should pupils understand?*

A Percentages were invented so that people did not have to use decimals. They are just numbers that are 100 times as big as the decimal. The word 'percentage' means 'out of 100'.

**Evaluation guide**

Pupils to:

- Express given numbers as a percentage of another.
- Calculate given numbers expressed as a percentage of another.
- Solve problems on percentage increase involving HIV/AIDS infection and other related problems.

Lesson 1 *Pupil's Book page 68***Preparation**

You will need to have: Pupil's Book; metre ruler.

**Starter activity**

Use a metre ruler and call it 100%.

Pupils measure shorter lengths in centimetres, and express them as a fraction of the length of the ruler by using 100ths. The numerator is the percentage. Example: A book is 30 cm tall. It is $\frac{30}{100}$ of the metre. So it is 30% of the metre.

Pupils measure lengths over 1 metre in centimetres, and express them as an improper fraction of the length of the ruler by using 100ths.

Example: A boy is 160 cm tall. He is $\frac{160}{100}$ of the metre. So he is 160% of the metre.

**Lesson focus****Use of a 10 × 10 square – page 68 Pupil's Book**

The 10 × 10 square = 1 = 100%. So any fraction of the 100 squares on it shows a percentage.

Make similar squares for your pupils. Pupils should colour in some of the squares, maybe in an attractive pattern. They can name the percentage coloured. Allow pupils to colour in the whole 100% and also more squares. Pupils should state the percentage of the 10 × 10 square that is now coloured. The answer will be over 100%.

**Answers****Exercise 1**

- | | |
|----------------------------|----------------------------|
| a) $\frac{9}{100} = 9\%$ | b) $\frac{65}{100} = 65\%$ |
| c) $\frac{35}{100} = 35\%$ | d) $\frac{70}{100} = 70\%$ |
| e) $\frac{95}{100} = 95\%$ | f) $\frac{45}{100} = 45\%$ |
- | | | |
|--------|--------|--------|
| a) 25% | b) 58% | c) 72% |
| d) 89% | e) 84% | f) 27% |
| g) 13% | h) 60% | i) 10% |
- | | |
|-------------------------------------|-------------------------------------|
| a) $\frac{5}{100} = \frac{1}{20}$ | b) $\frac{45}{100} = \frac{9}{20}$ |
| c) $\frac{65}{100} = \frac{13}{20}$ | d) $\frac{38}{100} = \frac{17}{50}$ |
| e) $\frac{80}{100} = \frac{4}{5}$ | f) $\frac{98}{100} = \frac{49}{50}$ |

Assessment

Pupils should be able to demonstrate the meaning of 100%, and both smaller and larger percentages, using both lengths (e.g. compare to 1 metre) and area (compared to a 10×10 square as the 100%).

Assess their performance in Exercise 1 and the hands-on activities, including the card game.

Extension activity

Interactive learning diagram: Percentage fraction

Support activity

Interactive learning diagram: Percentage square, Percentage line, Percentage full, Percentage pie

Lesson 2 *Pupil's Book page 70*



Preparation

You will need to have: Pupil's Book.



Starter activity

Ask a group of four girls and two boys to come to the front of the class.

What percentage is the number of boys of the number of girls?

(It is 2 compared to 4. This is the fraction $\frac{2}{4}$, which is $\frac{1}{2}$ or 50%. Note that we can also call it the decimal 0.5 which is 50 hundredths.)

What percentage is the number of boys of the number of girls?

(It is 4 compared to 2. This is the fraction $\frac{4}{2}$, which is 2 or 200%.)



Lesson focus

Groups of pupils repeat the activities for other numbers of boys and girls. Put this table on the board.

Number of boys	2	2	4	5	8
Number of girls	4	5	6	2	6
Boys as fraction of Girls	$\frac{2}{4}$				
Boys as percentage of Girls	50				
Girls as fraction of Boys	$\frac{4}{2}$				
Girls as percentage of Boys	200				

Go over the example on page 70, focusing on questions 1 and 2.

For question 3 we can recognise that 25% is $\frac{1}{4}$, and so we can quickly divide the 48 by 4.

This leads us to memorise a list of equivalent fractions and decimals.

fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{8}$
percent	50	25	75	$33\frac{1}{3}$	$66\frac{2}{3}$	$12\frac{1}{2}$

It is good if pupils memorise these as it speeds up getting the right answers, and enables them to find shorter quicker ways to solve maths problems.



Answers

Puzzle page 71

36 people preferred rugby

Challenge page 72

Depends on the chosen time given. Here is a list of a few % of 360:

1% = 3.6; 5% = 18; 20% = 72; 50% = 180

70% = 252; 99% = 356.4 etc.

Exercise 2

1. **a)** 20%; **b)** 80%; **c)** 76%; **d)** 90%; **e)** 70%;
f) 66.67%

2. **a)**

Subject	Marks Scores	Mark Obtainable	%
English studies	43	60	71.67%
Mathematics	54	70	77.14%
Basic science and technology	90	120	75%
Civics	45	80	56.25%
Computer studies	138	150	92%
Social studies	51	90	56.67%
Cultural and creative arts	30	50	60%

b) Computer studies; **c)** Civics

d) English studies, mathematics, basic science and technology and computer studies

3. **a)** 60%; **b)** 40%; **c)** 40%; **d)** 40%

4. **a)** 10%; **b)** 40%; **c)** 30%; **d)** 2.5%

Assessment

Pupils should be able to express one amount as a fraction and as a percentage of another amount. Assess their performance in the group activity.

Extension activity

Interactive learning diagram: Percentage converter

Support activity

Interactive learning diagram: Fraction to Percentage

Lesson 3 *Pupil's Book page 72; Workbook page 17*

Preparation

You will need to have: Pupil's Book; Workbook.

Starter activity

Make a Fraction-Decimal-Percentage sheet. Use this to review the equivalences of fractions and percentages and also decimals and percentages. Look at entries such as fifths, tenths, twentieths, sixths, and so on.

Lesson focus

Expressing one number (quantity) as a percentage of another requires that they are in the same units. If the numbers are 'messy' it is useful to use a calculator to perform the division. This gives the answer as a decimal, so we must know how to convert any decimal number to a percentage. There are two ways to think about it. They are:

- look at the number of hundredths:
for example 0.245 has 24.5 hundredths and so it is 24.5%.
- multiply the decimal by 100:
(move the digits left two places or the decimal point right) two places

Go over the example on page 71.

Answers

Exercise 3

1. 1%; 2. 0.25%; 3. 12.86%

Worksheet 9

1. a) 50%; b) 30%; c) $33\frac{1}{3}\%$; d) 60%; e) 40%;
f) 80% g) 30%; h) 25%; i) 90%; j) 30%;
k) 50%; l) 80%
2. a) 8; b) 76; c) 6 496; d) 30; e) 20;
3. a) 25%; b) 80%
4. 300 metres

Assessment

Pupils should be able to express one number (quantity) as a percentage of another.

Assess their performance in Exercise 3.

Extension activity

Interactive learning diagram: Percentage converter

Support activity

Interactive learning diagram: Fraction to Percentage

Homework activity

Complete WB Worksheet 9.

Lesson 4 *Pupil's Book page 74*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 74

1. a) $\frac{2}{100} = 2\%$ b) $\frac{33}{100} = 33\%$
c) $\frac{64}{100} = 64\%$ d) $\frac{95}{100} = 95\%$
e) $\frac{52}{100} = 52\%$
2. a) $\frac{11}{50}$; b) $\frac{43}{100}$; c) $\frac{13}{20}$; d) $\frac{47}{50}$; e) $\frac{17}{20}$
3. a) $\frac{70}{100}$; b) $\frac{83.3...}{100}$; c) $\frac{12.5}{100}$; d) $\frac{65}{100}$; e) $\frac{44}{100}$
4. a) 20%; b) 50%; c) 33.3...%; d) 25%;
e) 2.6...%; f) 20%; g) 5%; h) 10%; i) 50%
5. a) 9%; b) 91%
6. a) 62.5%; b) 37.5%
7. a) 140 girls; b) 60% boys

Objectives

By the end of this unit, each pupil should be able to:

- Solve problems on percentage increase and decrease
- Correctly calculate percentage increase such as in population growth
- Solve quantitative aptitude problems on percentage increase and decrease.



Suggested resources

counters



Common errors that pupils make

This topic is complex and can result in many misunderstandings. It is important to make sure that pupils understand the basic concepts.



Evaluation guide

Pupils to:

1. Calculate percentage increase of a given population growth.
2. Calculate percentage decrease of a given population growth.
3. Solve quantitative aptitude problems on percentage increase and decrease.

Lesson 1 *Pupil's Book pages 75 and 76*



Preparation

You will need to have: Pupil's Book; counters.



Starter activity

Pose this problem: Two counters 'magically' increase to three counters.

What is the increase? (1)

What is the increase *as a percentage of the original 2 counters*? ($\frac{1}{2}$ or 50%)

Pose another problem: 4 counters 'magically' increase to 5 counters.

What is the increase? (1)

What is the increase *as a percentage of the original 4 counters*? ($\frac{1}{4}$ or 25%)



Lesson focus

Groups of pupils work together on similar problems in this table.

Original	2	4	4	5	5	5
New	3	5	7	6	7	8
Increase	1					
Fraction	$\frac{1}{2}$					
Percent	50%					

Finding the increase and new amount

Some questions in Exercise 1 ask us to find the new price after a particular increase. First we must find the increase, and then add it.

Simple model: (You could use pupils for counters!)

Increase 20 counters by 30%.

Step 1: find 30% of 20. Now $30\% = \frac{3}{10}$ so we are finding $\frac{3}{10}$ of 20. $\frac{3}{10} \times 20 = 6$. This is the increase.

Step 2: add the increase to the original.

$20 + 6 = 26$. This is the new amount.

Use the examples on pages 75 and 76.



Answers

Challenge page 77

a) 1 500%; b) 66.67%

Exercise 1

1. a) 600; b) 10 metres; c) 272.8 litres; d) ~~4~~ 320

2. a) 6.25%; b) 25%; c) 136.84%; d) 11.1...%

3.

Name	Salary per month	Increase	% Increase
Mr Abiola	₦30 000	₦4 500	15%
Mrs Efuk	₦96 000	₦8 000	8.3...%
Mrs Bako	₦48 000	₦2 400	5%
Mrs Chukwu	₦250 000	₦10 000	4%

4. 9.524%; 5. ₦5 650; 6. ₦12 540

Assessment

Pupils should be able to find a percentage increase given two amounts.

Assess their performance in group activities and Exercise 1.

Extension activity

Interactive learning diagram: Percentage problems

Support activity

Interactive learning diagram: Percentage increase

Lesson 2 *Pupil's Book page 77*

Preparation

You will need to have: Pupil's Book.

Starter activity

This lesson involves the same ideas as the previous lesson, and so is another chance for you to make sure that all pupils understand these ideas. We are finding the increase as a percentage of the original. If you know both populations you can find the increase by subtracting the smaller from the larger. Then you find the percentage by first finding the fraction $\frac{\text{increase}}{\text{original}}$ and multiplying by 100 to make it into a percentage.

Lesson focus

Dual number line

This approach to understanding this topic might be helpful. We use two parallel number lines. For example, let us look at 4 increasing to 5.

0	1	2	3	4	5
0%				100%	?

The top number line has 0 up to the increased amount. The lower number line has 0% and 100% under the original amount.

This helps us to imagine how the percentages relate to the numbers. In this case we can see that the 5 is 25% more than the 4 (100%) and is at 125%.

It is possible to skip the first step described in the starter activity of finding the increase by subtracting. You could just divide the new value by the old value and the difference from 100% gives the increase. $5 \div 4 = 125\%$, so the increase (from 100%) is 25%.

Use this approach for the problem in the example on page 77.

For the example on page 78 we want to find the increase. So we find 5% of the original, and add it onto the original number.

Answers

Exercise 2

1. a) 20 950; b) 4.75% 2. 20%; 3. 77%

Assessment

Pupils should be able to find percentage increase.

Assess their performance in Exercise 2 and the game.

Extension activity

Interactive learning diagram: Percentage problems

Support activity

Interactive learning diagram: Percentage increase

Lesson 3 *Pupil's Book page 78; Workbook pages 18 and 19*

Preparation

You will need to have: Pupil's Book; Workbook; counters.

Starter activity

Pose this problem: Two counters 'magically' decrease to one counter.

What is the decrease? (1)

What is the decrease as a percentage of the original 2 counters? ($\frac{1}{2}$ or 50%)
 Pose another problem: 4 counters 'magically' decrease to 3 counters.
 What is the decrease? (1)
 What is the decrease as a percentage of the original 4 counters? ($\frac{1}{4}$ or 25%)



Lesson focus

Groups of pupils work together on similar problems in this table.

Original	3	5	7	6	7	8
New	2	4	4	5	5	5
Decrease	1					
Fraction	$\frac{1}{2}$					
Percent	50%					

Pupils should quickly see the similarity to percentage increase.

Dual number line

Again, this approach to understanding this topic might be helpful. We use two parallel number lines. For example, let us look at 4 decreasing to 3.

0	1	2	3	4
0%			100%	?

The top number line has 0 up to the original amount. The lower number line has 0% under 0 and 100% under the original amount.

This helps us to imagine how the percentages relate to the numbers. In this case we can see that the 3 is 25% less than the 4 (100%) and is at 75%.

Note that you can skip the first step of finding the decrease by subtracting. You just divide the new value by the old value and the difference from 100% gives the decrease. $3 \div 4 = 75\%$, so the decrease (from 100%) is 25%.



Answers

Exercise 3

1. **a)** 33.3...%; **b)** 25%; **c)** 10%; **d)** 9.19%;
e) 11.1...%; **f)** 50%

2.

Patient	Initial weight	Present weight	% decrease in weight
A	60.5 kg	40 kg	33.88%
B	50 kg	25 kg	50%
C	85 kg	59.5 kg	30%
D	96 kg	66 kg	31.25%
E	51.43 kg	45 kg	12.5%

3. 86.4 km/h; 4. 16%; 5. 25%

Challenge page 80

₹1 250

Exercise 4

1. **a)** 10.71%; **b)** 16.6...%; **c)** 22.2...%; **d)** 90%
 2. 231 players
 3. **a)** 280 people, 3 080 people; **b)** 2 926 people

Worksheet 10

1. **a)** 9 387 013 in 2007; 9 668 624 in 2008
b) 2010 **c)** 25.64% **d)** 3:2
 2. ₹5 850; 3. ₹1 350; 4. 37 %; 5. 12.5 %
 6. Approximately 82
 7. **a)** ₹36 000 **b)** ₹27 390
c) ₹9 630 **d)** ₹96 000
e) ₹421 800 **f)** ₹140 490

Assessment

Pupils should be able to find percentage decreases given two amounts.

Assess their performance in Exercises 3 and 4.

Extension activity

Interactive learning diagram: Percentage problems

Support activity

Interactive learning diagram: Percentage decrease

Homework activity

Complete WB Worksheet 10.



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 81

1. 20%;
2. a) 11 200 people; b) 12% increase;
3. 150%;
4. a) 4; b) 0.4; c) 30; d) 100; e) 192; f) 139.65;
5. a) ~~₹~~96; b) 504 kg; c) 1.8 m; d) 1 hr 36 min;
6. a) ~~₹~~360; b) 108 kg; c) 31.5 km; d) 5.85 £;
7. 18.75%;
8. ~~₹~~2 285.71;
9. ~~₹~~144;
10. 25%

Pupil's Book page 82

Objectives

This project requires pupils to work with ratio, percentage and applications of these concepts in the world, and write a report based on their findings. They then create a poster to go with their reports. The project is based on the figures for people living with HIV/AIDS in Africa.

**Guidelines**

You will need to have the following resources:
Pupil's Book, Internet access or library resources, and materials to make a poster.

Pupils must work on their own and for the first part of the project, pupils need to research and then follow these steps, as mentioned on page 82 of the PB.

1. Pupils must use resources in a library or on the Internet to find the following statistics:
 - a) The population of the different states of Nigeria.
 - b) The numbers of the population that live with HIV/AIDS in these states.
 - c) The numbers of the people that live with HIV/AIDS that are women and that are men.
 - d) The population of the ten other most populated states in Africa and arrange them in order of the largest population to the smallest population. Compare them to the figures for Nigeria.
 - e) The number of people in those states that live with HIV/AIDS.
 - f) The number of children in all those states (and Nigeria) that live with HIV/AIDS.
2. Pupils calculate ratios where possible, for example, the ratio of people that live with HIV/AIDS in Nigeria to the total population of Nigeria. Pupils must do this for all the countries for which they have data.
 3. Pupils calculate percentages, using the ratios.
 4. Pupils must try to compare results for more than one year or period so that they can see if there are developing trends.
 5. Pupils then write a report that shows all the information that they found and the conclusions they were able to make.
 6. Pupils make a poster to go with their reports. They can use pictures and drawings to illustrate their findings. Pupils may find it helpful to organise their findings in a table or tables.

Pupils' work must be informative and as accurate as possible.

Assessment

Because the pupils' findings will be different, assess pupils on:

- researching the correct information using reliable and credible sources
- collecting data
- calculating ratios
- calculating percentages using ratios
- comparing results
- writing a report
- creating a poster to represent their data.

Objectives

For this investigation pupils are required to investigate numbers and number patterns.



Guidelines

Pupils work on their own and write down their answers for this Investigation.



Answers

1. a) $33\ 334 \times 33\ 334 = 1\ 111\ 155\ 556$; $333\ 334 \times 33\ 334 = 111\ 111\ 555\ 556$; $3\ 333\ 334 \times 3\ 333\ 334 = 11\ 111\ 115\ 555\ 556$
- b) Each time you add a 6 to each whole number, the answer increases by adding a 4 and an 8. $6\ 667 \times 6\ 667 = 44\ 448\ 889$; $66\ 667 \times 66\ 667 = 4\ 444\ 488\ 889$
2. a) $1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$; $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$; $1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36$; $1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 49$; $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 64$; $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 81$
- b) The sums are all perfect squares of the highest number in the pattern.
- c) 10th line = 100; 11th line = 121
3. a) $1\ 111 \times 1\ 111 = 123\ 4321$; $11\ 111 \times 11\ 111 = 123\ 454\ 321$; $111\ 111 \times 111\ 111 = 12\ 345\ 654\ 321$; $1\ 111\ 111 \times 1\ 111\ 111 = 1\ 234\ 567\ 654\ 321$; $11\ 111\ 111 \times 11\ 111\ 111 = 123\ 456\ 787\ 654\ 321$; $111\ 111\ 111 \times 111\ 111\ 111 = 12\ 345\ 678\ 987\ 654\ 321$

Each time you add a 1, for example five 1s, the number starts as 12345 (up to 5), and then the next digit in the answer works its way back from 5, i.e. 4321 – the first half of the sum builds up, and the second half of the sum it breaks down

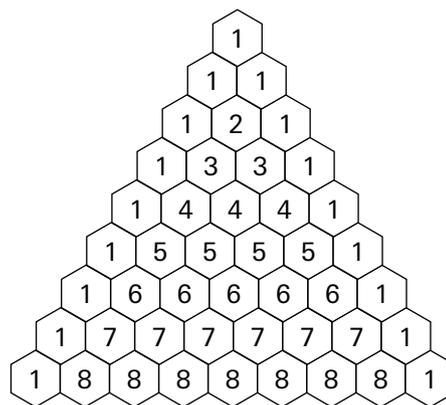
b) For the 7th line it reaches up to 10 – this will be expressed as 00 – then it will work itself

back again, i.e. 12345678900987654321.

For the 8th line, after the 0, the number starts at 1 again, but because it's the second set of numbers being built up again it takes a 2 and a 0, then the number is broken down again, i.e. 1234567890120987654321.

For the 9th line, it will then be 123456789012320987654321

4. a)



- b) Each time you increase in number, i.e. from 3 to 4, you increase the number of that number by 1.
- c) The drawback will be that you might not have enough space; each time you increase, the hexagon gets bigger and so does the number.

Assessment

Because the pupils' findings will be different, assess pupils on:

- investigating number patterns
- estimating and predicting the number patterns.

Pupil's Book page 84

Objectives

This assessment is a summative assessment of work that pupils have covered in Units 1 to 10. It has been designed to assess the pupils' mathematical understanding. It is also important that it is completed by individuals and not with the support of other pupils as this would not uncover any difficulties a pupil may be having with particular concepts.

 **Guidelines**

Pupils must work through the questions on their own. However, observing pupils while they are completing the assessment provides further information about each pupil and his or her level of understanding.

 **Answers**

1. a) 52 000 000; 53 000 000; 54 000 000;
55 000 000; 56 000 000; 57 000 000;
58 000 000
- b) 356 132 000; 456 132 000; 562 132 000;
762 132 000; 862 132 000; 962 132 000
2. a) nine hundred and sixty-five million three
hundred and forty-two thousand seven
hundred and eighty-one.
- b) 2 000; 40 000; 60 000 000; 80
- c) 60 042 080; d) 59 700 000
3. a) 4: 400; 7: 70; 5: 5
3: 0.3 or $\frac{3}{10}$; 7: 0.07 or $\frac{7}{100}$;
2: 0.002 or $\frac{2}{1000}$
- b) 4.7
4. a) 24: 24, 48, 72, 96, 120
32: 64, 96, 128, 160
- b) 24: 2, 3; 32: 2
- c) 96; d) $3 \times 8 \times 2 \times 2 = 96$; e) 8
5. a) Nigeria, 125 750 356; Egypt, 76 117 421;
Ethiopia, 71 336 571; Congo, DR,
58 317 930; South Africa, 44 448 470;
Algeria, 32 129 324
- b) 342 897 158
6. a) 18; 316; 12; 58; 34 b) $\frac{25}{50}$; $\frac{6}{10}$; $\frac{3}{4}$; $\frac{4}{5}$

7. a) $\frac{2}{5} < \frac{3}{7}$; b) $\frac{3}{4} > \frac{4}{7}$; c) $\frac{5}{8} < \frac{7}{9}$
8. a) $\frac{10}{20} = \frac{1}{2} = 0.5$; b) $\frac{12}{18} = \frac{2}{3} = 0.66 \dots$
- c) $\frac{10}{25} = \frac{2}{5} = 0.4$; d) $\frac{12}{16} = \frac{3}{4} = 0.75$
- e) $\frac{15}{20} = \frac{3}{4} = 0.75$, $\frac{12}{16}$ and $\frac{15}{20} = 0.75$
9. a) 3.1; b) 5.7; c) 9.7; d) 11.3; e) 19.5
10. a) $\frac{2}{10}$; b) $\frac{1}{4}$; c) $4\frac{3}{20}$; d) $6\frac{17}{20}$; e) $12\frac{3}{5}$
11. 50 m freestyle:
Women's qualifying time = 26.02s
Men's qualifying time = 23.3s
100 m freestyle:
Women's qualifying time = 56.89s
Men's qualifying time = 51.09s
12. a) 1:5; b) 4:5; c) 3:3
d) 180 ml butter; 45 ml baking powder
13. 23 460 000
- 14.

Number of Days	1	2	3	4	5	6	7	8	9	10
Number of Litres	50	100	150	200	250	300	350	400	450	500

- a) 350 litres;
- b) $100\ell = \text{N}1.5$; $1\ 000\ell = \text{N}15$; $5\ 000\ell = \text{N}75$
15. a) 10 km; b) 1 hour and 45 minutes
16. 11:14
17. a) 49%; b) 23%; c) 70%; d) 52.5%
18. a) $\frac{7}{10}$; b) $\frac{21}{50}$; c) $\frac{11}{50}$; d) $\frac{13}{20}$
19. a) $\text{N}105\ 000.00$; b) $\text{N}9\ 300$
20. a) 24:1; b) 54 pupils
c) 414 pupils and 15 teachers
d) 15.38%

Assessment

On completion of the assessment, teachers should look for correct answers and mistakes made by pupils. They should also check to see if there is a pattern in terms of any particular question causing a significant number of pupils' difficulties. By analysing the results of an assessment, they can identify weaknesses in individuals and provide the necessary support, and also strengths of individuals and provide them with more challenging activities. They will also be able to identify any weaknesses in their teaching programme and make adjustments where, or if, necessary.

Objectives

By the end of this unit, each pupil should be able to:

- Add any set of numbers
- Solve problems on subtraction of whole numbers
- Solve word problems involving addition and subtraction of whole numbers.

**Suggested resources**

playing cards

**Frequently asked questions**

Q *Why is estimation important?*

A Estimation is the ability to judge whether or not an answer is reasonable. It is usually done by rounding the numbers to one digit, and adding (or subtracting). If the place values of the digits are different, the larger one is important. If they are the same, add.

Q *What mathematical skills are needed?*

A Remind pupils to be careful with place value.

**Evaluation guide**

Pupils to:

1. Solve given problems on addition of whole numbers.
2. Solve given problems on subtraction of whole numbers.
3. Solve given word problems involving addition and subtraction.

Lesson 1 *Pupil's Book page 87; Workbook page 20***Preparation**

You will need to have: Pupil's Book; Workbook; playing cards.

**Starter activity****Card game – Add numbers to millions**

Shuffle the cards. There may be up to three players. For this game, the 10, J, Q and K all show 0.

Deal out seven cards to make a number of millions. Underneath, deal out another number of millions.

Add the two numbers. The person with the biggest total wins.

**Lesson focus**

This lesson may be extended over three or four days. Use the ideas presented and plan your lesson(s) accordingly. Remind pupils that addition and subtraction are inverse operations. Take time to work through the word problems. You can judge how much time this lesson will require depending on your pupils ability.

**Answers****Exercise 1**

1. a) 7 066 634 b) 8 966 634
2. a) 8 794 631 b) 17 904 631
3. a) 9 364 363 b) 12 054 063
4. a) 3 854 585 b) 7 632 985

Exercise 2

1. a) 1 285 864 b) 1 596 964
2. a) 1 297 949 b) 5 697 859

Challenge page 89

(Pupils should show working out in columns)

1. $560\,260 + 1\,845\,762 + 1\,827\,194 = 4\,233\,216$
 $502\,180 + 290\,393 + 1\,510\,427 = 2\,303\,000$
 The first three countries use less than half the amount of oil than the last three.

Exercise 3

1. a) 640 884 b) 202 449
2. a) 1 232 820 b) 91 436
3. a) 771 091 b) 652 029
4. a) 888 465 b) 291 138
5. a)

Country	Oil production – oil usage
Venezuela	2 521 240
Saudi Arabia	7 629 358
UK	247 806
South Africa	-272 235
Nigeria	244 809 700
Iran	2 469 214

- b) 245 330 038; c) 3 356 189; d) 2 387 454

Challenge page 92

1. a) 1 311 km; b) 2 196 km
2. 3 106 km in total

Exercise 4

1. 4.326 kg; 2. 0.545 ℓ
3. a) 4 492 km
 - b) Nearest city to Lagos is Accra
Furthest city from Lagos is Durban
 - c) Johannesburg to Durban is 4 491 km closer than from Lagos to Durban
Johannesburg to Dakar is 4 239 km further from Lagos to Dakar
Johannesburg to Accra is 4 245 km further than from Lagos to Accra

Worksheet 11

1. a)

12	41	22
35	25	15
28	9	38
Magic total: 75		

2. a) 9 734

3. a) 91 824 888

4. 3

5. 15

- b)

34	35	30
29	33	37
36	31	32
Magic total: 99		

- b) 4 905

- b) 21 397 085

Assessment

Pupils should be able to add and subtract whole numbers.

Assess their performance in games and Exercises 1 to 4.

Extension activity

Interactive learning diagram: Magic square, Estimate

Support activity

Interactive learning diagram: 0–99 patterns, Backwards adding

Homework activity

Complete WB Worksheet 11.

Lesson 2 *Pupil's Book page 92*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 92

1. a) 7 374 247 b) 3 216 211
2. a) 3 210 003 b) 4 354 064
3. a) 447 060 b) 706.274
4. a) -6 389 443 b) 36 577
5. a) 68 857.609 b) 218 010
6. 6 875 m
7. a) 758 090 km²; b) 2 344 527 km²
8. a) 1 665 km; b) 2 631 km
9. 2 771 km
10. Durban – 215 000
11. South Africa
12. Cape Town – 3 824 000; Durban – 3 241 000; Ekurhuleni – 3 614 000; Johannesburg – 4 127 000 – Total: 14 806 000

Objectives

By the end of this unit, each pupil should be able to:

- Multiply a 3-digit number by a 3-digit number
- Apply multiplication to daily life activities
- Solve problems on quantitative aptitude on multiplication
- Multiply decimal by decimal (to one decimal place)
- Multiply fractions by fractions.



Suggested resources

fraction strips; multiplication flash cards



Common errors that pupils make

Pupils can get confused when multiplication by a fraction or a decimal fraction makes the answer less than the number being multiplied. They have the idea that multiplying makes numbers bigger! Of course this is true when multiplying by numbers bigger than 1, but not for fractions or decimal fractions.



Evaluation guide

Pupils to:

1. Solve given problems involving multiplication by a 3-digit number.
2. Solve problems on quantitative aptitude involving multiplication.
3. Multiply decimals by decimals.
4. Solve problems involving multiplication of fractions by fractions.

Lesson 1 *Pupil's Book page 94*



Preparation

You will need to have: Pupil's Book; multiplication flash cards.



Starter activity

Multiplication by powers of 10

Ask pupils to multiply 435 by 10. Watch what happens. Some, who do not understand place value, will do this as a long multiplication, even multiplying by 0 and 1 and then adding.

When we multiply by 10 the digits get one place bigger. All they need to do is to move the digits left, and put a zero into the units place.

Let's look at some examples on a place value chart:

	<u>hundreds</u>	<u>tens</u>	<u>ones</u>
changes to		2	0
and		1	0
changes to	1	0	0
Also		1	2
changes to	1	2	0

The digits move left one place and we write 0 in the empty place on the right.

Please do not talk about 'adding a zero', we want pupils to know that adding 0 does not change the value of a number.



Lesson focus

You will probably have some pupils who cannot recall some of the multiplication facts. This is a worldwide problem. One solution is to narrow down the learning to what they need to remember.

Multiplication flash cards

Make small cards with the question (e.g. 6×7) on one side and the correct answer (42) on the back.

$$6 \times 7$$

$$42$$

Pupils can self-test, and find out which facts they need to set to memory. They then focus on those facts only and do the activity daily gradually reducing the number they still need to recall.

Pupils need to know that 6×7 is the same as 7×6 , and so there are only about half as many facts to learn as they might have thought.



Answers

Challenge page 95

- 1 483 948; 2 279 904; 3 2 953 080;
- 4 2 759 744; 5 10 238 540

Exercise 1

1. a) 19 100; b) 211 950; c) 290 320; d) 77 832;
e) 183 180; f) 34 375
2. a) 3 480; b) 19 866; c) 35 340; d) 22 156;
e) 17 526; f) 14 388

Assessment

Pupils should be able to multiply whole numbers.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Mirror products

Support activity

Interactive learning diagram: Multo Random, Multiplication in a table

Lesson 2 *Pupil's Book page 96; Workbook page 21*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Estimation with multiplication

In real life it is useful to be able to mentally estimate the answer to a multiplication.

Do that with one-digit rounding.

Replace every digit (except the one furthest to the left – the highest place value) with a 0. Round up or down to the nearest value of that digit.

Then multiply the single digits mentally and add the number of zeroes.

Example $3\ 456 \times 8\ 765$ becomes $3\ 000 \times 9\ 000$.
this is roughly 27 000 000.

The actual answer is 30 291 840, so 27 million is fairly close.



Lesson focus

Discuss situations in real life where we use multiplication. Make up suitable numbers and estimate the answers.

Note that for Exercise 3, Question 3 can be misleading. It seems that 6 cubes builds a staircase with 3 steps, so you might multiply by 4 to make one with 12 steps. But this is not so.



Answers

Exercise 2

1. a) ~~£~~1 500; b) ~~£~~3 600; c) ~~£~~45 000
2. a) k675; b) k810; c) k270
3. $6 \times \text{~~£~~87.50} = \text{~~£~~525}$

Exercise 3

1. 86 760 plants; 2. 8 760 hours; 3. 72 cubes;
4. $\frac{1}{3}$ of the pizza left after lunch

Worksheet 12

1. Answers will vary.

Assessment

Pupils should be able to recognise multiplication situations in real life and estimate answers.

Assess their performance in discussions and Exercises 2 and 3.

Extension activity

Interactive learning diagram: Estimate multiplication

Support activity

Interactive learning diagram: Long multiplication

Homework activity

Complete WB Worksheet 12, Question 1.

Lesson 3 *Pupil's Book page 98*



Preparation

You will need to have: Pupil's Book.



Starter activity

Ask pupils to answer this question. $0.2 \times 0.3 = ?$

As well as the correct answer, you should be asking how they know it is correct, i.e. why?

Do not be satisfied with answers about adding number of decimal places.

There are several answers that make some sense.

$\frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$ is one of them.

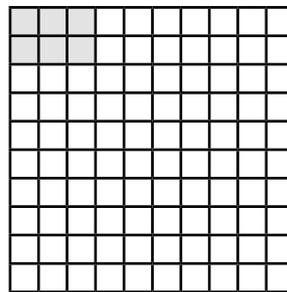
$\frac{1}{5}$ th of 0.3 metres is 6 cm (0.06 m) is another.



Lesson focus

Here is one explanation of why 0.2×0.3 is 0.06, based on area.

Draw a decimal square (10×10) on the board.



We will say that this has an area of 1, so the side lengths are each 1, and the small length along the sides are tenths.

Shade in an area 2 tenths by 3 tenths (that is 0.2×0.3). It is made from 6 squares, but these small squares are clearly hundredths. 0.06.

Change the multiplication

The factors of 0.2×0.3 are $2 \div 10 \times 3 \div 10$.

We can change the order of the operations, to get $2 \times 3 \div 10 \div 10$, which is $6 \div 100$. This is what is done in the example on page 98.



Answers

Exercise 4

- | | |
|-----------|-----------|
| a) 0.15 | b) 0.24 |
| c) 0.42 | d) 0.025 |
| e) 0.04 | f) 0.07 |
| g) 0.008 | h) 0.0028 |
| i) 0.0030 | |

Assessment

Pupils should be able to multiply tenths by tenths.

Assess their performance in Exercise 4.

Extension activity

Interactive learning diagram: Multiply by tenths

Support activity

Interactive learning diagram: Tenths \times tenths



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 101

1. **a)** 61 380; **b)** 260 650; **c)** 723 503; **d)** 998 001
2. 9 996 cm²
3. 9 999.49 cm²
4. 1.5964 km
5. **a)** 0.1; **b)** 0.4; **c)** 0.7; **d)** 0.4; **e)** 0.48; **f)** 0.44
6. **a)** $\frac{3}{8}$; **b)** $\frac{3}{4}$; **c)** $\frac{686}{25}$; **d)** $\frac{152}{9}$

Objectives

By the end of this unit, each pupil should be able to:

- Calculate squares of numbers up to 500
- Calculate the square roots of perfect squares.

**Frequently asked questions**

Q *What do the pupils need to understand?*

A There are many patterns with square numbers; the PB suggests a few of them. Pupils should be alert to these patterns.

**Evaluation guide**

Pupils to:

1. Calculate squares of given numbers up to 500.
2. Find the square roots of given perfect squares.

Lesson 1

Pupil's Book page 102; Workbook page 23

**Preparation**

You will need to have: Pupil's Book; Workbook.

**Starter activity**

Find the factors of different numbers, including squares. Pupils will find that squares always have an odd number of factors.

Factors of 9 are 1, 3, 9

...of 16 are 1, 2, 4, 8, 16

...of 36 are 1, 2, 3, 4, 6, 9, 12, 36 and so on.

**Lesson focus**

Pupils do not need to learn to multiply a number by itself. But to better understand how to find square roots, they should review how to write square numbers as the product of powers of prime factors.

Revise prime factors, using factor trees.

For example $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

Each prime factor is there twice. So this can be written as the product of two identical numbers.

$$36 = (2 \times 3) \times (2 \times 3) = 6 \times 6$$

Patterns involving squares

The exercise reveals several patterns involving square numbers. The example shows that to get from one square number (e.g. 100) to the next (121) you need to add twice the first number that is squared (10) plus 1 more. This is explored in question 1b). It is extended to other patterns in question 5.

When pupils list the square numbers in question 1, they will certainly find some pairs of squares that add to other squares. We will meet them again in Unit 20 Length, where we meet the Pythagorean relationship ($a^2 + b^2 = c^2$).

Another relationship to be met later is the difference between perfect squares (question 8). If the numbers being squared differ only by 1, then the difference between their squares is their sum.

**Answers****Challenge page 104**

a) $\sqrt{36} = 6$; b) $\sqrt{81} = 9$; c) $\sqrt{144} = 12$;

d) $\sqrt{225} = 15$

The square root of the product of two square numbers is equal to the product of the two initial numbers. e.g. $2^2 \times 3^2 = 36$ square root of $36 = 2 \times 3 = 6$

Challenge page 106

$$256 = 16 \times 16; 144 = 12 \times 12$$

Exercise 1

1. 1, 3; 2. 4, 5; 3. 9, 7; 4. 16, 9; 5. 25, 11; 6. 36, 13; 7. 49, 15; 8. 64, 17; 9. 81, 19; 10. 100, 21; 11. 121, 23; 12. 144, 25; 13. 169, 27; 14. 196, 29; 15. 225, 31; 16. 256, 33; 17. 289, 35; 18. 324, 37; 19. 361, 39; 20. 400, 41; 21. 441, 43; 22. 484, 45; 23. 529, 47; 24. 576, 49; 25. 625, 51; 26. 676, 53; 27. 729, 55; 28. 784, 57; 29. 841, 59; 30. 900, 61; 31. 961, 63; 32. 1024, 65; 33. 1089, 67; 34. 1156, 69; 35. 1225, 71; 36. 1296, 73; 37. 1369, 75; 38. 1444, 77; 39. 1521, 79; 40. 1600, 81; 41. 1681, 83; 42. 1764, 85; 43. 1849, 87; 44. 1936, 89; 45. 2025, 91; 46. 2116, 93; 47. 2209, 95; 48. 2304, 97; 49. 2401, 99; 50. 2500
2. a) 225; b) 289; c) 400; d) 2 025
3. a) 6.25; b) 121; c) 225; d) 400
4. a) 4^2 , 6^2 , 15^2 , 25^2
b) 0.15^2 , 0.25^2 , 0.4^2 , 0.6^2
5. a) 9; b) 25; c) 16; d) 36
6. a) 13^2 ; b) 14^2 ; c) 16^2 ; d) 19^2 ; e) 21^2 ; f) 24^2 ; g) 26^2 ; h) 31^2
7. Answers vary
8. a) 3; b) 5; c) 7; d) 9
9. The numbers are 10 and 11, their squares are 100 and 121.

Worksheet 13

1. a) 1 369; b) 4 096; c) 27 889; d) 209 764; e) 15 876; f) 76 729; g) 26 896; h) 71 289; i) 209 764; j) 153 664; k) 110 889; l) 215 296; m) 197 136; n) 128 164; o) 149 769

Assessment

Pupils should be able to find the prime factors of any square number, and explore patterns involving square numbers.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Differences between squares

Support activity

Interactive learning diagram: Square numbers

Homework activity

Complete WB Worksheet 13, Question 1.

Lesson 2 *Pupil's Book page 105; Workbook page 23*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Ask pupils to name the square roots of two-digit numbers. They should know these from their multiplication fact knowledge.



Lesson focus

Use a simple example to demonstrate the method. For example $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

Each prime factor is there twice. So this can be written as the product of two identical numbers.

$$36 = (2 \times 3) \times (2 \times 3) = 6 \times 6$$

So the square root of 36 is 6. It is the number that is multiplied by itself to give 36.

The book uses the continued division method to find the primer factors. Your pupils might like to use factor trees.



Answers

Puzzle page 105

$$16 = 7; 25 = 7; 36 = 9; 49 = 13; 64 = 10; 81 = 9; 100 = 1; 121 = 4; 144 = 9$$

Exercise 2

1. a) 28; b) 31; c) 40; d) 44; e) 64
2. a) Not perfect square; b) Not perfect square; c) Not perfect square; d) Not perfect square; e) Perfect square

3. a) 15; b) 20; c) 22; d) 13; e) 30; f) 60

4. a) 35; b) 30; c) 9; d) 48

Worksheet 13

2. a) 18; b) 256; c) 325; d) 219; e) 500; f) 197;
g) 277; h) 343; i) 423; j) 88; k) 73; l) 442;
m) 358; n) 299; o) 137

Assessment

Pupils should be able to find the square root of a perfect square using factorisation.

Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Prime factor products

Support activity

Interactive learning diagram: Factor trees

Homework activity

Complete WB Worksheet 13, Question 2

Lesson 3 *Pupil's Book page 107*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 107

1. The difference of their squares equals the sum of the odd numbers doubled.
2. The difference of their squares equals four times the odd number in between them
3. a) 40; b) 80; c) 120; d) 160
4. a) 21; b) 27; c) 42; d) 49; e) 56
5. a) 225; b) 36; c) 18; d) 90; e) 63

6. 156.25 cm^2

7. $20^2 - 19^2 < 2^2 + 6^2$

8. a) 400; b) 504

9. $(25^2 - 20^2) + (25^2 + 20^2) = 1\,250$

Objectives

By the end of this unit, each pupil should be able to:

- Divide whole numbers by 1-digit numbers
- Divide whole numbers by 2-digit numbers with factors
- Divide whole numbers and decimals by 2-digit and 3 digit numbers.



Suggested resources

base 10 sheets



Common errors that pupils make

Pupils do not know the division facts derived from a multiplication fact.

When learning or revising multiplication tables, always try to link the multiplication facts with the related division fact, so that pupils get used to seeing the facts as pairs. They also need to be constantly reminded of the commutative laws of multiplication, so that if they know $7 \times 8 = 56$, they must also know $8 \times 7 = 56$.



Evaluation guide

Pupils to:

1. Solve given problems on division by 2-digit and 3 digit numbers.
2. Solve problems on quantitative aptitude involving division.

Lesson 1 *Pupil's Book page 108*



Preparation

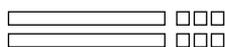
You will need to have: Pupil's Book; base 10 sheets.



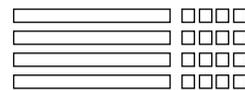
Starter activity

Use the base 10 sheets to demonstrate these divisions:

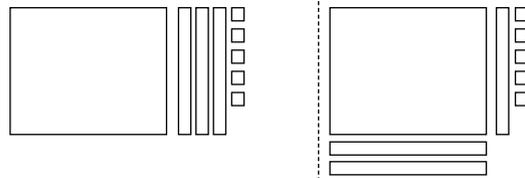
$26 \div 2$ (Make a rectangle with 26 with a width of 2, and find its length.)



$42 \div 3$ (Make a rectangle with 42 with a width of 3, and find its length. This means that one of the tens is changed to ten units.)



Another example: $135 \div 3$



$$\begin{aligned} 135 \div 3 \\ 120 \div 3 + 15 \div 3 \\ 40 + 5 = 45 \end{aligned}$$



Lesson focus

Short division

When we do the 'short method' we can multiply and subtract mentally.

Division by factors

This reduces a long division to two (or more) short divisions. It is quick and easy, but of course the divisor must have factors.



Answers

Exercise 1

1. a) $\pounds 6$; b) $\pounds 4$; c) $\pounds 3$; d) $\pounds 2$; e) $\pounds 2.5$; f) $\pounds 8$; g) $\pounds 6$; h) $\pounds 4$; i) $\pounds 4$; j) $\pounds 3\frac{1}{3}$; k) $\pounds 28$; l) $\pounds 20$

2. a) 7; b) 13; c) 11; d) 15; e) 73; f) 84

3. a) $15 = 5 \times 3$.

$$200 \div 5 = 40. \quad 40 \div 3 = 13\frac{1}{3}$$

$$200 \div 15 = 13\frac{1}{3}$$

b) $26\frac{1}{14}$

c) $42\frac{7}{18}$

d) $416\frac{8}{12} = 416\frac{2}{3}$

e) $41\frac{16}{24} = 41\frac{2}{3}$

f) $41\frac{32}{48} = 41\frac{2}{3}$

Assessment

Pupils should be able to perform correct short divisions by single digits, and divide by factors. Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Estimate to divide
Pupils may attempt the puzzle on page 108.

Puzzle page 108

2 468 fits the number description. It follows a pattern so you can keep on going infinitely.

Support activity

Interactive learning diagram: Division facts,
Dividing with remainders

Homework activity

Complete Exercise 1.

Lesson 2 *Pupil's Book page 109; Workbook page 24*

Preparation

You will need to have: Pupil's Book.

Starter activity

Show pupils how to quickly create a table of multiples of a number. Use 51, the divisor from the example on page 109.

Double it, and double it again, and again.

This gives $2\times$, $4\times$, and $8\times$.

For $3\times$, add $1\times$ and $2\times$. For $5\times$, add $4\times$ and $1\times$ (or divide $10\times$ by 2).

For $6\times$, add $2\times$ and $4\times$. For $7\times$ add another $1\times$.

For $9\times$, add $8\times$ and $1\times$. Here are the answers.

1	2	3	4	5	6	7	8	9
51	102	153	204	255	306	357	408	459

As you work through the example, show them how this saves time selecting the required multiple of 51 to solve the problem.

Lesson focus

Long division by two digits: $6\ 159 \div 34$

1 Estimate the answer.

$$6\ 000 \div 30 = 600 \div 3 = 200$$

2 Write the problem in this way.

$$34 \overline{)6159}$$

3 Since 34 does not divide into 6, divide 61 by 34. Since $34 \times 1 = 34$ and $34 \times 2 = 68$, it is 1. Write this here.

$$\begin{array}{r} 1 \\ 34 \overline{)6159} \end{array}$$

4 Multiply the 34 by the 1, and write it under the 61. Subtract, to give the remainder 7.

$$\begin{array}{r} 1 \\ 34 \overline{)6159} \\ \underline{-34} \\ 27 \end{array}$$

5 Note that there are 5 tens in the number. so with the 27 remainder there are 275 tens.

$$\begin{array}{r} 1 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \end{array}$$

6 Divide 275 tens by 34. This is where you need your 34 times table! Write it down.

$$1 \times 34 = 34$$

$$2 \times 34 = 68 \text{ (double)}$$

$$3 \times 34 = 102 \text{ (add the two above)}$$

$$4 \times 34 = 136 \text{ (double the } 2 \times 34)$$

$$5 \times 34 = 136 + 34 = 170 \text{ (half of } 34)$$

$$6 \times 34 = 170 + 34 = 204 \text{ (too small),}$$

$$7 \times 34 = 204 + 34 = 238 \text{ (too small), and}$$

$$8 \times 34 = 238 + 34 = 272. \text{ (just under) Got it!}$$

So $34 \times 8 = 272$, it is 8. Write the 8 (tens).

$$\begin{array}{r} 18 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \end{array}$$

- 7 Since $34 \times 8 = 272$, write 272 under 275, and subtract to find remainder 3.

$$\begin{array}{r} 18 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \\ \underline{-272} \\ 3 \end{array}$$

- 8 Note that 6 159 also has 9 units with the 3 tens there are 39 units. Write 9 in the units place.

$$\begin{array}{r} 18 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \\ \underline{-272} \\ 39 \end{array}$$

- 9 Divide 39 units by 34. Since $34 \times 1 = 34$, it is 1. Write the 1 in the units place.

$$\begin{array}{r} 181 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \\ \underline{-272} \\ 39 \end{array}$$

- 10 Since $34 \times 1 = 34$, write the 34 under the 0.39 and subtract leaving a remainder of 0.01.

$$\begin{array}{r} 181 \\ 34 \overline{)6159} \\ \underline{-34} \\ 275 \\ \underline{-272} \\ 39 \\ \underline{-34} \\ 5 \end{array}$$

The answer is 181, remainder 5.



Answers

Challenge page 110

1. 56.25; 2. 49.4; 3. 253.44...
4. 123.33...; 5. 59.73...

Exercise 2

1. a) 62; b) 53; c) 104; d) 77; e) 136; f) 197
2. a) 67.786; b) 89.2; c) 76.059; d) 151.806;
e) 247.5; f) 175.128

Worksheet 14

1. Own answers.

Assessment

Pupils should be able to divide by a two-digit number.

Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Division in a table

Support activity

Interactive learning diagram: Division in pictures

Homework activity

Complete WB Worksheet 14, Question 1, and check answers with a calculator.

Lesson 3 *Pupil's Book page 110*



Preparation

You will need to have: Pupil's Book.



Starter activity

Talk with the class about when they might need to divide by a three digit number. In real life most people use a calculator, usually on a phone. But it is still important to make an estimate, to have a rough idea of what the answer should be.



Lesson focus

Divide by a three-digit number

This uses the same approach, but it may be helpful to write the multiples of the divisor (1 to 9) first. Work through the example in the PB.

Answers

Exercise 3

- a) 22.748; b) 26.585; c) 9.911; d) 23.249;
e) 10.41; f) 18.364
- a) 55.201; b) 8.861; c) 5.959; d) 20.026;
e) 21.522; f) 20.029

Assessment

Pupils should be able to perform correct three-digit divisions.

Assess their performance in Exercise 3.

Extension activity

Interactive learning diagram: Division in a table

Support activity

Interactive learning diagram: Division in pictures

Homework activity

Complete Exercise 3, and check answers with a calculator.

Lesson 4 *Pupil's Book page 112; Workbook page 24*

Preparation

You will need to have: Pupil's Book.

Starter activity

Use equal multiplication or division by 10 to change the divisor into the simplest whole number.

Lesson focus

Equal division or multiplication by 10

Here is the idea of 'equal division by 10'.

$$\begin{array}{r} 150 \div 30 \\ \text{Divide by } 10 \Downarrow \Downarrow \text{Divide by } 10 \\ 15 \div 3 \end{array}$$

You can divide (or multiply) the two numbers in a division question by the same number and the answer is the same.

To work out $1.5 \div 300$, you must divide both sides of the divide by 100. You divide by 10 two times to get $0.015 \div 3$. The answer is 0.005, **with the same number of decimal places** as the 0.015.

Here is an example of 'equal multiplication by 10', done three times.

$$\begin{array}{r} 150 \div 0.003 \\ \text{Multiply by } 1\,000 \Downarrow \Downarrow \text{Multiply by } 1\,000 \\ 150\,000 \div 3 \end{array}$$

The answer is 50 000, **with the same number of zeroes** as the 150 000.

Answers

Challenge page 112

$3\,400 \div 24 = 141$ each and 16 will be left over.

Exercise 4

- a) 9.09...; b) 18.18...; c) 27.27...;
d) 63.63...; e) 72.72...; f) 81.81...
- a) 0.416; b) 0.416; c) 0.416; d) 0.416

Exercise 5

- a) 0.4; b) 2.4; c) 0.9; d) 4; e) 33.28; f) 4.25;
g) 36.654; h) 27.73
- 96.17 minutes; 3. 87.5 shoelaces; 4. 13.125 trays. So 13 trays can be filled.

Worksheet 14

- Answers will vary.

Assessment

Pupils should be able to convert a multiplication involving a division by a decimal number to an equal division by a whole number (using equal multiplication by 10).

Assess their performance in Exercises 4 and 5

Extension activity

Interactive learning diagram: Estimate to divide

Support activity

Interactive learning diagram: Divide with powers of 10

Homework activity

Complete WB Worksheet 14, Question 2.

Lesson 5 *Pupil's Book page 112***Preparation**

You will need to have: Pupil's Book.

**Lesson focus**

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

**Answers****Revision exercise page 113**

1. **a)** 7; **b)** 13.714; **c)** 11; **d)** 23
2. **a)** 56; **b)** 85; **c)** 26; **d)** 68
3. **a)** 104.187; **b)** 69; **c)** 30.48; **d)** 49.216
4. **a)** 0.56; **b)** 0.085; **c)** 0.026; **d)** 6.8
5. 45 students per class
6. 43 750 paperclips
7. 142.857 ml each

Objectives

By the end of this unit, each pupil should be able to:

- Use basic operations in the right order
- Solve quantitative aptitude problems involving BODMAS.

X Common errors that pupils make

Pupils forget the zero in the answer line when multiplying by the multiple of ten.

Constantly remind them, by partitioning, what they are multiplying by. Question them: “What is the effect of multiplying a whole number by a multiple of ten? What is the approximate answer to this calculation?”

Evaluation guide

Pupils to:

1. Solve problems using BODMAS.
2. Solve quantitative aptitude problems involving BODMAS.

Lesson 1 *Pupil's Book page 114; Workbook page 25***Preparation**

You will need to have: Pupil's Book.

Starter activity

How many answers might be possible for this question if there was no agreed order?

$$12 \div 3 \times 2 - 1 + 5$$

The correct answer is $4 \times 2 - 1 + 5 = 12$

If we did multiplication first: $12 \div 6 - 1 + 5 = 6$

If we did addition before subtraction:

either $4 \times 2 - 6 = 2$ or $12 \div 6 - 6 = 2 - 6 = -4$

There are several; other possible answers. This reinforces the need for an agreement.

Lesson focus

Explain how to get the correct answer for $12 \div 3 \times 2 - 1 + 5$.

There are no brackets,

We do multiplication and division in the order they come from left to right (so $12 \div 3 \times 2$ becomes $4 \times 2 = 8$).

We do addition and subtraction in the order they come from left to right (so $8 - 1 + 5$ becomes $7 + 5 = 12$).

Go over the examples on page 114.

Look at the example on page 115, to see why the brackets make a difference. Stress that brackets must be worked out first.

Not BODMAS but BI, where the I stands for Indices. (For example 3^2 must be changed to 9 after the brackets and before the rest.)

Note that the order applies to all types of numbers, including fractions and decimals.

Answers**Puzzle page 116**

1. 24; 2. 24; 3. 24; 4. 24; 5. 24; 6. 24; 7. 24

Exercise 1

1. a) 24; b) 12; c) 8; d) 6
2. a) 8; b) 4; c) 9; d) 2; e) 1; f) 10; g) 13; h) 12
3. a) 0.7; b) 0.21; c) 2.2; d) 0.28; e) 1.62; f) 0.02
4. a) $\frac{7}{10}$; b) $\frac{7}{40}$

Exercise 2

- Total sale = $(4 \times 35 - 6) \times \cancel{N}20 = (140 - 6) \times \cancel{N}20 = 134 \times \cancel{N}20 = \cancel{N}2\ 680$
- Score = $3 \times 3 \text{ points} + 2 \times (6 + 1 \text{ points}) = 9 + 2 \times 7 = 9 + 14 = 23 \text{ points}$
- Answers will vary. One possibility:
 $8 \div (6 - 4) \div 2 = 2$

Worksheet 15

- 544; 2. 88; 3. $8\frac{4}{5}$; 4. 0; 5. 701; 7. -657; 8. 36
- 134,4; 10. 1; 11. 28; 12. 1

Assessment

Pupils should be able to use basic operations in the right order.

Assess their performance in Exercises 1 and 2 and the game.

Extension activity

Interactive learning diagram: Biggest number

Pupils may complete the puzzle on page 114.

Puzzle page 114

1, 4, 8

1. 13; 2. -11; 3. -3; 4. 5; 5. 11

Five possible answers

Support activity

Interactive learning diagram: Order of operations

Homework activity

Complete WB Worksheet 15.

Lesson 2 *Pupil's Book page 114*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could

form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 117

- a) 1; b) 5; c) 1; d) 5; e) 3; f) 5; g) 3; h) 5
- a) 11; b) 14; c) 11; d) 36; e) 6; f) 8; g) 8; h) 8; i) 22; j) 1; k) 1
- a) 72; b) 7; c) 5; d) 2; e) 14; f) 39; g) 14; h) 9
- a) 0.07; b) 0.14; c) 0.08; d) 0.15
- a) $\frac{2}{3}$; b) $\frac{1}{4}$

Objectives

By the end of this unit, each pupil should be able to:

- Write numbers in index form
- Solve problems involving powers (indices)
- Solve problems on quantitative reasoning involving indices.

**Frequently asked questions**

Q *What do pupils need to understand in this unit?*

A The basic idea is power or 'index form', the use of a base and an exponent to tell how many times the base has been multiplied by itself. The PB uses both 'index' (plural 'indices') and 'exponent' for the little raised number that is part of a power.

**Evaluation guide**

Pupils to:

1. Write given squares in index form.
2. Express given numbers in index form.
3. Multiply given numbers using index form.
4. Solve problems on quantitative reasoning related to indices.

Lesson 1 *Pupil's Book page 118***Preparation**

You will need to have: Pupil's Book.

**Starter activity**

Review the use of factor trees to find the prime factors of a number. Choose numbers such as 24, 48, 72, 96, 400.

**Lesson focus**

In a previous unit pupils learned to express numbers as the product of their prime factors, using exponents.

Use this lesson to review that idea.

**Answers****Exercise 1**

1. 243; 2. 128; 3. 1 000; 4. 1; 5. 0

Puzzle page 119

1. 48 (8); 2. square numbers

Exercise 2

1. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
2. 2 and 3. 5 is not a factor of 48, because there is no whole number we can multiply by 5 to get 48.
3. a) $6 = 2 \times 3$
b) $12 = 2 \times 2 \times 3$
c) $16 = 2 \times 2 \times 2 \times 2$
d) $18 = 2 \times 3 \times 3$
e) $25 = 5 \times 5$
f) $36 = 2 \times 2 \times 3 \times 3$

Puzzle page 120

- a) false; b) true

Exercise 3

1. a) $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$
b) $126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$
c) $480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^5 \times 3 \times 5$
d) $245 = 5 \times 7 \times 7 = 5 \times 7^2$
e) $378 = 2 \times 3 \times 3 \times 3 \times 7 = 2 \times 3^3 \times 7$
f) $675 = 3 \times 3 \times 3 \times 5 \times 5 = 3^3 \times 5^2$
2. a) $180 \times 126 = (2^2 \times 3^2 \times 5) \times (2 \times 3^2 \times 7) = 2^3 \times 3^4 \times 5 \times 7$
b) $480 \times 378 = (2^5 \times 3 \times 5) \times (2 \times 3^3 \times 7) = 2^6 \times 3^3 \times 5 \times 7$

- c) $245 \times 675 = (5 \times 7^2) \times (3^2 \times 5^2) = 3^2 \times 5^3 \times 7^2$
3. a) $1\,000 \times 25 = (2 \times 2 \times 2 \times 5 \times 5 \times 5) \times (5 \times 5) = 2^3 \times 5^5$
- b) $75 \times 27 = (3 \times 5 \times 5) \times (3 \times 3 \times 3) = 3^4 \times 5^2$

Assessment

Pupils should be able to express numbers in index form.

Assess their performance in Exercises 1 to 3.

Extension activity

Interactive learning diagram: Prime factor products

Support activity

Interactive learning diagram: Factor trees

Homework activity

Complete Exercises 1, 2 and 3.

Lesson 2 *Pupil's Book page 121; Workbook page 26*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

This idea is most useful for multiplying multiples of powers of 10, as in the first example on page 121.

The important understanding here is to grasp *why* the 'number of zeroes' is the same as the index. For example 10^3 is 1 000 and has 3 zeroes. Ask the pupils to tell you why.



Lesson focus

The major idea is that when multiplying powers of the *same* base number you can just add the indices.

For example, $(2^3 \times 5^3) \times (2^4 \times 5^1) = 2^7 \times 5^4$

It would be good to check this using other methods.

So $(2^3 \times 5^3) = 8 \times 125 = 1\,000$

and $(2^4 \times 5^1) = 16 \times 5 = 80$. The product should be 80 000.

$2^7 \times 5^4 = 128 \times 625 = 80\,000$ (Check with long multiplication).



Answers

Exercise 4

- a) 1 500 000; b) 1 400 000; c) 20 000
- a) 2^7 ; b) 2^{11} ; c) 2^{13}
- a) $2^4 \times 3$; b) $2^6 \times 3^2$; c) $2^7 \times 3^3$
- a) $2^2 \times 5^4$; b) $2 \times 3 \times 5^4$; c) $2^4 \times 3^2 \times 5^2$
- a) $2^5 \times 2^4 = 2^9$; b) $3^4 \times 3^2 \times 7^2 = 3^6 \times 7^2$; c) $3^1 \times 1^{20} = 3$

Puzzle page 122

Yes, as 6 is a multiple of 2 and they are both even multiples. Any number which has 6 as a multiple will be an even number, which means that two will be a factor too.

For example, 36: 6×6 as well as $18 \times 2 =$ both 2 and 6 are factors

Exercise 5

- Yes, because 2 is a factor of 6
- Yes, because 2 is a factor of 4
- a) odd; b) even; c) odd; d) odd; e) even
- a) There are lots of possibilities for this answer. Six examples are:
 - $2 \times 3 = 6$
 - $2^2 \times 3 = 12$
 - $2 \times 3^2 = 18$
 - $2^2 \times 3^2 = 36$
 - $2^3 \times 3^2 = 72$
 - $2 \times 3^3 = 54 \quad \dots \text{etc.}$
- b) Add one to each of the exponents of 2 and 3. Then multiply these two numbers to find out how many factors there are altogether.

Worksheet 16

- | | |
|--------------------------|--------------------------------|
| a) $72 = 2^3 \cdot 3^2$ | b) $60 = 2^2 \cdot 3 \cdot 5$ |
| c) $32 = 2^5$ | d) $98 = 2 \cdot 7^2$ |
| e) $50 = 2 \cdot 5^2$ | f) $150 = 2 \cdot 3 \cdot 5^2$ |
| g) $100 = 2^2 \cdot 5^2$ | h) $48 = 2^4 \cdot 3$ |

- i) $63 = 3^2 \cdot 7$ j) $245 = 5 \cdot 7^2$
 k) $175 = 5^2 \cdot 7$ l) $225 = 3^2 \cdot 5^2$
 m) $36 = 2^2 \cdot 3^2$ n) $108 = 2^2 \cdot 3^3$
2. a) 72 b) 4 500
 c) 9 261 d) 5 929
3. a) done
 b) $8 \times 16 = 2^3 \cdot 2^4 = 2^7 = 128$
 c) $9 \times 54 = 3^2 \cdot 2 \cdot 3^3 = 2 \cdot 3^5 = 486$
 d) $49 \times 64 = 7^2 \cdot 2^6 = 3 136$
 e) $125 \times 49 = 5^3 \cdot 7^2 = 6 125$
4. $3 \times 3 \times 3 = 3^3 = 27$ blocks
5. a) 100; b) 128; c) 1 000; d) 1 024;
 e) 1 000 000; f) million; g) billion;
 h) trillion

Assessment

Pupils should be able to multiply powers by adding the indices of the same base numbers.

Assess their performance in Exercises 4 and 5.

Extension activity

Interactive learning diagram: Prime factor products

Let pupils try the Puzzle and Challenge on page 121 of the PB.

Puzzle page 121

$6 = 4 + 2$; $244 = 242 + 2$; $66 = 64 + 2$ etc.
 Yes – as even numbers are determined by their ability to be divisible by 2.

Challenge page 121

$256(16 \times 16)$ and $144(12 \times 12)$ are perfect squares

Support activity

Interactive learning diagram: Factor trees

Homework activity

Complete WB Worksheet 16.

Lesson 3 *Pupil's Book page 121*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 123

1. a) $72 = 2 \times 2 \times 2 \times 3 \times 3$
 b) $125 = 5 \times 5 \times 5$
 c) $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$
 d) $350 = 2 \times 5 \times 5 \times 7$
 e) $960 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
2. a) $51 = 3 \times 17$
 b) $61 = 1 \times 61$ Prime number
 c) $71 = 1 \times 71$ Prime number
 d) $311 = 1 \times 311$ Prime number
 e) $99 = 3 \times 3 \times 11 = 3^2 \times 11$
 f) $63 = 3 \times 3 \times 7 = 3^2 \times 7$
 g) $543 = 3 \times 181$
 h) $443 = 1 \times 443$ Prime number
 i) $113 = 1 \times 113$ Prime number
3. a) 180 000; b) 560 000; c) 4 000 000
4. a) 2^{13} ; b) 2^{11} ; c) 2^{11}
5. a) $2^2 \times 5^3$; b) $2^6 \times 5^3$; c) $2^{11} \times 5^4$
6. No, because 1's only factor is itself (i.e. it only has one factor).
7. a) 1 605 km; b) 78×10^7

Objectives

By the end of this unit, each pupil should be able to:

- Solve problems on ratios
- Express a number as a percentage of another number
- Solve problems on profit and loss percent
- Apply the profit and loss percent to business
- Solve problems on quantitative aptitude on ratio, percentage, profit and loss.

**Suggested resources**

playing cards; counters

**Common errors that pupils make**

Pupils give the equivalent of 0.5 as 5% instead of 50% and $5\% = 0.5$ instead of 0.05.

Plot some percentages and decimals on number lines marked in tenths, one under the other, so that pupils can see that 0.5 is half way and is much further along than 5%, and that 0.05 is half way between 0 and the first marker, as is 5%.

**Evaluation guide**

Pupils to:

1. Calculate ratios of given numbers.
2. Express a number as a percentage of another.
3. Calculate profit and loss percentages of given problems.
4. Solve problems on quantitative aptitude on ratio, percentage, profit and loss

Lesson 1 *Pupil's Book page 124***Preparation**

You will need to have: Pupil's Book; counters

**Starter activity**

This section reviews ratio and direct proportion.

Use counters to remind them about equivalent ratios.

For example the left pile has 4 counters, and the right pile has 3.

The ratio left to right is 4 : 3.

Double the number in each pile.

Ask: "Is 8 : 6 still in the ratio 4 : 3?"

Yes, because it can be split into two lots that are 4 : 3.

Make two piles in the ratio 20 : 16. What is a simpler form of this ratio?

We can divide both numbers by 4, making 5 : 4.

Make a ratio 3 : 2. The first number is multiplied by 5, to make 10. What is now the second number to make equal ratios?

**Lesson focus**

This approach was introduced in Unit 8.

Dual number line method

This approach to understanding this topic might be helpful. We use two parallel number lines.

For example, let us look at the ratio 2 : 5.

0	1	2	3	4	5
0	A	5	B	C	D

The top number line has 0 up to the largest required amount.

The lower number line has 0 and 5 under the 2 (on the top line).

Now we have a picture about how the numbers in equal ratios are related.

For example, C is 10, because when one is doubled the other also doubles.

Ask the pupils to find A (2.5), B (7.5) and D (12.5).

Unit ratio method

In this method we make a unit ratio (comparing the numbers to 1) and then multiply.

In the case of the ratio 2 : 45, we divide by 2 to make 1 : 2.5. This immediately gives answer A.

Then we multiply, by 3 to get B (7.5), by 4 to get C (10) and by 5 to get D (12.5).

Increasing or decreasing in a given ratio (a : b)

The ratio will tell you whether the amount will

- increase (if a is bigger than b)
- or decrease (if a is smaller than b).

New amount = Start amount \times first number in ratio (a) \div second number in ratio (b).

Answers

Exercise 1

- 11 : 17;
- 9 : 1;
- 26 boys; 40 girls;
- 24 cm;
- 28 blue bikes

Assessment

Pupils should be able to find equivalent ratios and use this to increase or decrease a quantity in a given ratio

Assess their performance in Exercise 1 and the game.

Extension activity

Interactive learning diagram: Find 1 or 100%

Support activity

Interactive learning diagram: Find fractions, decimals and percentages on a number line

Homework activity

Complete Exercise 1.

Lesson 2 *Pupil's Book page 125*

Preparation

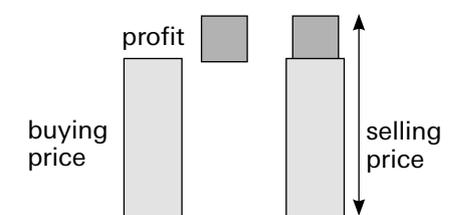
You will need to have: Pupil's Book; playing cards.

Starter activity

Explain the concept of buying goods for resale. This should be a familiar concept to most pupils.

Profits

If the selling price is **higher** than the buying price, you make a **profit**.



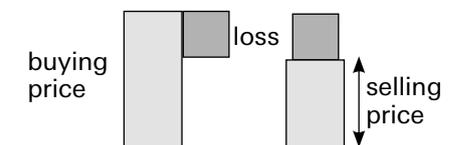
The **profit** is the money you get **more** than the buying price.

You buy 50 oranges for ₦300 and eat two. You sell the rest at ₦10 each. What is your profit?

Losses

Sometimes an item has to be sold at a low price. There is no profit, and the seller loses money.

You buy goods at some price and then sell them. If the selling price is **lower** than the buying price, you make a **loss**.



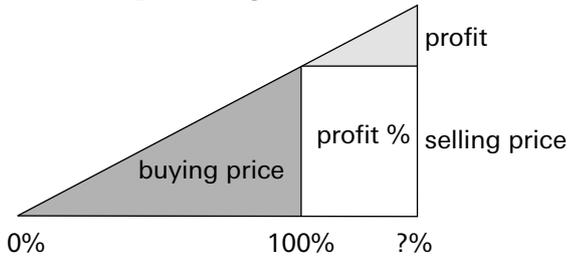
The **loss** is the money you get **less** than the buying price.

You buy 100 eggs for ₦3 000 and break 4. You sell the rest at ₦30 each. What is your profit or loss?

Lesson focus

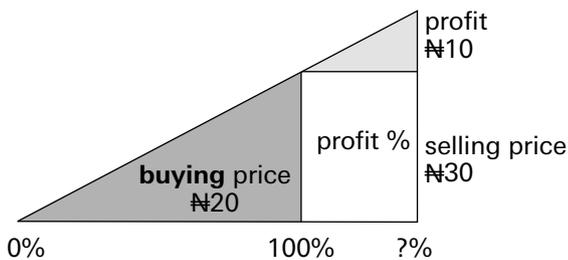
You might find this diagram approach helps your pupils to understand what is going on. It is offered as an alternative to the 'dual number line' approach.

Profits as percentages



The **profit percentage** is the percentage you have **more** than the 100% buying price.

For example, you buy something at ₦20 and sell it at ₦30. You make a profit of ₦10. What is that as a percentage of the buying price?



Compare ₦10 profit to ₦20 buying price. The percentage profit is 50%.

Losses as percentages



The **loss percentage** is the percentage you have **less** than the 100% buying price. For example, you buy something at ₦20 and sell it at ₦15. You make

a loss of ₦5. What is that as a percentage of the buying price?

Compare ₦5 to ₦20 and get 0.25 or 25%. So the loss percentage is 25%.

Card game – Estimating percentage increases or decreases

You need a pack of cards. J = 11, Q = 12 and K = 13. Shuffle the cards.

Choose two cards and make a two-digit number. This is your old price. Choose another two cards for another two-digit number – this is your new price. The difference is increase or decrease. All players estimate this as a percentage of the old price. Work it out with a calculator, and the person nearest the correct answer wins a point.

Answers

Exercise 2

- a) 5; b) 19; c) 6.9; d) 234.5 g; e) 15; f) 80; g) 64; h) 21.2 km; i) 11; j) 30; k) 14; l) 175; m) 236; n) 235; o) 3 250; p) 0.7 kg; q) 300; r) 210; s) 2 400; t) 3 ℓ
- 22% × 380 = 83.6
23% × 400 = 92
54% × 240 = 129.6
58% × 250 = 145
- ₦4 500

Exercise 3

- a) ₦1 000
b) ₦320
c) ₦10 000
d) ₦120 000
- ₦4 320
- He made ₦112.50
His new selling price was ₦562.50
- ₦77 000
- ₦416 000
- a) ₦24
b) ₦54
c) ₦114
- a) Actual decrease = ₦30 – ₦24 = ₦6
Percentage decrease = 80%
b) 20%
c) 20%

Assessment

Pupils should be able to find profits or losses as percentages of the original price.

Assess their performance in Exercises 2 and 3 and the game.

Extension activity

Interactive learning diagram: Percentage problems

Support activity

Interactive learning diagram: Percentage increase, Percentage decrease

Homework activity

Complete Exercises 2 and 3.

Lesson 3 *Pupil's Book page 127*



Preparation

You will need to have: Pupil's Book.



Starter activity

Play the card game 'Estimating percentage increases or decreases' (in Lesson 2 above) to review the basic ideas.

Ask pupils to give examples of increases in prices, or discounts on prices, particularly when they are expressed as percentages.



Lesson focus

This lesson contains the same ideas as in the previous lesson. However in the games we can extend these ideas to home budgeting and to running a small business, where daily or weekly profits and losses are carried over to the next day or week. This should make it clear to pupils that these ideas are important to survival, either as a householder or as a small business. If any of the pupils have parents who run a small business you might invite them to speak to the class about this aspect of business.



Answers

Exercise 4

1.

Original Price	Selling Price	Profit or loss	Percentage profit or loss
₦7 000	₦6 300	Loss	10%
₦2 500	₦3 000	Profit	20%
₦10 000	₦7 500	Loss	25%
₦15 000	₦7 500	Loss	50%
₦60 000	₦75 000	Profit	25%
₦240 000	₦288 000	Profit	20%

2. Percentage loss = 20%
3. Percentage profit = 20%
4. Percentage profit = 25%
5. Percentage profit = 25%

Assessment

Pupils should be able to find profits or losses in business situations

Assess their performance in Exercise 4.

Extension activity

Interactive learning diagram: Hire purchase

Let pupils attempt the challenge on page 129.

Challenge page 129

- a) 30 April 2012 it had a book value of ₦216 000
30 April 2013 it had a book value of ₦172 800
- b) 46% loss

Support activity

Interactive learning diagram: Personal budget

Homework activity

Complete Exercise 4.

6. No, in 7 : 10 the first quantity is less than the second quantity.
 7. 68.7%; 8. 70%; 9. 200%; 10. 13.5%;
 11. $\frac{44}{60} = 73 \frac{1}{3}\%$ is better than $\frac{58}{80} = 72,5\%$
 12. 54.9%; 13. 0.07%; 14. 33.9%; 15. 40%
 16. ~~₦7 700~~; ~~₦6 160~~; 17. 18.4%

Assessment

Pupils should be able use their understanding of ratio and percentage to figure out the relationships.

Assess their performance in Exercises 5 and 6.

Extension activity

Interactive learning diagram: Find 1 or 100%

Support activity

Interactive learning diagram: Percentage problems

Homework activity

Complete WB Worksheet 17.

Lesson 5 *Pupil's Book page 130*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 130

- Profit = ₦5, Selling price = ₦55
 - Profit = ₦10, Selling price = ₦60
 - Profit = ₦25, Selling price = ₦75
 - Profit = ₦50, Selling price = ₦100
- New price = ₦55, percentage mark-up = 10%

- New price = ₦70, percentage mark-up = 40%
 - New price = ₦90, percentage mark-up = 80%
 - New price = ₦110, percentage mark-up = 120%
- Mark-up = ₦5, new price = ₦55
 - Mark-up = ₦7.50, new price = ₦57.50
 - Mark-up = ₦10, new price = ₦60
 - Mark-up = ₦12.50, new price = ₦62.50

4.

Old price	₦40	₦40	₦40	₦40	₦50	₦50	₦50
New price	₦50	₦30	₦44	₦36	₦30	₦70	₦100
% increase or decrease	25% increase	25% decrease	10% increase	10% decrease	40% decrease	40% increase	100% increase

- ₦40; b) ₦45; c) ₦40
- ₦196 263.94
- ₦4 950
- ₦31 500
- 480 ml
- $30 \div 5 = 6$ counts, so the globe will rotate
 6×150 degrees = 900 rotations
- 6 pens

Objectives

By the end of this unit, each pupil should be able to:

- Solve problems expressed as open sentences
- Interpret words in open sentences and solve them
- Solve related problems on quantitative aptitude.

**Suggested resources**

bags and counters

**Frequently asked questions**

Q *Can the unknown (\square) be any number at all?*

A Yes, but only one number makes the equation true.

Q *What mathematical skills are needed?*

A Pupils use trial and error or systematic working backwards to find the number that fits the equation.

**Evaluation guide**

Pupils to:

1. Interpret word problems into open sentences and solve them.
2. Pupils are to solve quantitative aptitude problems involving at least three arithmetic operations.

Lesson 1 *Pupil's Book page 132***Preparation**

You will need to have: Pupil's Book; bags and counters.

**Starter activity**

We start with a way to represent an equation, or open sentence.

Use a cloth or paper bag and put some counters into it, say 4. Tell the pupils how many are in the bag.

Now ask pupils to make up number sentences that are true for the number of counters in the bag. On this page we will use \square to show the bag.

For example, these are true for $\square = 4$.

$$\square + 3 = 7, \square - 3 = 1, 2 \times \square = 8, 3 \times \square + 5 = 17, \\ 5 \times (\square - 1) = 15, 2 \times (3 \times \square + 1) = 26$$

Check their answers.

Repeat for a different number of counters in the bag. This should give them the idea of what an open sentence is.

**Lesson focus****Guess and check**

For this approach you just try numbers you think might work. For example, for $\square + 1 = 4$ we could try $\square = 4$.

Then we see that $4 + 1 = 5$. It is too big, so you might then try $\square = 3$, and $3 + 1 = 4$. It works.

**Answers****Exercise 1**

1. a) 13; b) 26; c) 3; d) 8
2. a) 8; b) 13; c) 26; d) 4; e) 15; f) 31

Assessment

Pupils should be able to find the solutions to open sentences

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Guess and check

Support activity

Interactive learning diagram: Missing numbers

Homework activity

Complete Exercise 1.

Lesson 2 *Pupil's Book page 132; Workbook page 30*



Preparation

You will need to have: Pupil's Book.



Starter activity

Ask pupils to discuss in groups how they might find the answers to these.

a $4 \times \square + 1 = 17$ b $19 - 5 \times \square = 9$
c $4 \times (\square + 3) = 40$ d $4 \times (\square - 3) = 32$

Get a member of each group (or some of them) to report on the methods they used.



Lesson focus

Review the 'Guess and check' method for more complex equations.

Guess and check

For this approach you just try numbers you think might work. For example, for $4 \times \square + 1 = 13$, we could try $\square = 4$.

Then we see that $4 \times 4 + 1 = 17$. It is too big, so you might then try $\square = 3$, and $4 \times 3 + 1 = 13$. It works.

Working backwards

Another way to help pupils find the missing number is called 'backtracking'. It can be done using a very 'hands-on' way, using your hand to cover the part of the question you are trying to work out.

So, for $4 \times \square + 1 = 13$, you can cover the $4 \times \square$ part with your hand.

Then all you can see is $\square + 1 = 13$, so the covered part, \square , is 12.

Now you know that $4 \times \square = 12$. So $\square = 3$.

Here are two more examples using this idea.

$\square \times 5 - 8 = 32$. Cover the $\square \times 5$.

Then $\square - 8 = 32$. So $\square = 40$.

We know that $\square \times 5 = 40$. So $\square = 8$.

$11 = \square \times 3 - 4$. Cover the $\square \times 3$.

Then $11 = \square - 4$. So $\square = 15$

We know that $\square \times 3 = 15$. So $\square = 5$.



Answers

Exercise 2

- a) 9; b) 6; c) 4; d) 6
- a) 9; b) 6; c) 4; d) 6
- a) 9; b) 6; c) 4; d) 6

Worksheet 18

- a) 2; b) $\frac{1}{3}$; c) 47; d) 0; e) 7; f) 10; g) $\frac{3}{4}$; h) $\frac{1}{8}$;
i) $\frac{3}{4}$; j) -36; k) 52; l) 98; m) 1; n) 32; o) 150;
p) 16; q) 30; r) 100; s) 3

Assessment

Pupils should be able to solve equations with at least two operations

Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Backtracking

Support activity

Interactive learning diagram: Guess and check

Homework activity

Complete WB Worksheet 18, Question 1.

Lesson 3 *Pupil's Book page 133; Workbook page 31*



Preparation

You will need to have: Pupil's Book.



Starter activity

For this lesson we need to make sure that pupils understand a fraction as a division, and that adding or subtracting in the numerator means that these operations are 'in brackets' and are to be worked out first, once you know the number for the \square .

Find the value of these:

$$\frac{10}{5} \quad \frac{7+3}{5} \quad \frac{10-4}{2}$$



Lesson focus

We have learned two methods for solving equations: guess and check, and working backwards.

Ask pupils in groups to solve these using both methods, and decide which way they prefer.

$$\frac{\square}{5} = 4; \quad \frac{\square+1}{5} = 4; \quad \frac{\square-1}{5} = 4; \quad \frac{2 \times \square}{5} = 4$$

For working backwards, you will start by covering the top of the fraction.

For example $\frac{\square-1}{5} = 4$ becomes $\frac{\bullet\square}{5} = 4$, so $\square = 20$.

This means that $\square - 1 = 20$, so $\square = 21$.



Answers

Challenge page 134

$$P = 450 \div 5; \quad P = 50$$

\therefore 50 children will be on each bus

Exercise 3

1. a) $\frac{1}{2}$; b) $2\frac{1}{2}$; c) $\frac{1}{3}$; d) $1\frac{2}{3}$; e) $4\frac{1}{2}$; f) $6\frac{1}{2}$; 2. a) 9; b) 6; c) 9; d) 6; 3. a) 1; b) $\frac{1}{2}$; c) 5; d) $2\frac{1}{2}$

Worksheet 18

7. answers will vary.; 8. Yes

Assessment

Pupils should be able to solve equations involving the division operation and at least one other.

Assess their performance in Exercise 3.

Extension activity

Interactive learning diagram: Backtracking

Support activity

Interactive learning diagram: Guess and check

Homework activity

Complete WB Worksheet Questions 7 and 8.

Lesson 4

Pupil's Book page 135; Workbook page 30



Preparation

You will need to have: Pupil's Book.



Starter activity

The words used in problems actually tell you what open sentence to write. You just have to know what the English words mean.

You could say these words (or write them on the board) in a random order, and ask the class to sort them into those that represent adding, subtracting, multiplying or dividing.

Adding:

'the sum of', 'added to', 'as well as', 'and', 'gained', 'more than', 'altogether', 'increased by', 'total',

Subtracting:

'take away', 'less than', 'minus', 'the difference between' (always bigger – smaller), 'lost', 'decreased by', 'fewer than',

Multiplying:

'of', 'times', 'the product of'. 'by a factor of',

Dividing:

'shared by', 'per', 'the quotient of', 'out of', 'divided between', 'how many _ are there in _'



Lesson focus

One approach to explaining word problems is to work backwards. Give the pupils the equations, and ask them to make up a word problem that fits the equation.

a $2 \times \square + 4 = 5$

b $2 \times \square - 4 = 5$

c $4 \times (\square + 3) = 28$

d $3 \times (\square - 4) = 28$

e $\square = 4$

f $\square + 3 = 2$

After each group has worked at it, ask them to share what they have written. In this way they will hear a variety of word problems and see that they can all be represented by the same equation. Finally, if time allows, solve the equations.



Answers

Exercise 4

1. 60 eggs per box; 2. 48 years old; 3. 7 pieces; 4. 10 220

Exercise 5

- Answers to this question will vary. Possible answers are listed below.
a) C and D; b) A and B; c) A and B; d) C and D
- Answers to this question will vary. Possible answers are listed below.
a) $\square = 3$ $\Delta = 2$; b) $\square = 2$ $\Delta = 8$;
c) $\square = 6$ $\Delta = 7$; d) $\square = 2$ $\Delta = 8$
e) $\square = 2$ $\Delta = 12$

Worksheet 18

- 20; 3. 3:4:6
- Anna 54, Beth 36 and Cathy 27
- Alan 47,5 % and John 55 %
- Father earns ₦1 000 000, mother earns ₦1 125 000 and brothers earn ₦125 000 each.

Assessment

Pupils should be able to write an equation for a given word problem.

Assess their performance in Exercises 4 and 5 and the 'working backwards' exercise.

Extension activity

Interactive learning diagram: Backtracking

Support activity

Interactive learning diagram: Guess and check

Homework activity

Complete WB Worksheet 18, Questions 2 to 6.

Lesson 5 *Pupil's Book page 135*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 137

- Answers will vary. Suggested answers below.
 - The sum of \square and Δ is eight; \square added to Δ is eight
 - Δ subtracted from \square gives eight; The difference between \square and Δ is eight
 - \square multiplied with Δ gives 8. The product of \square and Δ is 8.
 - \square divided by Δ is eight. \square is eight times bigger than Δ .
- Answers will vary. Possible answers below.
 - Three times \square plus 1 equals Δ . Δ is one more than three times \square . $\square = 2$; $\Delta = 7$
 - Δ is one less than three times \square
One subtracted from three times \square is Δ
 $\square = 3$; $\Delta = 8$
 - Δ is one bigger than a third of \square
 \square divided by three plus one is Δ
 $\square = 9$; $\Delta = 4$
 - Subtract one from \square and divide by three to get Δ
 Δ is one third of \square , less one
 $\square = 3$; $\Delta = 0$
- a) 32; b) 38; c) 20; d) 50
- Answers will vary. Possible answers below.
 - Three times \square incremented by 1 equals Δ
 Δ is one more than \square tripled
 $\square = 2$; $\Delta = 9$
 - Δ is one less than \square tripled
Three times \square decremented by 1 equals Δ
 $\square = 9$; $\Delta = 24$
 - A third of \square plus one is Δ
 Δ equals \square plus one, divided by three
 $\square = 11$; $\Delta = 4$
 - Δ equals \square minus one, divided by three
A third of one less than \square is Δ
 $\square = 7$; $\Delta = 2$
- a) 5; b) 8; c) 12; d) 3; e) 1
- Let the number be N
 $N + 5 + 8 = 17$, so $N + 13 = 17$, then $N = 4$

Objectives

By the end of this unit, each pupil should be able to:

- Solve problems on taxes and rates
- Solve problems involving buying and selling of shares and dividends
- Solve problems on quantitative aptitude based on taxes and rates as well as selling of shares and dividends
- Use a calculator to convert from one currency to another
- Solve problems on quantitative aptitude based on converting from one currency to another.

Common errors that pupils make

Pupils do not keep careful track of income tax calculations.

Encourage them to set out their working as shown in the worked example.

Pupils carry out the wrong operation when converting currencies.

Teach them to estimate whether their answer should be smaller or larger than the original amount. This is a useful check on answers!

Evaluation guide

Pupils to:

1. List various forms of taxes.
2. Calculate taxes, rates and share dividends.
3. Solve quantitative aptitude problems.
4. Use a calculator to convert currencies.

Lesson 1 *Pupil's Book page 138; Workbook page 32*

Preparation

You will need to have: Pupil's Book; Workbook.

Starter activity

Discuss direct taxes, such as income tax, and indirect tax, such as VAT. Discuss who collects them, when and how it is done and how the money is used. For example income tax will be taken out of the regular pay of an employer. Discuss

the purpose of a tax return, and the calculation of 'taxable income' after deducting allowances. Discuss the example of tax rates on page 138. In this system the wealthy (those who earn more) have to pay a higher percentage of their income as tax.

Lesson focus

The main focus of this lesson is the calculation of the income tax for people on certain incomes.

Answers

Exercise 1

1.

	Total income	Non-taxable allowance	Taxable income	Tax to be paid
a	₺ 6 700	₺ 2 500	₺ 4 200	₺ 540
b	₺ 8 300	₺ 1 950	₺ 6 350	₺ 987.50
c	₺ 13 500	₺ 4 100	₺ 9 400	₺ 1 820
d	₺ 16 500	₺ 4 600	₺ 11 900	₺ 2 760
e	₺ 20 000	₺ 6 200	₺ 13 800	₺ 3 520

2. a) ₺12 600; b) ₺3 040

3. a) ₺38 300; b) ₺17 560

Worksheet 19

1. Abdul ₺567 000; Beth ₺105 000;
Calvin ₺1 200 000; David ₺14 000;
Ephraim ₺60 000; Fred ₺285 000

Assessment

Pupils should be able to calculate an income tax using a tax table.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Credit card

Support activity

Interactive learning diagram: Percentages of money, Income tax

Homework activity

Complete WB Worksheet 19, Question 1.

Lesson 2 *Pupil's Book page 140; Workbook page 33*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Have a discussion about the different kinds of taxes that your pupils are aware of. Do they know about VAT? Are they aware of prices given with or without VAT and how this affects how much they pay?



Lesson focus

Exercise 2 are examples requiring no more than multiplications followed by additions, but they are good examples of situations that might be used for word problems!

Exercise 5 involves percentages in these calculations. You should make sure that pupils understand the words of the questions, as an understanding of the real-life context is just as important as the calculations.



Answers

Exercise 2

- a) ₦87 500; b) ₦315 000; c) ₦455 500;
d) ₦787 500
- a) ₦2 010; b) ₦9 060
- a) ₦920 000; b) ₦280 200

Exercise 5

- ₦174 960; 2. ₦100 116; 3. ₦888 320

Worksheet 19

- ₦21 180
- a)

Lamps	Cost price at ₦700	Selling price at ₦900	Rent	Profit
100	70 000	90 000	45 000	-25 000
150	105 000	135 000	45 000	-15 000
200	140 000	180 000	45 000	-5 000
250	175 000	225 000	45 000	5 000
300	210 000	270 000	45 000	15 000
350	245 000	315 000	45 000	25 000

- b) 225; 350 c) 375
- a) ₦17 500 b) ₦34 750
c) ₦121 000 d) ₦169 000
- Income, water, electricity, refuse etc.
- ₦5 760 000

Assessment

Pupils should be able to calculate rates and taxes, using percentages where required.

Assess their performance in Exercises 2 and 5.

Extension activity

Interactive learning diagram: Hire purchase

Homework activity

Complete WB Worksheet 19, Questions 4 to 6, 8 and 9.

Lesson 3 *Pupil's Book page 141; Workbook page 33*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Discuss the concept of 'shares' in a company on the 'share market'. There are many companies listed, and their shares can be bought and sold. This unit does not concern itself with the market, but with the basic idea of what shares are, and what dividends are.

Lesson focus

Exercise 3 is concerned only with multiplication or division, but the concept of shares must be understood.

Exercise 4 involves percentages in these calculations. Make sure pupils understand the terms 'dividend', 'value of shares', and 'dividend rate'.

Answers

Exercise 3

1. a) ₦189.05; b) ₦1 250; c) ₦1 350
2. a) 400 shares; b) 500 shares; c) 600 shares; d) 1 800 shares
- 3.

	Number of shares	Cost at 50k each	Cost at 70k each	Cost at 85k each	Cost at ₦1.35 each	Cost at ₦2.75 each
a	100	₦ 50	₦ 70	₦ 85	₦ 135	₦ 275
b	350	₦175	₦245	₦297.50	₦472.50	₦962.5
c	2 500	₦1250	₦1750	₦2125	₦3375	₦6875

Exercise 4

1. a) ₦10; b) ₦112.5
2. a) 43 000 shares; b) 56 000 shares

Worksheet 19

3.

Number of shares	Cost at 50k each	Cost at 60k each	Cost at 75k each	Cost at ₦1.25	Cost at ₦1.70
100	₦50	₦60	₦75	₦1.25	₦170
250	₦125	₦150	₦187.50	₦312.50	₦425
500	₦250	₦300	₦375	₦625	₦850
1 500	₦750	₦900	₦1 125	₦1 875	₦2 550

- a) 6 400 b) 2 000 c) 5 000

7. 555 555 A shares and 266 666 B shares yields ₦54 999.91 in dividends.

Assessment

Pupils should be able to calculate the cost of a number of shares at a given price, and the dividend due to a shareholder who owns shares of a given value, at a given dividend rate.

Assess their performance in Exercises 3 and 4.

Homework activity

Complete WB Worksheet 19, Questions 3 and 7.

Lesson 4 *Pupil's Book page 143*

Preparation

You will need to have: Pupil's Book.

Starter activity

How many different currencies can the pupils think of? There are five on page 143, but each country has its own.

Discuss the fact that currencies have different values, depending on the wealth of the country, and other factors. Some people buy and sell currencies, and try to make a profit from this activity.

Lesson focus

Every currency now has a main note (or coin) and a smaller coin that is one hundredth of the main one. (See Exercise 6.)

Discuss the use of exchange rates. Pupils often get these confused, and use them in reverse. Use the two examples above Exercise 7 to make sure they can see when to multiply and when to divide.

Answers

Exercise 6

1. a) ₦20; b) ₦3.5; c) ₦200; d) ₦42.50
2. a) ₦1.50 × 100 = 150k; b) 14 250k; c) 50k; d) 325 000k
3. a) R1.55; b) R14.50; c) R100; d) R5.55
4. a) 50c; b) 15 000c; c) 350 000c; d) 350c
5. a) \$2.50; b) \$2852.00; c) \$42.50; d) \$1.99

Exercise 7

1. a) ₦420; b) R514.75; c) \$375; d) £7
2. R5 355; 3. ₦6 000

Assessment

Pupils should be able to understand and use exchange rates.

Assess their performance in Exercises 6 and 7.

Homework activity

Complete Exercises 6 and 7.

Lesson 5 *Pupil's Book page 144; Workbook page 32*



Preparation

You will need to have: Pupil's Book; Workbook.

Pence	Naira	Pence	Naira	Pound	Naira	Pound	Naira	Pound	Naira	Pound	Naira
1	₦ 2.10	10	₦ 21.00	1.00	₦ 210.00	10	₦ 2 100.00	100	₦ 21 000.00	1000	₦ 210 000.00
2	₦ 4.20	20	₦ 42.00	2.00	₦ 420.00	20	₦ 4 200.00	200	₦ 42 000.00	2000	₦ 420 000.00
3	₦ 6.30	30	₦ 63.00	3.00	₦ 630.00	30	₦ 6 300.00	300	₦ 63 000.00	3000	₦ 630 000.00
4	₦ 8.40	40	₦ 84.00	4.00	₦ 840.00	40	₦ 8 400.00	400	₦ 84 000.00	4000	₦ 840 000.00
5	₦ 10.50	50	₦ 105.00	5.00	₦ 1 050.00	50	₦ 10 500.00	500	₦ 105 000.00	5000	₦ 1 050 000.00
6	₦ 12.60	60	₦ 126.00	6.00	₦ 1 260.00	60	₦ 12 600.00	600	₦ 126 000.00	6000	₦ 1 260 000.00
7	₦ 14.70	70	₦ 147.00	7.00	₦ 1 470.00	70	₦ 14 700.00	700	₦ 147 000.00	7000	₦ 1 470 000.00
8	₦ 16.80	80	₦ 168.00	8.00	₦ 1 680.00	80	₦ 16 800.00	800	₦ 168 000.00	8000	₦ 1 680 000.00
9	₦ 18.90	90	₦ 189.00	9.00	₦ 1 890.00	90	₦ 18 900.00	900	₦ 189 000.00	9000	₦ 1 890 000.00
10	₦ 21.00	100	₦ 210.00	10.00	₦ 2 100.00	100	₦ 21 000.00	1000	₦ 210 000.00	10000	₦ 2 100 000.00

Note how the numbers in columns 7 and 8 are simply ten times those in columns 5 and 6. The other pairs of columns work the same way.



Lesson focus

Explain how to use the reckoner, by finding the dollar (\$) amounts in odd numbered columns and converting to Naira (₦) from the numbers next to them, and then adding. Use the first example on page 145. (This is equivalent to doing the multiplication.) Pupils do Question 1 in Exercise 8.

Go over how to use the reckoner backwards to convert Naira to dollars. Use the second example on page 145. Pupils do Question 2 in Exercise 8.

If time allows they can also do Questions 3 and 4.



Starter activity

The 'ready reckoner' on page 145 may be used instead of the calculator and an official exchange rate. However Fedex offices and banks always use the most recent conversion rates and a calculator.

Note how the reckoner is structured in pairs of columns. Find the basic rate at the top of columns 5 and 6 (\$1 = ₦120).

Note how numbers in the rows below that are calculated as simple multiples.



Answers

Challenge page 146

For this challenge it depends on which currencies they research. Here is one example:

$$\$1 = R10.76$$

For example, if I have \$7 how many Rands will that be?

$$7 \times 10.76 = R75.32$$

Exercise 8

- a) ₦30 480; b) ₦776 400; c) ₦1 180 320; d) ₦3096; e) ₦69 900; f) ₦367 014
- a) \$210; b) \$1008; c) \$3920; d) \$974.50; e) \$341; f) \$5428

3. Pupils design a ready reckoner
4. a) ₦73 500 b) ₦1 348 200
 c) ₦1 570 800 d) ₦16 485
 e) ₦89 376 f) ₦76 534.50

3. a) ₦2 560 b) ₦935 c) ₦375
 4. a) ₦37.5 b) ₦233.75
 5. a) ₦660 b) 260 800 Cedis
 c) \$3 d) £2
 6. a) £6 b) \$2
 c) \$3 d) £2

Worksheet 19

2. a) ₦25 600 b) ₦67 500
 c) ₦17 550 d) \$6 250
 e) ₦150 000 f) £83.33
 g) 1 575 cedis h) €3 333.33

Assessment

Pupils should be able to use a ready reckoner to convert currencies in both directions.

Assess their performance in Exercise 8.

Homework activity

Complete WB Worksheet 19, Question 2.

Lesson 6 *Pupil's Book page 144*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 147

1.

	Total income	Non-taxable allowance	Taxable income	Tax to be paid
a	₦ 7 500	₦ 2 500	₦5 000	₦700
b	₦ 8 600	₦ 1 950	₦6 650	₦1062.5
c	₦ 14 200	₦ 4 100	₦10 100	₦2040
d	₦ 17 300	₦ 4 600	₦12 700	₦3080
e	₦ 22 400	₦ 6 200	₦16 200	₦4540

- a) ₦11 800 b) ₦2 720

Objectives

By the end of this unit, each pupil should be able to:

- Use Pythagoras' rule to find the unknown length of a right-angled triangle
- Solve quantitative aptitude problems involving Pythagoras' rule.

**Suggested resources**

plain paper; cm grid paper

**Frequently asked questions**

Q *What is the Pythagoras' theorem?*

A Pythagoras' theorem (the area of the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides) is a fact about areas. However it can also be used to relate the side lengths of right-angled triangles in the expression $a^2 + b^2 = c^2$.

**Evaluation guide**

Pupils to:

1. Find the unknown length of a given right-angled triangle using Pythagoras' Rule.
2. Solve quantitative aptitude problems on Pythagoras' Rule.

Lesson 1 *Pupil's Book page 148***Preparation**

You will need to have: Pupil's Book; plain paper.

**Starter activity**

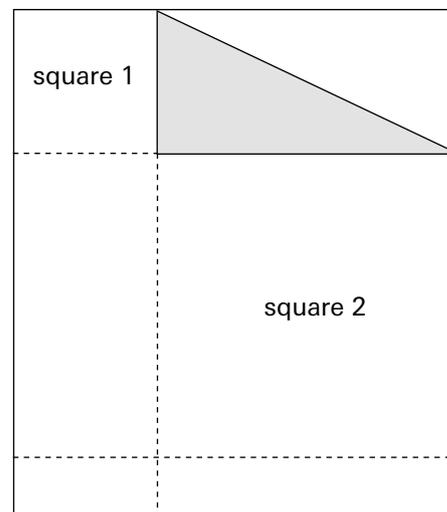
Draw a right-angled triangle, and draw squares on sides. Discuss the fact that the biggest side (the hypotenuse), opposite the biggest angle (the right angle), also has the biggest square area.

**Lesson focus**

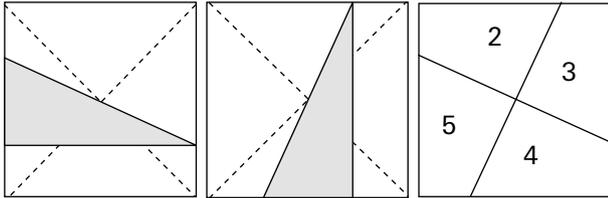
The lesson has a short statement of the result. However pupils might enjoy being shown a simple proof that it is true.

Paper folding proof

1. Mark any point on the short side of an A4 sheet. Fold through that point parallel to the long side.
2. Measure the length from your mark to the nearest long side, and mark that point down the longer side. Fold parallel to the short side. You now have a small square in one corner of the paper. You have created square 1.
3. Measure the length from the first mark to the other long side of the A4 sheet. Below the second fold you made (the one parallel to the short side of the A4 sheet) mark this length. Fold parallel to the short A4 side so you create square 2.
4. Draw the hypotenuse of the right-angled triangle and clearly mark the two squares on its shorter sides. You now have this diagram. Cut out the triangle and two squares. Move the rest of the paper out of the way.



- 5 Find the midpoint of the larger square, using two folds. Mark it.
- 6 Slide the triangle down so that its hypotenuse goes through the midpoint you have marked. Draw along the hypotenuse.
- 7 Turn the triangle through 90° , keeping the midpoint at the centre. Draw along the hypotenuse again.



- 8 You now have this diagram with five parts, square 1 and the four parts cut from square 2. Cut them out. Put the five parts together to make a square that fits on the hypotenuse of the triangle. (This is a puzzle; it can always be done, but pupils might take a while to get it done.)

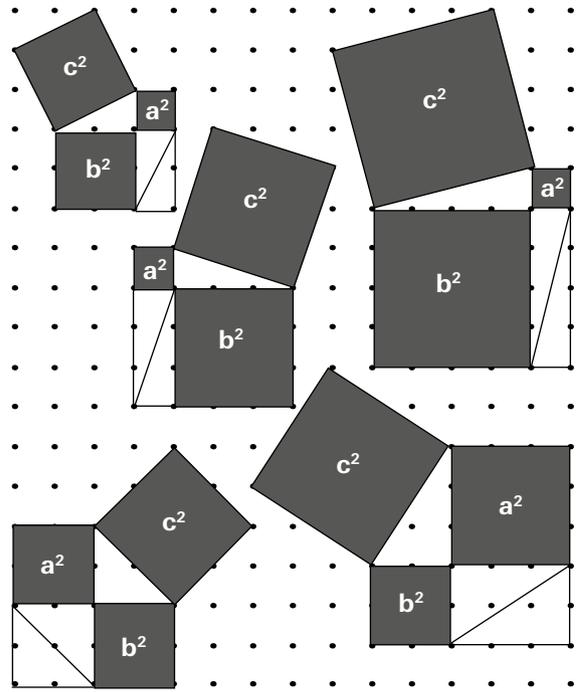
It will work no matter what sizes of square or shapes of triangle you have. This proves that the area of the square on the hypotenuse is equal to the area of the squares on the other two sides, for any right-angled triangle.

Drawing right-angled triangles on grid paper

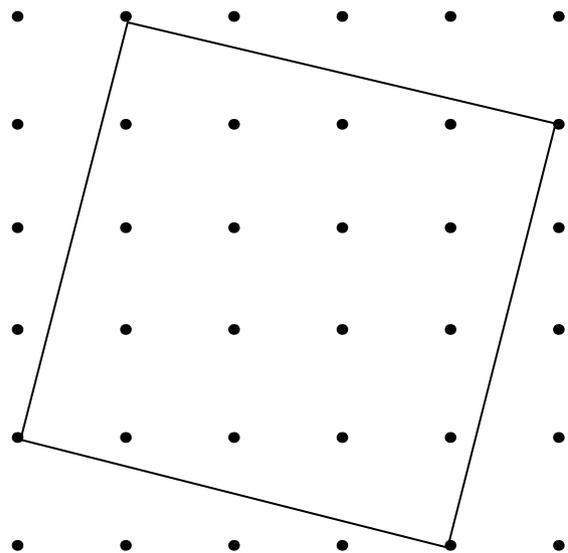
These right-angled triangles should be drawn with the two shorter sides (numbers given) horizontal and vertical.

(1, 2), (1, 3), (1, 4), (2, 2), (2, 3)

It will look like this.



Pupils can then check whether or not $a^2 + b^2 = c^2$ is true, for these cases. Note that the area of most of the squares on the hypotenuses is found as in this example.



9 in the middle, put the triangle together in two pairs each of which is 4. so $9 + 4 + 4 = 17$ units.

Assessment

Pupils should be able to understand the Pythagorean rule involving the areas of three squares.

Assess their performance in the activity.

Extension activity

Interactive learning diagram: Pythagoras' theorem

Support activity

Interactive learning diagram: Squares and triangle

Lesson 2 *Pupil's Book page 148*



Preparation

You will need to have: Pupil's Book; cm grid paper.



Starter activity

Review how to find square roots by the factorisation method (Unit 13).



Lesson focus

The theorem is true for all right-angled triangles. One way to check this is to draw any right-angled triangles, and measure the side lengths and check by squaring the numbers.

We do this on grid paper to make sure the triangles are right-angled.

You could do it for other situations where you are sure the angle is 90° . You only need the two perpendicular lines. You can mark any length along each of them, measure the lengths along the shorter sides, and the length of the hypotenuse. Find the squares of the numbers for these lengths, and check them against $a^2 + b^2 = c^2$. This activity matches Example 1 on page 149.

This leads to using the relationship to find missing sides.

Find the hypotenuse by adding the squares of the two shorter sides, and taking the square root – see Example 2 on page 149.

Find a shorter side by taking the square of the other shorter side from the square of the hypotenuse, and take the square root – see Example 3 on page 149.



Answers

Note: These questions rely on factorising perfect squares to derive their route. However some of these squares are not perfect and therefore prime factorisation cannot be implemented.

Exercise 1

- a) yes; b) yes; c) yes; d) yes; e) yes; f) no
- a) 15; b) 61; c) 41; d) 85; e) $(3 \times \sqrt{13})$
(**note:** not a perfect square, therefore cannot use prime factorisation); f) 109
- Remember Pythagoras' rule: $a^2 + b^2 = c^2$.
Therefore $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$.
a) 12; b) 60; c) 40; d) 84; e) 80; f) 60
- a) 10; b) 5; c) 13; d) 15

Assessment

Pupils should be able to find the length of a missing side in a right-angled triangle, in the case of an exact square root.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Prime factor products

Support activity

Interactive learning diagram: Prime factor products

Homework activity

Complete Exercise 1.

Lesson 3 *Pupil's Book page 151; Workbook page 36*

Preparation

You will need to have: Pupil's Book; Workbook.

Starter activity

Find some rectangles (preferably outdoors) and measure the sides and the diagonal. Check them against $a^2 + b^2 = c^2$.

Lesson focus

This section deals with real situations where we need to calculate the length of a diagonal, or a shorter length in a right-angled triangle.

Question 1 involves a rectangle, and question 3 a ladder leaning against a vertical wall. If possible use real things to illustrate these.

You might use a wall of your classroom. Find its height and length, and work out its diagonal length. If possible, check the answer.

Answers

Exercise 2 (Quantitative aptitude problems)

1. AC = 29 cm; 2. 33 cm
3. Hypotenuse = the length of the ladder = 10 m
 - a) One of the smaller sides = the distance from the ladder to the wall = 6 m
 - b) 8 m
4. 208 m

Worksheet 20

1. Hypotenuse
2. a) 5 b) $\sqrt{194}$ c) not possible
d) 25 e) 35

Assessment

Pupils should be able to solve word problems related to Pythagoras' rule.

Assess their performance in the quantitative aptitude problems and the activities.

Extension activity

Use a sheet of paper with right-angled corners. Measure 5 cm along one of the sides at right angles, and 12 cm long the other. Calculate the distance between these two points. Check this by measuring.

Support activity

Find some right-angled corners (e.g. in the classroom or outdoors). Measure 3 metres along one of the sides at right angles, and 4 metres long the other. The distance between these two points should be 5 metres. Check this.

Homework activity

Complete WB Worksheet 20.

Lesson 4 *Pupil's Book page 152*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 152

1. a) no; b) no; c) yes
2. a) 10; b) 26; c) 40
3. a) $3 \times \sqrt{41}$ (note: not a perfect square)
b) 26; c) 40
4. a) 5; b) 60; c) 20
5. 10

Objectives

By the end of this unit, each pupil should be able to:

- Express the same weight in different units: grams, kilograms, tonnes
- Solve word problems on weights.



Suggested resources

number slide; bathroom scales (Ask pupils to bring scales to school so they can weigh each other and other objects.); variety of empty containers (Ask pupils to bring empty bottles, cans, jars, boxes, etc. so they can find their capacities.)



Common errors that pupils make

In converting, pupils sometimes multiply when they should divide and vice versa.

When converting to a small unit there will be more of them, so the numbers are multiplied (by 1 000).

When converting to a larger unit there will be fewer of them, so the numbers are divided (by 1 000).



Evaluation guide

Pupils to:

1. Convert weights expressed in tonnes to kilograms and vice versa.
2. Solve word problems involving weights.

Lesson 1 *Pupil's Book page 153; Workbook page 37*



Preparation

You will need to have: Pupil's Book; Workbook; number slide; bathroom scales.



Starter activity

Discuss the difference between weight and mass. Pupils will know that you 'weigh less' on the moon – about one sixth of your weight on Earth. But the mass of the moon-walker has not changed, just the force pulling the person to the centre of the moon.

This is because the moon has a far smaller mass than the Earth.

Note: Pupils will have heard about 'weightlessness' in outer space. This is not lack of mass or weight, but free fall, when you cannot tell that you are falling because the space ship also falls with you. (You do not really fall by getting closer to the Earth's surface, but you fall just enough to keep you moving in a circle around the Earth.)



Lesson focus

Use the number slide for converting mass units.

If possible, use bathroom scales to find the mass of each pupil. Then find the total mass of the class and convert to tonnes.

Pupils might ask about smaller units of mass. Milligrams ($\frac{1}{1000}$ th of a gram) are often used in medicines, and even micrograms ($\frac{1}{1000}$ th of a milligram).

Larger units are kilo-tonnes kT (the mass of about 1 000 cars) and mega-tonnes MT (1 000 kT)



Answers

Exercise 1

1. **a)** 3 742 g; **b)** 5 605 g; **c)** 6 540 g; **d)** 8 120 g;
e) 3 500 g; **f)** 7 400 g; **g)** 900 g; **h)** 99 g; **i)** 9 g
2. **a)** 9.586 kg; **b)** 2.907 kg; **c)** 5.007 kg; **d)** 5.570 kg;
e) 0.280 kg; **f)** 0.5 kg; **g)** 0.075 kg; **h)** 0.06 kg;
i) 0.002 kg
3. **a)** 7 tonnes; **b)** 11 tonnes; **c)** 0.3 tonnes;
d) 0.049 tonnes

4. a) 1 587 kg; b) 4 560 kg; c) 350 kg; d) 600 kg

Worksheet 21

1. a) 3 200 kg; b) 5 000 kg; c) 3 256 g; d) 500 g;
e) 1 100 000 g; f) 0.76 kg; g) 0.001 kg;
h) 2 650 kg; i) 0.3 kg; j) 3.456 t
2. 1.9 t; 4.725 t; 3. 3.15 t; 4. 194 kg; 0,194 t

Assessment

Pupils should be able to convert units of weight (mass).

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Metric conversions

Let pupils attempt the challenges on pages 154 and 155.

Challenge page 154

1. (1 tonne = 1 000 000 g: so divide by 1 000 000)
a) 0.47; b) 0.296; c) 0.280; d) 0.500
2. (1 000 000 g = 1 tonne so multiply by 1 000 000)
a) 286 286; b) 946 000
c) 230 000; d) 800 000

Challenge page 155

Each answer will be different depending on which weight they choose.

For example, 60 kg for 1 person
 60×30 (if there are 30 students in the class)
 = 1 800 kg in total

Support activity

Interactive learning diagram: Weight conversions

Homework activity

Complete WB Worksheet 21, Questions 1 to 4.

Lesson 2 *Pupil's Book page 155; Workbook page 37*

Preparation

You will need to have: Pupil's Book; Workbook; variety of containers.

Starter activity

If possible, find a container with a capacity of one litre. In front of the class, weigh the container, and then add 1 litre of water. The weight should rise by 1 kilogram. Discuss the relationships between mass and capacity.

Lesson focus

The new idea in this lesson is that for water the units of capacity and mass are linked.

millilitre	\longleftrightarrow	litre	\longleftrightarrow	kilolitre
	$\times/\div 1000$		$\times/\div 1000$	
gram	\longleftrightarrow	kilogram	\longleftrightarrow	tonne

Note that capacity is covered again in Unit 25, when we concentrate on the distinction between capacity and volume, and focus mainly on volume. The emphasis here is to link mass units to the capacity units.

As much as possible this should be a practical lesson:

Pupils should find the capacity of different containers (bottles, cans, boxes) in millilitres. Imagine they are full of water. Work out the mass of that water.

When your empty classroom is 'empty' it is really filled with air. Measure your classroom in metres. Find its volume in cubic metres by multiplying length \times width \times height. Each hollow cubic metre has a capacity of 1 kilolitre. What is the capacity of your classroom?

If your classroom were to be filled with water, what would be the mass of that water? (Note: 1 000 kilograms is a tonne.)

Answers

Exercise 2

1. a) 4 000 kg; b) 40 000 kg; 2. 900 000 kg;
3. 31 kg; 4. 3.82 kg; 5. 700 kg heavier;

6. 755 g lighter; 7. 2.25 kg

Worksheet 21

5. 13 g
6. 5.845 kg
7. a) A: 4 t; B: 5.4t; C: 7.1 t
b) no, total weight is 16.5 t
c) A and B
8. a) 1 200 ℓ water; b) 1 500 ℓ petrol;
c) 1 080 kg load
9. 2.335 kg

Assessment

Pupils should be able to solve word problems related to weight or mass, and to use capacity units to find weights of quantities of water.

Assess their performance in hands-on activities and Exercise 2.

Extension activity

Interactive learning diagram: Metric conversions

Support activity

Interactive learning diagram: Weight conversions

Homework activity

Complete WB Worksheet 21, Question 5 to 9.

Lesson 3 *Pupil's Book page 156*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 156

1. Mass is the amount of matter contained within a body, and it measures how heavy an object is. Weight is the force of gravity acting on the mass of an object.
2. a) 100 g; b) 60 g
3. a) i) 2 592 g ii) 7 650 g iii) 8 900 g
b) i) 3.856 kg ii) 3.76 kg iii) 4.5 kg
c) i) 3.859 t ii) 3.45 t iii) 5.05 t
d) i) 3 000 kg ii) 3 600 kg iii) 5 280 kg
4. 6 kg; 5. 7.139 tonnes; 6. 1.365 kg

Objectives

By the end of this unit, each pupil should be able to:

- Tell time in hours, minutes and seconds
- Solve quantitative aptitude problems on time
- Read timetables of journeys
- Use time to plan daily activities.



Suggested resources

playing cards; local timetables



Common errors that pupils make

Calculations with time have many possible sources of error. For example to figure how many days between the 1st and the 10th, we should not count both days, only one of them. Pupils often make errors when calculating with 60 seconds in a minute.



Evaluation guide

Pupils to:

1. Measure the time of tasks.
2. Solve quantitative aptitude problems involving timing.
3. Be able to plan for a week or two.
4. Obtain a train timetable and estimate the time to reach a specific destination.

Lesson 1 *Pupil's Book page 157; Workbook page 39*



Preparation

You will need to have: Pupil's Book; Workbook; playing cards.



Starter activity

Discuss the analogue and digital clocks on page 157. Make sure that all pupils can successfully read analogue clocks.

Discuss the use of am (before noon) and pm (after noon), and the use of 24-hour notation. Pupils should complete Exercise 1.



Lesson focus

Calculating with time includes some new ideas. Adding seconds (or minutes) uses place value as in other numbers, but changing units uses 60s. For example 45 minutes + 30 minutes is 75 minutes, but we call it 1 minute and 15 seconds.

It might be good for pupils to make up problems of these kinds for each other to solve. However they should find the answer themselves, so they can check each other's work.



Answers

Exercise 1

1. a) 25 minutes to 7
b) 16 minutes past 3
c) 10 minutes past 10
d) 5 minutes to 8
2. a) 40 minutes and 3 seconds past 1
b) 50 minutes and 30 seconds past 9
c) 15 minutes and 45 seconds past 12
d) 8 minutes and 32 seconds past 11
3. a) 03:00; b) 18:00; c) 12:00; d) 22:00
4. a) 5:00 am; b) 11:00 am; c) 2:00 am;
d) 2:00 pm; e) 6:00 pm; f) 3:00 pm;
g) 9:00 pm; h) 7:00 pm; i) 11:00 pm;
j) 12:00 pm

Exercise 2

1. 6 days and 5 hours
2. 102 years old
3. 312 years

Worksheet 22

1. a) 02:30:10 b) 07:20:45 c) 03:45:15
d) 11:58:20 e) 06:30:00 f) 02:45:15
2. a) quarter past 4 in the morning
b) 12 minutes, 12 seconds past 12 in the morning
c) 25 minutes, 36 seconds past 1 afternoon
d) 25 minutes, 5 seconds past 10 in the morning
e) 1 second to midnight
f) 14 minutes, 59 seconds to 4 afternoon

Assessment

Pupils should be able to convert units of time in calculations with hours, minutes and seconds

Assess their performance in Exercises 1 and 2 and the game.

Extension activity

Interactive learning diagram: Time calculations

Pupils may attempt the challenge on page 159.

Challenge page 159

This exercise will depend on the newspapers read. But look at their conversion between 24 hr and 12 hr clocks.

Support activity

Interactive learning diagram: Clock

Homework activity

Complete WB Worksheet 22.

Lesson 2 *Pupil's Book page 160*



Preparation

You will need to have: Pupil's Book; local timetables; playing cards.



Starter activity

Discuss the way to read the timetable, and also use local timetables if you have them. Use the example on page 160 to help you.



Lesson focus

Discuss the idea of duration and how it can be calculated. Discuss conversion of units. The relationship between the units often uses the number 60. (This number was chosen thousands of years ago because 60 has many factors, so you can get many fractions of an hour, or a minute.)

For many people it is easier to subtract (or find a duration) by starting at the earlier time and adding up to the later time. Go over the example on page 159, and make up other examples. These could involve numbers of days between different dates, involving months in the same year, or across numbers of years (don't forget leap years).



Answers

Exercise 3

1. 12; 2. 4; 3. 06:15; 4. 21:10
5. 2 hours and 55 minutes
6. Bus number 35; 7. Bus number 12
8. Bus number 35

Assessment

Pupils should be able to use a time table, including calculating the duration of an event, given the starting and finishing times.

Assess their performance in Exercise 3.

Extension activity

Play the following game as an extension activity:

Card games – More duration calculations

Use the numbers 1 to 12. (Use J for 11 and Q for 12. Do not use K.)

Duration in days – same year

Put out one card to show the month (using the numbers 1 to 12 to show January to December.) Put out two more cards and add them to show the date of the month. (This will be from 2 to 24.)

Repeat for another date.

Now find the number days that have elapsed between the earlier date and the later one. You will

need to take account of the number of days in each month: 28 for February (or 29 for leap years, that are multiples of 4); 30 for April, June, September, November; 31 for January, March, May, July, August, October, December.

Support activity

Interactive learning diagram: Time calculations

Lesson 3 *Pupil's Book page 161*



Preparation

You will need to have: Pupil's Book; playing cards.



Starter activity

Discuss the purpose of daily planning. Pupils are at an age where it is easy to waste much time in idle activity with friends, or – if they have a computer – in wasteful 'surfing' of the internet.



Lesson focus

Pupils should work together in a group to plan what they think is a good way to spend a school day, and a non-school day. These can then be compared as each group reports to the class.



Answers

Exercise 4

Answers will vary.

Assessment

Pupils should be able to make a daily and weekly plan.

Assess their performance in Exercise 4.

Extension activity

Play the following game as an extension activity:

Card games – More duration calculations

Use the numbers 1 to 12. (Use J for 11 and Q for 12. Do not use K.)

Duration in days – different years

Put out one card to show the month (using the numbers 1 to 12 to show January to December.) Put out two more cards and add them to show the date of the month. (This will be from 2 to 24.) Now put out two cards (between 1 and 12) and add them to show the year in this century.

Repeat for another date and year.

Now find the number of days that have elapsed between the earlier date and the later one.

Try the challenge on page 161.

Challenge page 161

Dependent on each pupil.

Support activity

Interactive learning diagram: Calendar

Lesson 4 *Pupil's Book page 161*



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 162

- 5 minutes, 6 seconds
- Twelve in the morning
 - thirty-five minutes and eleven seconds past ten pm
 - twenty-five minutes past six
 - twenty minutes to twelve
- 21:15; 4. 21:24; 5. 1 year and 22 days
- 25 minutes; b) A quarter past ten; c) 2 hours
- 225 km/hour

Pupil's Book page 164

Objectives

This project requires pupils to work with time in real-life situations and applications of these concepts in the world. They work with time differences, create a timeline and a calendar and plan an event, using time as the basis from which to work and plan.

**Guidelines**

You will need to have the following resources: Pupil's Book, Internet, library resources, an atlas, a time difference chart, and materials for creating a calendar.

Pupils need to research and then follow the steps, as mentioned on page 164 of the PB.

For the first part of the project, pupils arrange the athletes' times in order from the winner down: 43.94; 44.46; 44.52; 44.79, 44.81; 44.83; 44.98, 45.14 – this would be recorded in seconds and fractions of seconds.

They then find the results of other athletic teams and compare the times, as well as how the winning time was given. They also find the records for an event and compare how the records have changed between the period of 1990 to present day. Pupils must represent this information on a timeline. Pupils can access this information from the Internet or from their nearest library.

For the second part of this project, pupils are required to create a daily calendar for two weeks, showing every hour of the day. Pupils need to fill the calendar with activities planned for every time slot over this period of time. You could perhaps create a calendar for one 24-hour day and show pupils exactly what is required of them, right down to hours you allocate to sleeping and rest. Encourage pupils to be productive and creative with their time.

For the third part of this project, pupils must make use of an atlas and a time difference chart. Pupils work with GMT and compare the different time differences from where the pupils are, and other places in the world.

For the last part of this project, pupils plan an event, calculating time differences and using travelling time to work out their estimated arrival times and GMT. Perhaps you could do an example on the board for pupils to understand fully what is required of them.

Pupils' work must be informative and as accurate as possible, especially when working with the time differences.

Assessment

Because the pupils' findings will be different, assess pupils on:

- researching and comparing record times
- telling time in seconds and minutes
- solving quantitative aptitude problems on time
- read timetables of journeys and GMT
- using time to plan daily activities creating a calendar
- planning an event to calculate the estimated time differences, incorporating travelling time.

Pupil's Book page 165

Objectives

For this investigation pupils are required to investigate taxes, rates and rents, and other financially-related matters.

**Guidelines**

You will need to have the following resources: Pupil's Book, Internet, newspapers, and materials for their posters.

Pupils work on their own, research and write down their answers for this Investigation. It is important to start this investigation by pointing out to pupils that Governments (local and national) levy rates and taxes. They get money for the projects they develop for the good of the community. At local level the money pays for electricity or water availability and refuse removal, etc.

For this investigation, pupils need to find out about the types of rates and taxes levied by their local government or municipality. This may include annual rates on property owned and monthly levies for services, such as refuse removal, availability of water or electricity.

Pupils need to find out how the taxes and rates are applied; meaning they need to find out how and how much people have to pay.

For the next part of the investigation, pupils need to make up a situation where they draw up the expenses of an imaginary family situation and calculate the rent, rates and taxes that the head of the family has to provide for. Pupils need to make account slips for every item as if it would be posted.

For the next part of the investigation, pupils must investigate the income tax system. They need to use the following questions to assist them in their investigation:

- a) Do people pay a percentage of their salary, or is it a fixed amount? (Income tax is usually calculated as a percentage of a person's income.)
- b) Does everyone pay the same percentage (if applicable) or do people who earn more pay a higher percentage?
- c) How much does a person earning an average salary have to pay?
- d) What is taxable income?
- e) Are there possible deductions from the salary to make the taxable income less?
- f) Pupils must do calculations for different amounts of income.

Pupils then find out what is meant by shares and dividends. They can use newspapers to obtain information about share prices. Pupils need to find out the following information for their investigation:

- a) Find out why companies issue shares. (Remember that when someone buys shares in a business, it means that they become part-owners of the business. Companies also sell their shares to members of the public. Shares can be bought and then sold, because they can often be bought at a low price and then sold for a lot more later on.)
- b) What is meant by a dividend and why do companies pay dividends? (Companies pay dividends to their shareholders, based on their profits.)
- c) Pupils then need to make up a situation where a person has shares (in one or more companies). Pupils must choose the shares from a newspaper. They then work out how

many shares this person will have and what the cost will be. Pupils must follow the movement of the shares over a period of time and decide when to sell. They then calculate the profit or loss made on the shares.

Pupils must represent their findings in a poster. Remind pupils that they need to work neatly and as accurately as possible. They also need to keep updated with the rising and falling of share prices.

Assessment

Because the pupils' findings will be different, assess pupils on:

- researching, collecting and recording information
- investigating taxes and rates
- investigating the buying and selling of shares and dividends
- solving problems on quantitative aptitude based on taxes and rate as well as selling of shares and dividends.

Pupil's Book page 166

Objectives

This assessment is a summative assessment of work that pupils have covered in Units 11 to 22. It has been designed to assess the pupils' mathematical understanding. It is also important that it is completed by individuals and not with the support of other pupils as this would not uncover any difficulties a pupil may be having with particular concepts.



Guidelines

Pupils must work through the questions on their own. However, observing pupils while they are completing the assessment provides further information about each pupil and his or her level of understanding.



Answers

- 174 080; 15 374 868; 459 111; 1 785.092; 57 420; 521 664
 - 273 728; 24 642
- 975 003.46; 281 280 5.7
 - 11 898 300.00; 46 244 327.86
- $\frac{1}{2}$; $1\frac{1}{6}$; $2\frac{2}{3}$
- 25; 25; 14
- 2.795; 38,25; 1.44;
 - 3.375; 0.0112; 225
- $\frac{7}{10}$; 0,7 b) 175 g; $\frac{28}{65}$ kg; 2.45 kg
- $\frac{1}{8}$; $\frac{2}{5}$; $\frac{5}{48}$; $2\frac{1}{3}$; 40;
 - 15; 150; 5.2
- ~~390.2385~~; ~~135.1614~~; ~~374.6175~~
- 144; 784; 22 500; 160 000
- 6; 14; 16; 50
- ~~35 295~~; ~~19 859~~
- $(20 + 8) \times 35$; $(30 - 2) \times 35$
 - $28 \times ((42 \div 6) \times 5)$; $28 \times (30 + 5)$
- $(8 - 2) \times 7 = 42$; $(9 + 6) \times 8 = 120$
 - $(\frac{2}{3} + \frac{1}{3}) \times 6 - (8 \div 2) + 7 = 11$
 - $24 \times (-3 + 6) + 5 = -7$
- 11^2 ; 13^2 ; 15^2 ; 8^2 ; 10^2 ; 20^2
- 2^5 ; $3^2 \times 5$; 2^7 ; $2^3 \times 5^2$
 - $3^2 \times 2^3$; $2^5 \times 3$
- $20 = 2^2 \times 5$; $64 = 2^6$;
 $20 \times 64 = 2^2 \times 5 \times 2^6 = 2^8 \times 5 = 1\ 280$
 - $44 = 11 \times 2^2$; $33 = 3 \times 11$
 $44 \times 33 = 11^2 \times 2^2 \times 3 = 1\ 452$
 - $36 = 2^2 \times 3^2$; $15 = 5 \times 3$
 $36 \times 15 = 2^2 \times 3^2 \times 5 = 540$
- 2.33%; 19. ~~3~~13.75
- 18.26%
 - In order to work this out you need to divide 93 by three. This will tell you how many scarves will be placed on each rack. There will be 31 scarves per rack.
- $x = 6$; $H = 25$
- $b^2 - y^2 + x^2$
Need to work out the hypotenuse
 $b^2 - 52 + 12.5^2$
 $b^2 - 25 + 156.25$
 $b^2 - 25 + 181.25$
 $H = 13.46$
 \therefore the ladder is not long enough to reach the cat, as the length needed is 13.46 m
- 70.77 kg;
 - Total lift can take = 920 kg
 $78\text{ kg} + 35\text{ kg} + (750 \times 1\text{ kg})$
 $= 863\text{ kg in total}$
 \therefore the lift will be strong enough to carry the combined weight
- The children are in class for a total of 4 hours and 55 minutes each day.

Assessment

On completion of the assessment, teachers should look for correct answers and mistakes made by pupils. They should also check to see if there is a pattern in terms of any particular question causing a significant number of pupils' difficulties. By analysing the results of an assessment, they can identify weaknesses in individuals and provide the necessary support, and also strengths of individuals and provide them with more challenging activities. They will also be able to identify any weaknesses in their teaching programme and make adjustments where, or if, necessary.

Objectives

By the end of this unit, each pupil should be able to:

- Discover that different rectangles with the same area have different perimeters.



Suggested resources

centimetre grid sheets; paper squares; measuring tapes



Frequently asked questions

Q *What do pupils need to understand?*

A The concepts of rectangle perimeters and areas are not the same, but they are clearly linked, as they both involve the width and height of the rectangle.

If the area is fixed, then the 'rectangle closest to a square' will have the smallest perimeter.

If the perimeter is fixed, then the 'rectangle closest to a square' will have the largest area.

Q *What mathematical skills are needed?*

A Pupils should be able to use the appropriate formulae, and have some problem solving skills (mainly trial and error) and persistence.



Common errors that pupils make

Pupils link the size of area and perimeter in the wrong way.

Many pupils become convinced that a rectangle that is 'bigger' in one way (area or perimeter) than another must also be bigger in the other way (perimeter or area). This lesson aims to correct that idea.



Evaluation guide

Pupils to:

1. Calculate perimeter of rectangles with equal areas but with different lengths and breadths.

Lesson 1

Pupil's Book page 169;

Workbook page 40



Preparation

You will need to have: Pupil's Book; Workbook; cm grid paper; paper squares.



Starter activity

Revise the three formulae for perimeter of a rectangle.

Revise the formula for area of a rectangle.

(Note: the terms 'width' and 'height' are used here for the sides of the rectangle. You or the pupils might prefer 'length' but this can be confusing. Both sides are lengths, in contrast to areas. 'Height' clearly refers to 'up the page' in a diagram.)



Lesson focus

Hands-on activity with rectangles

Each group needs about 20 cut-out paper squares. Ask the group to make at least three different 'filled-in' rectangles with 12 squares. For each they should find the perimeter. They record their answers on the centimetre grid page.

Conclusion: if area stays the same, the perimeter can be different. For rectangles with the same area, the closer the rectangle is to a square, the smaller is its perimeter.

Now ask the group to make at least three different rectangles with a perimeter of 12 units. For each they should find the area. They record their answers on the centimetre grid page.

Conclusion: If perimeter stays the same, the area can be different. For rectangles with the same

perimeter, the closer the rectangle is to a square the larger its area.

Repeat the activities with 20 squares instead of 12. Make two rectangles so that one of them has a larger perimeter and the smaller area, and the other has the smaller perimeter and the larger area. They record their answers on the centimetre grid page.

Hands-on activity with non-rectangles

Start with a rectangle 3×4 .

Find its perimeter and area.

Remove one square but keep the perimeter the same.

Now remove another square but increase the perimeter. Can you do it one more time? Make the shape have the largest perimeter you can with the smallest area at the same time.

With 9 squares what is the largest perimeter you can make? What is the smallest perimeter you can make?

Repeat with other numbers of squares.



Answers

Exercise 1

1.

	a	b	c	d	e
height	5	4	3	2	1
width	1	2	3	4	5
perimeter	12	12	3	12	12
area	5	8	9	8	5

2. and 3. Answers will vary.

4. a) perimeter: 12; area: 8
b) to d) answers vary

Puzzle page 170

$$A = 288 \text{ m}^2$$

Challenge page 171

1. 50 mm; 2. 6.2 mm; 3. 56 mm; 4. 3.3 mm;
5. 99 mm; 6. 46.5 mm; 7. 19.8 mm

Exercise 2

Rectangle A: Width = 20, Height = 10.
Perimeter = 60, Area = 200

Rectangle B: Width = 25, Height = 8.
Perimeter = 66, Area = 200

Worksheet 23

- ? = 9,375; perimeter = 80; perimeter = 98,75
- ? = 120; perimeter = 260
- Rectangles with different perimeters can have equal areas.
- ? = 45; perimeter = 100

Assessment

Pupils should be able to create shapes with different perimeters and areas.

Assess their performance in Exercises 1 and 2, and the activity.

Extension activity

Interactive learning diagram: Area and perimeter

Support activity

Interactive learning diagram: Rectangle perimeter and area

Homework activity

Complete WB Worksheet 23 and the hands-on activities.

Lesson 2 *Pupil's Book page 171*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 171

Answers will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Calculate the areas of figures which can be divided into rectangles and or triangles

- Calculate land areas in hectares



Suggested resources

plain white paper; sticky tape; sticks to mark the vertices of triangles on grass – three per group; long metric tapes (10 m, 20 m or even 50 m); examples of land areas measured in hectares (ha)



Frequently asked questions

Q *What should pupils understand?*

A Although there is a formula for the area of triangles and each of the quadrilaterals, instead of just memorising these, pupils should understand where the formulas come from.



Evaluation guide

Pupils to:

1. Find areas of figures which can be divided into rectangles and triangles.
2. Calculate the areas of the given shapes or figures.
3. Calculate given areas in square metres and convert to hectares.

Lesson 1 *Pupil's Book page 172*



Preparation

You will need to have: Pupil's Book; paper; measuring tape; sticks (or pegs).



Starter activity

Demonstration and practical work

Show how you can take a rectangle, draw a diagonal and cut along it. Show that the two

triangles fit on top of each other. They have the same area. So the area of the triangles is half the area of the rectangle from which they came. Pupils repeat the same activity (called Right-angled triangles on page 172 in the PB). They measure the base and the perpendicular height and find the area in square centimetres (m^2).



Lesson focus

Paper triangles – group work

Pupils repeat the other two activities (with acute-angled triangles and obtuse-angled triangles). They measure the base and the perpendicular height and find the area in square centimetres (m^2).

Because they all come from halving the area of the A4 sheet, they will all have the same area. This is the point on the top of page 173.



Answers

Exercise 1

1. Practical activity;
2. Answers will vary.
3. a) 20
b) 6
c) 4
d) 2
4. a) 20
b) 6
c) 8
d) 2

Assessment

Pupils should be able to explain why the area of a triangle is half the base times the perpendicular height.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Area of polygon
Let pupils attempt the challenge on page 174.

Challenge page 174

1. $A = 187 \text{ cm}$
2. The shape is divided into 2 rectangles; 1 square and 2 triangles.
It could be further divided into 8 triangles.

Support activity

Interactive learning diagram: Triangle area

Homework activity

Complete Exercise 1.

Lesson 2 *Pupil's Book page 174;* *Workbook page 41*



Preparation

You will need to have: Pupil's Book; Workbook; paper.



Starter activity

Demonstrate how to find the area of the parallelogram, by folding an A4 sheet to a parallelogram.



Lesson focus

Group work

Groups of pupils should now find the areas of the rhombus and the kite using the instructions on page 175.
To make the rhombus, they fold an A4 sheet in half both ways to make four thicknesses. Then they cut off one triangle that does not include any folds.

To make the kite, use the diagram on page 175. The kite has only one line of symmetry. The important idea is for pupils to realise that the areas of the rhombus and the kite are just the sum of two triangles. This leads to the formulas given.

Outdoor quadrilateral area

Use the teaching note on page 177 of the PB as a practical activity.



Answers

Exercise 2

1. 30 cm^2
2. a) 21 m^2 ; b) 60 cm^2
3. 1.62 l

Worksheet 24

1. $112,7 \text{ cm}^2$; 2. 28 cm^2 ; 3. 72 m^2 ; 4. 600 m^2

Assessment

Pupils should be able to find the areas of quadrilaterals and other shapes made from rectangles and triangles.
Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Quadrilateral areas
Pupils may attempt the puzzle on page 176.

Puzzle page 176

(Pupils will use trial and error for these, so each will have different working out.)

1. She could use an equilateral triangle with each side = 4 m
She could use a square with each side = 3 m
She could use a rectangle with length = 6 m each and width = 2 m each
2. The rectangle would give her the largest area.

Support activity

Interactive learning diagram: Some areas

Homework activity

Complete WB Worksheet 24, Questions 1 to 4.

Lesson 3 *Pupil's Book page 177; Workbook page 42*



Preparation

You will need to have: Pupil's Book; Workbook; newspaper articles showing land area.



Starter activity

Discuss the use of a larger area unit – the hectare. Provide examples of this from newspapers etc.



Lesson focus

Outdoor measurement

On a suitable space pupils measure out a square $100\text{ m} \times 100\text{ m}$ using long metric tapes.

If such an area is not available, find an equivalent area (if possible). Examples are $50\text{ m} \times 200\text{ m}$ or $20\text{ m} \times 500\text{ m}$.

If these are far too big for your school space, choose a smaller square. For example, $50\text{ m} \times 100\text{ m}$ is half of a hectare (0.5 hectare) or $50\text{ m} \times 50\text{ m}$ is a quarter of a hectare (0.25 hectare).

The important message is that 1 hectare = 10 000 square metres.



Answers

Exercise 3

- Area = 5 hectares.
- Square with each side being 100 m
Rectangle with one pair of sides being 20 m and the other pair being 500 m
Rectangle with one pair of sides being 10 m and the other pair being 1000 m
Rectangle with one pair of sides being 1 m and the other pair being 10 000 m
- 400 m
- base = 200 m height = 400 m
base = 100 m height = 800 m
base = 10 m height = 8000 m
base = 1 m height = 80 000 m
- 24 hectares

Worksheet 24

- a) Park: 1,65725 ha
b) Play garden: $0,068\text{ ha} = 680\text{ m}^2$
c) Park without play garden: $15\,847,5\text{ m}^2$
d) Cost = ₦1 505 512,50
- 2,595 ha
- Answers will vary.

Assessment

Pupils should be able to say what area is meant by 1 hectare.

Assess their performance in the outdoor activity and Exercise 3.

Extension activity

Interactive learning diagram: Land areas parts 2 and 3

Support activity

Interactive learning diagram: Land areas part 1

Homework activity

Complete WB Worksheet 24 Questions 5 to 7.

Lesson 4 *Pupil's Book page 178*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Revision exercise page 178

- a) 50 cm^2 ; b) 600 cm^2 ; c) $1\,600\text{ cm}^2$; d) 18 cm^2
- a) 8 cm; b) 6 cm; c) 4 cm; d) 2 cm
- a) 4 cm; b) 6 cm; c) 8 cm; d) 2 cm
- height = 8, base = 6 (example)
- 6. To be done in class, pupils' work will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Use standard units for capacity
- Solve word problems involving litres
- Calculate volume of prisms, cylinders and spheres
- Solve some quantitative aptitude problems on volumes of prisms, cylinders and spheres.

**Suggested resources**

measuring jugs for water – at least up to 1 litre (one for each group, maybe brought from home by pupils); water supply – maybe a bucket If water is a problem you may use sand, dried maize seeds or rice); rice, grain. seeds to measure capacity; many empty watertight containers, possibly brought from home by pupils; measuring tape; boxes; set of unit cubes for a visible demonstration (24); copies of net of triangular prism – page 20; copies of net of rhombic prism – page 20; scissors; sticky tape; A4 paper, drink cans and other cylinders; small ball or orange, can of water and measuring cylinder

**Common errors that pupils make**

Pupils confuse volume and capacity.

This is not helped when we say volume, and use litres. A capacity is used when we have a container. Only containers have capacity. Volume is the space taken up by something, including the liquid in the container.

**Evaluation guide**

Pupils to:

1. Calculate volumes of given triangular prisms, cylinders and spheres.
2. Find the capacities of common containers.
3. Solve word problems on capacities.

Lesson 1

Pupil's Book page 179

**Preparation**

You will need to have: Pupil's Book; measuring jugs; water; empty watertight containers.

**Starter activity**

Discuss the volume of solids, and capacity in containers. Discuss the units of capacity (on pages 179 to 181 of the PB) and conversion between them.

**Lesson focus****Practical work**

Organise pupils into groups. They will use water, measuring jugs and empty watertight containers to work on Exercise 1.

**Answers****Exercise 1**

To be completed in class, answers will vary

Assessment

Pupils should be able to understand capacity, measure it and convert cuboid volumes to capacities.

Assess their performance in Exercise 1.

Extension activity

Interactive learning diagram: Cuboids

Support activity

Pupils should look at home at any cooking utensils. Cups and measuring spoons and jugs will be marked in millilitres and litres. This is because they are containers. Some recipes use units of capacity, such as '100 ml of water', or '1 tbsp (tablespoon) of flour'.

Lesson 2

*Pupil's Book page 180;
Workbook page 44*



Preparation

You will need to have: Pupil's Book; Workbook; measuring tape; boxes.



Starter activity

Demonstrate that the volume of the cuboid is the area of its base multiplied by its height.

Make a layer of cubes 3×2 , 1 unit high for the base. The volume of this cuboid is simple $2 \times 3 \times 1$.

Add another layer: the volume is $2 \times 3 \times 2$.

Add another layer: the volume is $2 \times 3 \times 3$.

Add another layer: the volume is $2 \times 3 \times 4$.

The volume is *area of base* \times *height*.



Lesson focus

Practical work

Groups of pupils work on these tasks.

1. Measure cuboid containers (e.g. boxes) to find their volumes.
2. Measure the dimensions of the classroom, or other large volume, and find its capacity. Use suitable units. Note a hollow 1 cubic metre (1 m^3) has a capacity of 1 kilolitre (1 000 litre).



Answers

Challenge page 181

Volume is the measure of the space taken up by something whereas capacity is the amount a container can hold.

Exercise 2

1. a) Capacity = 0.305 ℓ
Volume = 305 cm^3
b) Capacity = 1 200 ℓ
Volume = $1\,200\,000 \text{ cm}^3$
c) Capacity = 0.7 ℓ
Volume = 700 cm^3
So 15 parcels have a capacity of 10.5 ℓ and a volume of $10\,500 \text{ cm}^3$
2. Capacity = $200 \text{ cm} \times 100 \text{ cm} \times 80 \text{ cm}$
 $= 1\,600\,000 \text{ cm}^3$
 $= 1\,600\,000 \text{ ml} = 1\,600 \ell$
3. Two examples:
 - i) Height = 2 cm, width = 30 cm, depth = 2 cm
 - ii) Height = 4 cm, width = 10 cm, depth = 3 cm

Worksheet 25

6. 3.5 cm; 9. $3\,500 \text{ cm}^3$

Assessment

Pupils should be able to find the volumes of cuboids

Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Boxes

Support activity

Interactive learning diagram: Cuboids

Homework activity

Complete WB Worksheet 25, Questions 6 and 9.

Lesson 3

*Pupil's Book page 181;
Workbook page 43*



Preparation

You will need to have: Pupil's Book; Workbook; copies of net of a triangular prism; copies of net of a rhombic prism.

➡ Starter activity

Make a triangular prism from a net. Use it to show the pupils a prism that is not rectangular. Discuss the general formula: Volume = area of base \times height.

🔍 Lesson focus

Practical work

Groups of pupils work on these tasks.

1. Make a triangular prism from its net and find its volume.
2. Make a rhombic prism from its net and find its volume.

📖 Answers

Exercise 3

1. a) 64 cm^3 ; b) $4\,096 \text{ cm}^3$; c) 614.125 cm^3
2. a) 105 cm^3 ; b) $1\,233.75 \text{ cm}^3$
3. 420 cm^3 ; 4. 300 m^3 and 300 kilolitres

Worksheet 25

1. 600 cm^3

Assessment

Pupils should be able to measure and calculate the volume of a non-rectangular prism.

Assess their performance in hands-on activities and Exercise 3.

Support activity

Interactive learning diagram: Prisms

Homework activity

Complete WB Worksheet 25, Question 1.

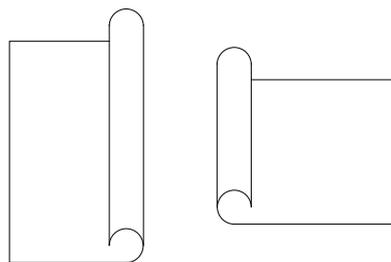
Lesson 4 *Pupil's Book page 183;*
Workbook page 43

⚙️ Preparation

You will need to have: Pupil's Book; Workbook; rice, grain, seeds for measurement; scissors; sticky tape; A4 paper; grid paper; cold drink cans and other cylinders.

➡ Starter activity

An A4 sheet can be rolled into a cylinder in two ways: giving different heights and diameters.



Do this and use sticky tape to make them look like cylinders.

Ask the class to guess which has the larger volume (or capacity). Which holds more? Take a vote. Don't tell them the answer – yet.

🔍 Lesson focus

Groups solve a puzzle

Use the cylinders formed in the starter activity above. Groups should now use their own sheets to make the cylinders, and then direct measurement to compare them. They may use rice, or dried seeds to find which is the larger.

Remind them of the basic formula:

Volume = area of base \times height.

Now they can measure the area of the circles at the base by tracing them onto centimetre grid paper and counting squares. If needed, tell them the areas.

(The short one has a circle with circumference about 30 cm, so its radius is about 4.8 cm. Hence the area is about 72 cm^2 .)

The tall one has a circle with circumference about 20 cm, so its radius is about 3 cm. Hence the area is about 32 cm^2 .)

Now each area has to be multiplied by its height: Short $72 \times 20 = 1\,440 \text{ cm}^3$ or 1 440 ml.

Tall $32 \times 30 = 960 \text{ cm}^3$ or 960 ml

The shorter cylinder is much bigger: 50% bigger in fact.

Volume (capacity) of drink cans etc.

Instead of reading the capacity on the label of a drink can, pupils can now trace the circular area of its base and count squares to find the area.

They can measure the height and multiply by the

base to find the volume (in cm^3), and convert that to millilitres.

Answers

Exercise 4

1. a) 60 cm^3 ; b) $22 \times 4 = 88 \text{ cm}^3$; c) 160 cm^3
2. a) $6\,000 \text{ cm}^3$; b) 6 l ; c) 4.5 l
3. 25 cm

Worksheet 25

2. $282\,600 \text{ cm}^3$; 3. 282.6 l ; 7. 11.1 cm ; 8. 77.9 cm

Assessment

Pupils should be able to find the volume (and capacity if hollow) of a cylinder, given the base area.

Assess their performance in practical work and Exercise 4.

Extension activity

Interactive learning diagram: Volume of a cylinder

Support activity

Interactive learning diagram: Cylinder

Homework activity

Complete WB Worksheet 25, Questions 2, 3, 7, 8.

Lesson 5 *Pupil's Book page 184;* *Workbook page 43*

Preparation

You will need to have: Pupil's Book; Workbook; scissors; sticky tape; A4 paper; cold drink cans and other cylinders; measuring jug; watertight containers; water; small ball or orange.

Starter activity

A sphere is the mathematical name for a ball-shape. Pupils can name some objects that are spheres or parts of spheres.

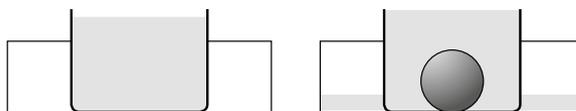
Lesson focus

Practical demonstration

The teacher can directly measure the volume of a sphere (or any other object) by displacement of water. Here is how it is done:

You need a small ball, orange or other small spherical object.

You need a container that can be completely filled with water. This is placed inside another container that can catch any water that comes out of the smaller container as the ball goes into it.



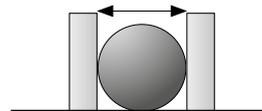
Then you carefully push the sphere under the water, displacing water as you do it. Do not displace more than the volume of the sphere. Pour the displaced water into the measuring jug. This will tell you the volume of the sphere. It will use millilitres, and these are also cm^3 .

Introduce the formula

Explain the use of the formula by using the example on page 184.

Then pupils should use the formula with the sphere used above.

First we find the diameter.



Place the ball between two books. Measure the distance between the books. This is the diameter. The formula uses the radius, so we must divide the diameter by 2.

Use this number – instead of 2 – for the radius in the formula as in the example.

Check that it is at least close to the answer measured by displacement of water.

Answers

Exercise 5

1. a) $36\frac{22}{7} \text{ cm}^3$ b) $\frac{440}{21} \text{ cm}^3$ c) $\frac{352}{21}$

Worksheet 25

4. $5\ 572.5\text{ cm}^3$; 5. 1 m^3

Assessment

Pupils should be able to understand what is meant by ‘the volume of a sphere’, and compute it given the formula and the radius.

Assess their performance in Exercise 5.

Extension activity

Find the volume of one drop of water. Get a slowly dripping tap and count the number of drops that make up an amount you can measure. Divide that volume by the number of drops to find the volume of one drop. (It is about one tenth of a millilitre.)

Homework activity

Complete WB Worksheet 25, Questions 4 and 5.

Lesson 6 *Pupil's Book page 185*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Revision exercise page 185

- a) 0.4 l b) 0.8 l c) 0.15 l
- $4 \times 5 = 20$ bottles
- Height = 2, width = 4, depth = 9 (example)
- a) 100 cm^2 ; b) $100 \times 3\ 000 = 300\ 000\text{ cm}^3$

Objectives

By the end of this unit, each pupil should be able to:

- Calculate average speed
- Solve quantitative aptitude problems on time and speed
- Estimate time to complete races.

**Suggested resources**

measuring tape (10 m, 20 m or 50 m); stopwatch

**Frequently asked questions**

Q *What are the important concepts?*

A There are three relationships that are closely related:

$$\text{distance} = \text{time} \times \text{speed}$$

$$\frac{\text{distance}}{\text{time}} = \text{speed} \quad \frac{\text{distance}}{\text{speed}} = \text{time}$$

Many people remember only the first. They can then divide distance by time to get speed, and divide distance by speed to get time.

**Evaluation guide**

Pupils to:

1. Calculate speed when time and distance are given.
2. Estimate the time a person will take to run 100 m.

Lesson 1

Pupil's Book page 186;

Workbook page 45

**Preparation**

You will need to have: Pupil's Book; Workbook; measuring tape (10 m, 20 m or 50 m); stopwatch.

**Starter activity****Outdoor activity**

In this activity we measure a speed using a tape for the distance, and a stopwatch for the time.

Take the pupils outdoors.

Ask a pupil to walk steadily for five seconds, timed using a watch.

Measure the distance. The speed will be distance divided by 5, the number of seconds, and it will be in metres per second.

Ask these questions, which use proportion:

If the pupil walked for 10 seconds, how far would he/she go?

How far in a minute? (Multiply by 60)

How far in an hour? (Multiply by 60.)

Then convert the distance (metres) to kilometres by dividing by 1 000. You now have the speed in kilometres per hour – km/h.)

**Lesson focus****Practical speed measurement**

If possible, groups of pupils should now repeat the activity. They will all need their own tapes and watches.

Alternatively, measure a 10-metre length. They can then walk or run this distance, and find their speeds by dividing 10 by the number of seconds. (This can be done on a calculator.)

They can now find the distance they would go in 10 seconds, 1 minute (60 seconds), and 1 hour (60 minutes). They convert this to km/h.

**Answers****Exercise 1**

1. $\frac{100}{20} = 5$ m/s; 2. 10 km/h; 3. 500 km/h;
4. 2.39 m/s

Exercise 2

1. 9 000 km; 2. 8 hours; 3. a) 80 km;
b) 4 hours; c) 20 km/h; 4. a) 176 km;
b) 3.5 hours; c) 50.3 km/h

Worksheet 26

1. a) 1.6 min/km b) 1.6 min/km
c) 1.8 min/km d) 2.2 min/km
e) 2.3 min/km f) 2.5 min/km
g) 1.7 min/km h) 1.8 min/km
i) 2 min/km j) 2.5 min/km
k) 2.6 min/km l) 2.8 min/km
2. a) 1:35.8 b) 1:44.9
3. a) 2.9 min/km b) 3.2 min/km
4. 52 sec; 5. 54 sec; 6. 1:48.1; 7. 3:30:59;
8. 4.4 min/km

Assessment

Pupils should be able to calculate speed in metres per second and convert this to kilometres per hour.

Assess their performance in practical activities and Exercises 1 and 2.

Extension activity

Interactive learning diagram: Distance, time, speed

Speed in metres per second can be converted to kilometres per hour by multiplying by 3.6. This is a good example of the need for multiplying by decimals.

Support activity

Interactive learning diagram: Dist-time-speed

Homework activity

Complete WB Worksheet 26, Questions 1 to 8.

Lesson 2 *Pupil's Book page 188;*
Workbook page 47



Preparation

You will need to have: Pupil's Book; Workbook.

➔ Starter activity

Discuss the fastest runners in the world. All the names on page 188 are African, or Afro-American (Michael Johnson). Other records held by Africans are:
Men: For every distance from 800 m to the marathon the world record is held by an African.
Women: 5 km, 15 km, one hour, 20 km to 30 km.

🔍 Lesson focus

For Exercise 3 (questions 3 and 4) pupils should use a calculator.

Pupils might like to find the speeds (in km/h) of different record races.

Distance (m)	100	200	400	800
Time (s)	9.58	19.19	43.18	100.91
Speed (km/h)	38	38	33	29

Distance (km)	1	1.5	2	3
Time (s)	132	206	285	440
Speed (km/h)	27	26	25	25

Distance (km)	5	10	20	40
Time (s)	757	1577	3312	7020
Speed (km/h)	24	23	22	21



Answers

Exercise 3

1. 21.285 km/h; 2. 18.517 km/h; 3. 9.26 m/s;
4. 7.58 m/s

Worksheet 26

9. Ann

Assessment

Pupils should be able to calculate speeds for races.
Assess their performance in Exercise 3.

Extension activity

Interactive learning diagram: Distance, time, speed

Support activity

Interactive learning diagram: Dist-time-speed

Homework activity

Complete WB Worksheet 26, Question 9.

Lesson 3 *Pupil's Book page 189***Preparation**

You will need to have: Pupil's Book.

**Lesson focus**

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

**Answers****Revision exercise page 189**

1. 50 km/h
2. 1.5 hours
3. a) 60 km b) 3 hours c) 20 km/h
4. 14.79 km/h
5. a) 6.6 m/s b) 6.34 m/s

Objectives

By the end of this unit, each pupil should be able to:

- Identify the basic properties of plane figures such as a rectangle and a square.

**Suggested resources**

A4 paper

**Frequently asked questions**

Q *What do pupils need to know about symmetry?*

A The two types of symmetry are a simple way to understand many of the properties of shapes. Rectangles have two perpendicular lines of symmetry and half-turn symmetry. This shows the properties of equal opposite sides and angles. Squares have four lines of symmetry, and quarter-turn symmetry (because they look the same after a quarter turn). This shows the properties of all equal sides and angles.

**Evaluation guide**

Pupils to:

- State properties of a square and a rectangle.

Lesson 1

*Pupil's Book page 190;
Workbook page 50*

**Preparation**

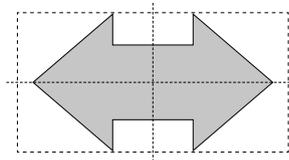
You will need to have: Pupil's Book; Workbook; paper; playing cards.

**Starter activity****Symmetry by folding and tearing paper**

Pupils fold a sheet of paper in half. The two halves will match. They can do this in two ways, thus showing that a rectangle has two lines of symmetry. Ask them to fold the paper along a diagonal, and see that the two halves do not match, so the diagonal is *not* a line of symmetry.

This kind of symmetry is like looking in a mirror, so it is often called 'mirror symmetry' and the lines are often called 'mirror lines'.

Finally fold the rectangle in both ways, so there are four thicknesses. Then tear a shape from the corners of the paper that do not include any fold lines. Open out the sheet to see the symmetrical pattern. The creases are *lines of symmetry*.

**Lesson focus****Half turn symmetry**

Each pupil takes their design and turns it upside down. They will see that it looks the same after a half turn. This means it also has half-turn symmetry. Rectangles have half-turn symmetry.

Sorting cards

Groups of pupils look at the designs on a pack of cards. Some clearly have half-turn symmetry, because they look the same upside down, but some do not, because the shapes, or the placement of the shapes, do not look the same upside down. Which cards have half-turn symmetry?

Sorting capital letters

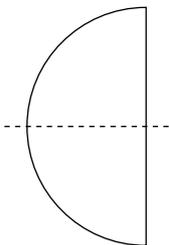
Some letters have reflection symmetry, such as **A**. Some letters have half turn symmetry, such as **Z**. Some letters have both, such as **X**; these also have two lines of symmetry.

Some letters have no symmetry, such as **F**. Groups of pupils decide which letters fit into each category.

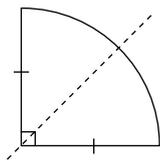
Answers

Worksheet 27

6.



1 line of symmetry



1 line of symmetry

Assessment

Pupils should be able to use symmetry to find properties of rectangles and the addition properties of squares.

Assess their performance in hands-on activities.

Extension activity

Interactive learning diagram: Rotating, Two reflections

Support activity

Interactive learning diagram: Reflection symmetry, Turn symmetry

Homework activity

Complete WB Worksheet 27, Question 6.

Lesson 2 *Pupil's Book page 190;*
Workbook page 48

Preparation

You will need to have: Pupil's Book; Workbook; paper.

Starter activity

Symmetry of squares

Show pupils how to make a square from a rectangle.

Fold one corner of a rectangular piece of paper over to meet the longer side. This will make the two sides equal. Cut off the extra paper.

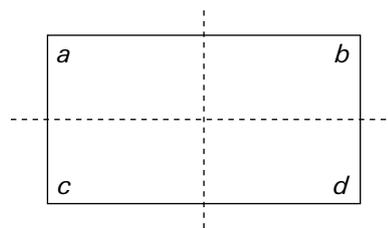
How many lines of symmetry does a square have? (There are four; this time the diagonals are also mirror lines.)

After what fraction of a turn does the square 'look the same'? (A quarter turn or 90° ?)

Lesson focus

Properties of rectangles

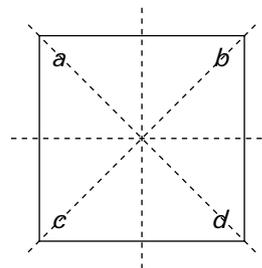
Groups of pupils use sheets of A4 paper and discuss the symmetry of rectangles. To help them to recognise the facts below, ask "Because of the symmetry, which sides and which angles of the rectangle are equal?"



- Opposite sides are equal (due to reflection symmetry and also half-turn symmetry).
- Adjacent angles are equal (due to reflection symmetry). $a = b$, $c = d$, $a = d$, $b = c$
- Opposite angles are equal (due to half turn symmetry). $a = c$, $b = d$

Properties of squares

Groups of pupils use sheets of A4 paper to make squares and discuss their symmetry. To help them to recognise the facts below, ask "Because of the symmetry, which sides and which angles of the square are equal?"



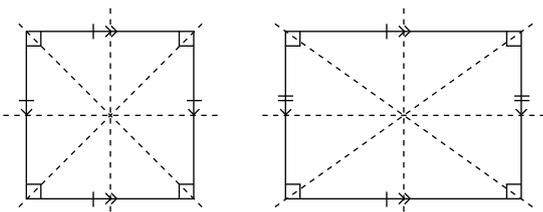
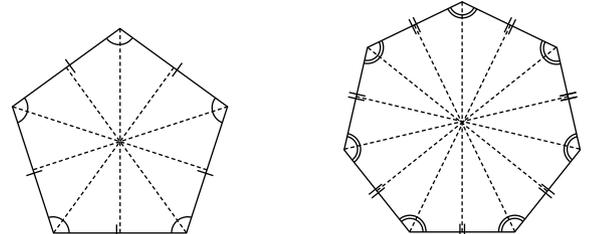
- All sides are equal (due to quarter-turn symmetry)
- All angles are equal (due to quarter turn symmetry). $a = b = c = d$

Answers

Exercise 1

1. 2 2. 2 3. 4 4. 2
5. They are equal. 6. yes
7, 8 own answers 9. 90°
10. yes 11. yes for both questions

Worksheet 27

1. 
3 lines of symmetry 2 lines of symmetry
5. 
Length of sides = 20 mm Length of sides = 18 mm
Angles = 72° Angles = 51°

Assessment

Pupils should be able to use symmetry to find properties of rectangles and the addition properties of squares.

Assess their performance in hands-on activities and Exercise 1.

Extension activity

Interactive learning diagram: Make a quadrilateral

Support activity

Interactive learning diagram: Squares

Homework activity

Complete WB Worksheet 27, Question 1.

Lesson 3 *Pupil's Book page 192*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 192

- Four lines of symmetry, has half turn symmetry and quarter turn symmetry
 - Has only one line of symmetry, down the middle.
 - Two lines of symmetry and has half turn symmetry.
- rectangle
- square

Unit 28 Angles

Objectives

By the end of this unit, each pupil should be able to:

- Measure angles in a plane
- Describe an angle as an amount of turning
- Measure angles in degrees
- Draw angles using a protractor.



Suggested resources

rulers; cardboard clock with movable hands; demonstration protractor; protractors – pupils should have their own



Frequently asked questions

Q *What should pupils understand?*

A Many pupils get confused about the difference between what an angle is (two arms meeting at a vertex) and what it measures (the amount of turning from one arm to the other). Be careful in your language, particularly at this stage.



Evaluation guide

Pupils to:

1. Measure given angles from a diagram.
2. Draw various plane shapes and measure resulting angles to the nearest degree.
3. Identify the edges of vertices and faces of three dimensional shapes.

Lesson 1 *Pupil's Book page 193*



Preparation

You will need to have: Pupil's Book; cardboard clock with movable hands.



Starter activity

Talk about things that turn. For example a wheel turns; it has a centre (or vertex) and the arms of the angle are the spokes in the wheel. Have the pupils stand and go through each of the activities in Exercise 1 with them.



Lesson focus

Use a large paper/cardboard clock – preferably with a moveable minute hand, to introduce the idea of relating number of minutes to fractions of a turn of the minute hand.

Allow pupils to work in groups to complete Exercise 2.



Answers

Exercise 2

1. $\frac{3}{4}$ of a whole turn. This is 270°
- 2.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{10}$	$1\frac{1}{2}$
Minutes	30	15	45	20	40	10	50	6	90
Degrees	180	90	270	120	240	60	300	36	540

Assessment

Pupils should be able to describe what an angle measures.

Assess their performance in activities and Exercise 2.

Extension activity

Interactive learning diagram: Angles

Support activity

Interactive learning diagram: Estimating angles

Lesson 2 *Pupil's Book page 195*

Preparation

You will need to have: Pupil's Book.

Starter activity

Pupils can turn the cover of their book to show the different sizes of angles: acute, right, obtuse, straight and reflex.

Lesson focus

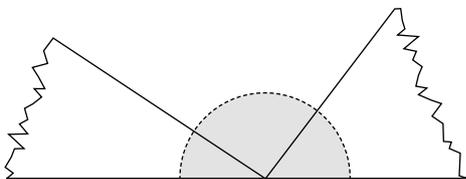
Drawing triangles

Pupils should draw these triangles, or where it is not possible, they should say so.

- triangle with three acute angles
- triangle with only two acute angles
- triangle with one right-angle
- triangle with two right angles
- triangle with one obtuse angle
- triangle with two obtuse angles
- triangle with one reflex angle.

Sum of the angles in any triangle

Each pupil should draw a triangle. As far as possible they should all be different.



Pupils should mark the angles of their triangle with dots. Each pupil tears off two of the angles of their triangle, and puts them next to the third angle. They will all find that they make a straight angle of 180° . The angle sum for any triangle is 180° . This will help them explain why some triangles (d, f, g) above are not possible.

Answers

Exercise 3

- Purple: AVN and VNA
Red: OHT and HTO
 - Purple: VAN
Red: TOH
- Answers will vary.

Assessment

Pupils should be able to draw any of the five types of angles.

Assess their performance in Exercise 3 and hands-on activities.

Extension activity

Interactive learning diagram: Target

Support activity

Interactive learning diagram: Types of triangles

Lesson 3 *Pupil's Book page 196; Workbook page 51*

Preparation

You will need to have: Pupil's Book; Workbook; protractor; ruler.

Starter activity

Show pupils how to use a protractor. The best method is this:

Firstly, decide whether the angle is acute or obtuse, and estimate the number of degrees.

Secondly, put the zero line on top of one arm of the angle, then count around the edge until you reach the other arm. Read the correct number, either less than 90° or over 90° . See the diagram on page 197.

If you are measuring, you have the answer. If you are drawing, put a small mark there and draw the second arm of the angle.

Lesson focus

Pupils use their ruler and protractor to answer Exercise 4. The angles in the shapes they draw must be large enough for them to use the protractor successfully. They should be at least as long as the radius of the protractor's semicircle. If the shape is not large enough – as in the revision exercise – they may lightly draw the extensions of the sides until they are long enough.

This will reinforce a basic idea from lesson 2 that the angle sum of a triangle is 180° . However you should expect some measurement errors.

Angle sum for other shapes

Exercise 4 asks about the angle sum of triangles and pentagons (five-sided shapes). But it does not mention 4 or 6-sided shapes. Pupils should draw these and make a table of the results.

Number of sides	3	4	5	6
Angle sum ($^{\circ}$)	180	360	540	720

They might be able to describe the pattern. Some might even be able to predict the angle sum of a decagon (10 sides).



Answers

Exercise 4

Check this in class, answers will vary.

Worksheet 28

1. $a = 90^{\circ}$ $b = 120^{\circ}$ $c = 180^{\circ}$
 $d = 83^{\circ}$ $e = 30^{\circ}$ $f = 90^{\circ}$
 $g = 45^{\circ}$ $h = 95^{\circ}$ $i = 147^{\circ}$

Assessment

Pupils should be able to draw and measure angles.

Assess their performance in Exercise 4 and activities.

Extension activity

Interactive learning diagram: Star polygons

Support activity

Interactive learning diagram: Regular polygons

Homework activity

Complete WB Worksheet 28, Question 1.

Lesson 4 *Pupil's Book page 201*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 198

Answers will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Measure heights of human beings, buildings, trees and distance.

**Suggested resources**

number slide; measuring tapes; chalk; stones; rulers

**Frequently asked questions**

Q *What do pupils need to understand?*

A Estimation, unit conversion and indirect measurement (with isosceles triangles and ratios) are used in this unit.

Q *What mathematical skills are needed?*

A Pupils should improve their estimation, and measurement skills.

**Evaluation guide**

Pupils to:

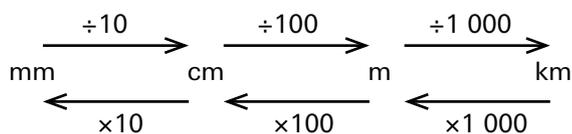
1. Measure and obtain the dimensions of classroom, house, play grounds, halls etc. as a project.

Lesson 1 *Pupil's Book page 199***Preparation**

You will need to have: Pupil's Book; number slide; chalk; stones; measuring tape.

**Starter activity**

Discuss the units of length. Remind them of the number slide for multiplication or division by 10, 100, 1000. Discuss conversion of the units.

**Lesson focus****Estimate and measure**

Each group draws a chalk line on the ground. They then toss a stone some distance from the line, say 2 to 4 metres away.

Each person estimates the distance of the stone from the line – for example 3.45 m.

Two pupils in the group measure the distance and tell the others. Each person works out the size of their error – the difference between their estimate and the real distance.

After doing this several times, the person with the lowest total error is the winner. Pupils should find that as their estimates get better so their errors get smaller.

Groups work on Exercise 1. The teaching note on the bottom of page 199 has more ideas if you have time.

**Answers****Exercise 1**

Class activities.

Assessment

Pupils should be able to estimate distances and convert length units.

Assess their performance in hands-on activities and Exercise 1.

Extension activity

Interactive learning diagram: Length conversions, Metric conversions

Support activity

Interactive learning diagram: Length conversions

Lesson 2 *Pupil's Book page 200;*
Workbook page 53



Preparation

You will need to have: Pupil's Book; Workbook; measuring tapes; rulers.



Starter activity

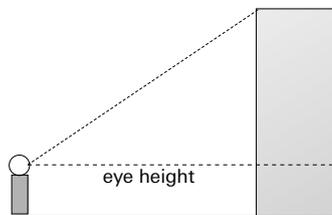
Go over the way each of the two activities (Exercises 2 and 3) work. Exercise 2 involves isosceles triangles of different sizes. Exercise 3 involves a ratio.



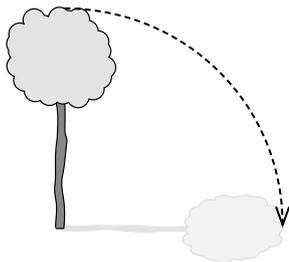
Lesson focus

This lesson involves practical work with tapes and rulers.

For Exercise 3, it is not necessary to lie on your stomach. Instead you need to use the method described to measure from your eye height to the top of the building. At the end, add your eye height.



Pupils can try an even simpler way to estimate the height of something. Imagine it 'falling over'. You can do this by 'sighting' the object with your hand, and then turn your hand through 90° . Then estimate, or even measure, from its base to where you think the top would be when it fell.



Answers

Exercise 2 and 3

Class activities, answers will vary.

Worksheet 29

Answers will vary.

Assessment

Pupils should be able to measure height and distances using a variety of methods.

Assess their performance in hands-on activities and Exercises 2 and 3.

Extension activity

Interactive learning diagram: Bike distances

Support activity

Interactive learning diagram: River width

Homework activity

Complete WB Worksheet 29.

Lesson 3 *Pupil's Book page 201*



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



Answers

Revision exercise page 201

Answers will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Solve more problems on two-dimensional and three dimensional shapes
- Identify vertices, faces and edges of three dimensional shapes
- Solve problems on the volumes of cylinders.



Suggested resources

A4 paper; tangram puzzle sheets; three-piece puzzles; skewers (or match-sticks or toothpicks) and clay (or equivalent); sheet of triangles and squares; scissors and sticky tape; cans and other cylinders



Frequently asked questions

Q *This unit includes both 2D and 3D shapes. What is the D?*

A The D stands for dimension. A line has only one dimension, but a square or any flat shape has two dimensions, often called width and length. The third dimension is height and is required for solid objects. Of course the flat faces of 3D shapes (polyhedra) can be thought of as 2D and the edges of a solid shape are just 1D.



Evaluation guide

Pupils to complete a chart to indicate:

1. Number of lines of symmetry.
2. Number of sides.
3. Number of angles contained in given shape.

Lesson 1

Pupil's Book page 202;

Workbook page 49



Preparation

You will need to have: Pupil's Book; Workbook; A4 paper.



Starter activity

Draw examples of different types of triangles. Discuss their symmetry.



Lesson focus

Two-way classification of triangles

Pupils try to fill all the cells in this table. They explain why it cannot be done.

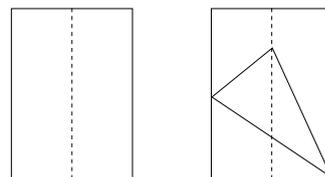
	scalene	isosceles	equilateral
acute-angled			
right-angled			
obtuse-angled			

(Equilateral triangles have only 60° angles. They are acute-angled, but cannot be right-angled or obtuse-angled.)

Fold equilateral triangle from A4

Use these steps to fold a rectangle of paper into an equilateral triangle.

- Fold the sheet in half vertically. Crease the paper and open it out. This will be a line of symmetry of the triangle. The shorter side will be the base of the triangle.
- Now fold one base corner over so that it touches the crease, and the new crease also goes through the other base corner. (The new crease will also be a line of symmetry.) The top point will be the top of the triangle.



Use a ruler to draw the sides of the equilateral triangle to the bottom corners of the sheet. Cut it out.

Pupils can now fold the lines of symmetry, and check that all the sides and angles are equal.

Answers

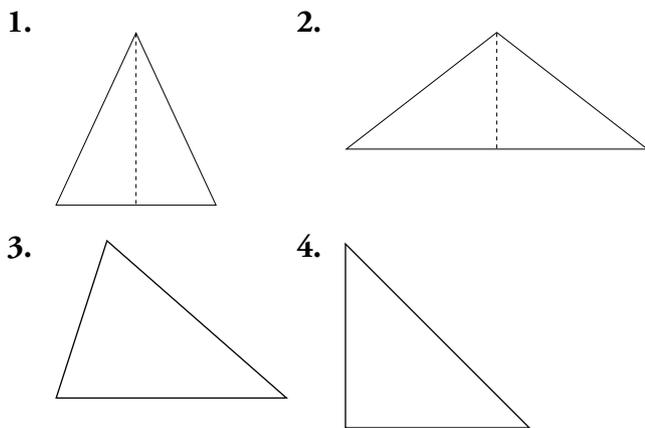
Puzzle 1 page 203

This is a trick question. You cannot have one side equal, because what is it equal to?

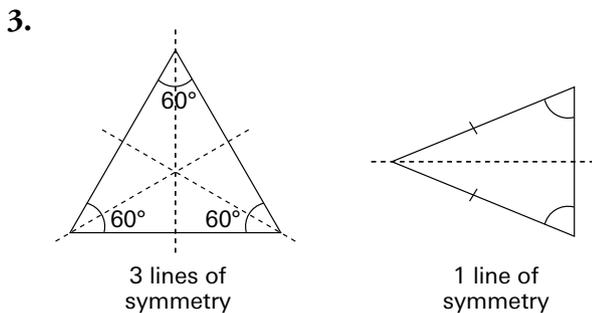
Puzzle 2 page 203

An equilateral triangle is dependent on the angle size as that depicts the length of the sides; therefore there is only one type of equilateral triangle.

Exercise 1



Worksheet 27



Assessment

Pupils should be able to sort triangles by number of equal sides or the size of their largest angle.

Assess their performance in hands-on activities and Exercise 1.

Extension activity

Interactive learning diagram: Sorting triangles

Support activity

Interactive learning diagram: Types of triangles

Homework activity

Complete WB Worksheet 27, Question 3.

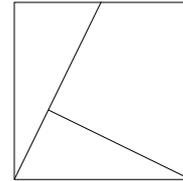
Lesson 2 *Pupil's Book page 203; Workbook page 48*

Preparation

You will need to have: Pupil's Book; Workbook; three-piece puzzle sheet.

Starter activity

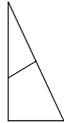
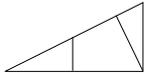
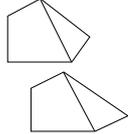
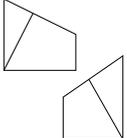
Pupils cut out the three shapes from the Three-piece puzzle sheet.

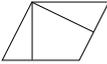
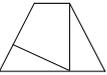
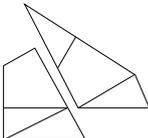


Lesson focus

Three-piece puzzle

Use the number of shapes stated to make the shape named. Then draw it in the space on the sheet. This activity will require pupils to think about each shape and its properties, both when solving the puzzles and when drawing the solutions. Here are the solutions.

2 pieces – triangle 	3 pieces – triangle 	2 pieces – trapezium 
2 – pentagon 	3 – quadrilateral 	2 – trapezium 

3 – square 	3 – rectangle 	3 – parallelogram 
3 – trapezium 	3 – pentagon 	3 – quadrilateral 



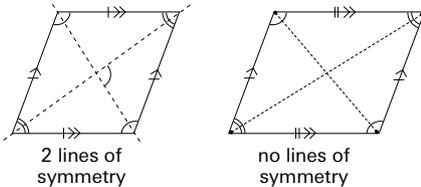
Answers

Exercise 2.

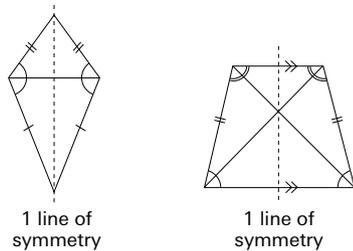
- A, B, C, D and F have lines of symmetry
A = 4 lines of symmetry
B = 4 lines of symmetry
C = 2 lines of symmetry
- A, B, C and D have turn symmetry
A = half and quarter-turn symmetry
B = half-turn symmetry
C = half-turn symmetry
D = half-turn symmetry
- A, C, E and H have right angles
- acute and/or obtuse
- B: yes; D: yes; E: yes; F: yes; G: possibly; H: yes
- It is not possible to have 4 acute angles as this would add up to be less than 360° .
- It is not possible to have 4 obtuse angles as this would add up to be more than 360° .

Worksheet 27

2.



4.



Assessment

Pupils should be able to identify types of quadrilaterals.

Assess their performance in puzzles and Exercise 2.

Extension activity

Interactive learning diagram: Hunting quadrilaterals

Support activity

Interactive learning diagram: Quad diagonals

Homework activity

Complete WB Worksheet 27, Questions 2 and 4.

Lesson 3

*Pupil's Book page 204;
Workbook page 55*



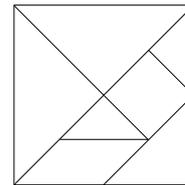
Preparation

You will need to have: Pupil's Book; Workbook; tangram square.



Starter activity

Pupils cut up the tangram square.



Lesson focus

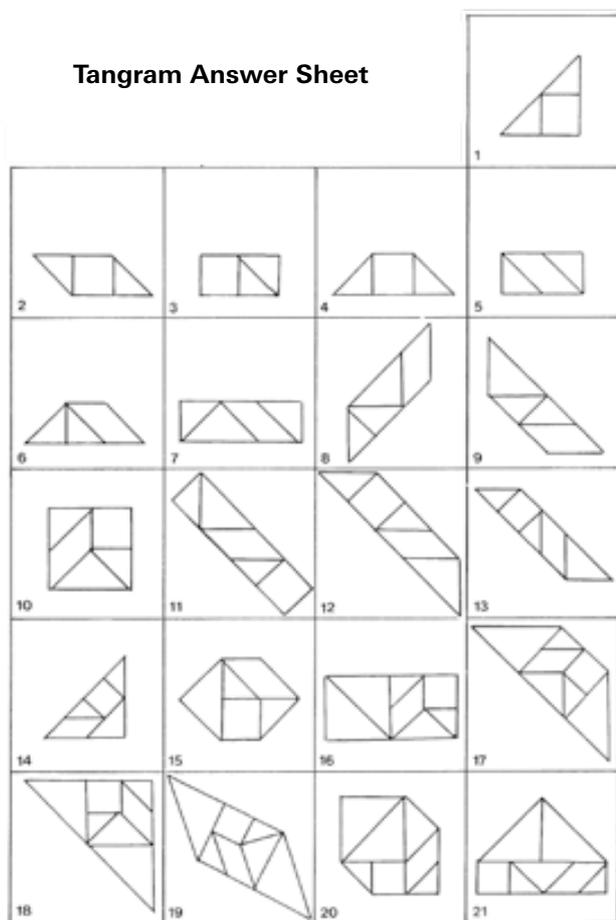
Tangrams

There are seven shapes within the tangram set. They are cut from the square. We have labelled them A to G. Use the shapes with the letters to make the shape named. Then draw it in the space on the sheet.

This activity will require pupils to think about the shape and its properties, both when solving the puzzles and when drawing the solutions.

Here are the solutions.

Tangram Answer Sheet



Answers

Exercise 3

1. Not necessarily, but it can.
2. Not necessarily, but they can.
3. Not necessarily, but they can.
4. no
5. yes, 2 lines of symmetry
7. no; 8. yes; 9. yes

Worksheet 30

1. Equilateral triangle: all angles 60° , all sides equal, 3 lines of symmetry, no parallel lines
 Square: all right angles, all sides equal, 4 lines of symmetry, 2 pairs parallel sides
 Regular pentagon: all angles 108° , all sides equal, 5 lines of symmetry, no parallel sides
 Regular hexagon: all angles 120° , all sides equal, 6 lines of symmetry, 3 pairs parallel sides
 Regular heptagon: all angles 128° , all sides equal, 7 lines of symmetry, no parallel sides

Regular octagon: all angles 135° , all sides equal, 8 lines of symmetry, 4 pairs parallel sides

Assessment

Pupils should be able to describe the properties of quadrilaterals.

Assess their performance in hands-on activities and Exercise 3.

Extension activity

Interactive learning diagram: Hunting quadrilaterals

Support activity

Interactive learning diagram: Properties of quadrilaterals

Homework activity

Complete WB Worksheet 30, Question 1.

Lesson 4 *Pupil's Book page 206;
Workbook page 51*

Preparation

You will need to have: Pupil's Book; Workbook; skewers (or match-sticks or toothpicks) and clay (or equivalent).

Starter activity

Name different types of three dimensional objects around the school.

Lesson focus

If possible engage pupils with making 'skeleton' models of 3D shapes using skewers (or matchsticks or toothpicks) and clay (or equivalent).



Answers

Exercise 4

- a) cylinder b) cuboid c) cube
d) tetrahedron e) hexagonal prism
f) pyramid g) triangular prism
h) pentagonal prism

Worksheet 28

2. 12 edges, 8 vertices, 5 faces and 18 pairs of parallel sides.
3. 8 edges, 5 vertices, 5 faces and 2 pairs of parallel sides.
4. Octahedron: 12 edges, 6 vertices, 8 faces and 2 pairs of parallel sides.
Tetrahedron: 6 edges, 4 vertices, 4 faces and 0 pairs of parallel sides.

Assessment

Pupils should be able to identify 3D shapes in the environment, naming cuboids, prisms, pyramids and spheres.

Assess their performance in Exercise 4 and activities.

Extension activity

Interactive learning diagram: Cube nets

Support activity

Interactive learning diagram: Boxes

Homework activity

Complete WB Worksheet 28.

Lesson 5 *Pupil's Book page 207*

Preparation

You will need to have: Pupil's Book; triangular prism net; sheet of triangles and squares; scissors; sticky tape.

Starter activity

Use the net on page 208 to make a triangular prism.

Lesson focus

Give pupils the sheet of triangles and squares so they may make nets (with scissors and sticky tape) to check the answers to Exercise 5.

Answers

Exercise 5

Class activity.

Assessment

Pupils should be able to visualise the net of a triangular prism.

Assess their performance in Exercise 5.

Extension activity

Interactive learning diagram: Plans and elevations

Support activity

Interactive learning diagram: Prisms

Homework activity

Complete Exercise 6.

Lesson 6 *Pupil's Book page 208;* *Workbook page 57*

Preparation

You will need to have: Pupil's Book; Workbook; cardboard model of tetrahedron; four paper equilateral triangles; sheet of triangles and squares; scissors; sticky tape; tangram square.

Starter activity

Provide a large cardboard model of a tetrahedron, so pupils can know what it looks like. Give pupils four equilateral triangles, and ask pupils to use them to make a net that folds to a tetrahedron – a triangular pyramid made from four equilateral triangles.

Lesson focus

Give pupils the sheet of triangles and squares so they may make nets (with scissors and sticky tape) to check the answers to Exercise 7.

Answers

Exercise 7

Nets 1 and 3 will make square pyramids.

Worksheet 30

Practical activities.

Assessment

Pupils should be able to visualise the nets of a square pyramid and a tetrahedron.

Assess their performance in hands-on activities and Exercise 7.

Extension activity

Interactive learning diagram: Pyramid volume

Support activity

Interactive learning diagram: Pyramid

Homework activity

Complete WB Worksheet 30, Questions 2, 3 and 4.

Lesson 7 *Pupil's Book page 209*

Preparation

You will need to have: Pupil's Book; cylindrical can; paper; bottle; water.

Starter activity

Obtain a cylindrical can. Put a rectangle of paper around it as the label on the curved surface. In front of the class, carefully remove the label, showing that the net of a cylinder is a rectangle and two circles (top and bottom).

Lesson focus

This lesson is a revision of the volume of a cylinder. The instructions in Exercise 8 should be followed for several different cylinders. Groups of pupils may use cans and other cylinders.

Note that the base and top areas are found by counting squares, but the area of the rectangle that forms the curved surface may be found by multiplying the distance around the top by the height of the can.

Answers

Exercise 8

Answers will vary, to be done in class.

Assessment

Pupils should be able to identify cylinders and describe how to calculate their volumes.

Assess their performance in Exercise 8.

Extension activity

The bottom part of a bottle is a cylinder. Fill the bottle with water using only the cylinder part. Measure the height. In this way calculate the area of the base of the cylinder.

Support activity

Interactive learning diagram: Cylinder

Lesson 8 *Pupil's Book page 211*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

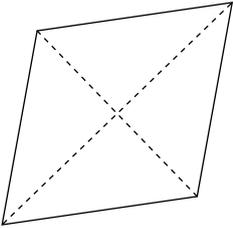


Answers

Revision exercise page 211

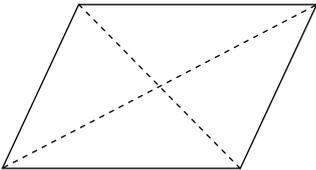
1 to 2. Answers will vary, practical activity.

3.



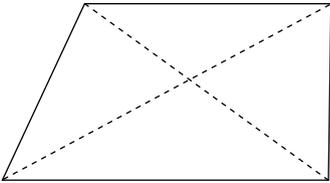
- a) The rhombus is symmetrical across both of its diagonals.
- b) There are two sets of parallel sides. All four sides are the same length.
- c) There are two angles of 60° and two angles of 120° (because they are made up from two 60° angles from the triangle).
- d) The diagonals bisect each other at a right angle.

4.



- a) The parallelogram is symmetrical across both of its diagonals
- b) There are two sets of parallel sides of the same length
- c) Two of the angles are 120° and the other two are 60°
- d) The two diagonals bisect each other at 90°

5.



- a) The trapezium does not have any lines of symmetry
- b) no

Objectives

By the end of this unit, each pupil should be able to:

- Draw plans according to a given scale
- Apply and use scale drawing in converting lengths and distances of objects in his/her environment to any scale.

**Suggested resources**

measuring tapes; rulers; plain paper; photos; house plans; maps

**Common errors that pupils make**

Pupils choose a scale that is too complex and difficult to use.

Keep the scale to powers of ten or times 1, 2 and 5. Even professional architects use simple scales.

**Evaluation guide**

Pupils to:

1. Convert given lengths and distances to a given scale.
2. Draw to scale school farms, play grounds etc. with their dimensions given.
3. Convert scale plans to real dimensions.

Lesson 1

*Pupil's Book page 212;
Workbook page 58*

**Preparation**

You will need to have: Pupil's Book; Workbook; measuring tapes; A4 paper.

**Starter activity**

Measure the length and width of the classroom.

**Lesson focus**

Make a sketch plan first.

Draw a good plan of the classroom.

Start by drawing a plan of the walls to scale. You need to take the measurements, and then choose a scale so that the plan will fit onto your A4 paper.

You should choose from one of these scales:

1 m = 1 cm, 1 m = 2 cm, 1 m = 5 cm,
1 m = 10 cm.

For example, if the room is 6 m on its longer side, and the paper is 18 cm wide, you need these lengths for each suggested scale:

1 m = 1 cm 2 cm 5 cm 10 cm
for 6 m 6 cm 12 cm 30 cm 60 cm

This means that we will use 2 cm = 1 m, and 6 m will be a line 12 cm long. (We double the number and change the units to cm.)

Fill in details through measuring.

**Answers****Exercise 1**

This is a practical activity.

Worksheet 31

2. Practical activity, answers will vary.

Assessment

Pupils should be able to make a good sketch of a room.

Assess their performance in the drawing they produce.

Extension activity

Using the drawing they have made, these pupils could predict a length they have not previously measured, and check if it is correct.

Support activity

Choose a simple rectangular shape to draw, and use a simple scale (1 cm = 10 cm, or 1 cm = 1 m).

Homework activity

Complete WB Worksheet 31, Question 2.

Lesson 2 *Pupil's Book page 213*

Preparation

You will need to have: Pupil's Book.

Starter activity

In this unit we will be estimating scale factors and checking by dividing. Follow the example on page 213.

Lesson focus

Work on Exercise 3.

Answers

Exercise 3

Answers will vary.

Assessment

Pupils should be able to estimate a scale factor. Assess their performance in the homework and Exercise 3.

Extension activity

Interactive learning diagram: Size change

Lesson 3 *Pupil's Book page 214*

Preparation

You will need to have: Pupil's Book; pictures/photos of people.

Starter activity

Show the class a photograph of a person from a newspaper. Assume that the person is 165 cm tall. Measure the height of the person in the photograph. Then find the scale by dividing the photo height by 165.

For example the photo is 12 cm high.

The scale is 12 cm = 165 cm. Divide by 12, to get 1 cm = 13.75 cm. Every centimetre on the photo shows 13.75 cm on the person.

Lesson focus

Use the example on page 214 to show how to do this with drawings that are 'to scale'.

Groups of pupils find the scales for photos of people, cars etc.

Assessment

Pupils should be able to use a known distance to find the scale on a scale drawing.

Assess their performance in activities.

Extension activity

Find house plans or aerial photos of locations from magazines or the internet. Estimate the scale factors.

Support activity

Pupils can find photos of people, and estimate the scale of the photo (as reductions) by dividing. For example, if a 150 cm tall person is reduced to 3 cm in a photo the reduction is $\frac{1}{50}$ (or 0.02).

Lesson 4 *Pupil's Book page 215;* *Workbook page 57*

Preparation

You will need to have: Pupil's Book; Workbook.

Starter activity

Help pupils to find the scale for the map using the scale line in the bottom right corner.

Measure the length of the line: it is 16 mm.

So 16 mm shows 200 km. Using proportion we can find other scales:

mm	16	8	4	2	1	10
km	200	100	50	25	12.5	125

So 10 mm (1 cm) shows 125 cm.

Lesson focus

‘Edge of paper’ method

Show pupils how to use the ‘edge of paper’ method to use a map scale.

You use the edge of a sheet of paper to make copies of the small scale in the bottom right corner.

It looks like this:



You continue it along the edge of the paper as far as you wish.

You may now put this edge of paper on the map between two places and read off the distance in km.

‘Use a known distance’ method

Question 1 in Exercise 2 uses the method introduced in the last lesson.

Now you have three methods to find a distance on a map. Use all three methods for question 2, and compare the answers.

Answers

Exercise 2

Answers will vary.

Worksheet 31

3. and 4. Answers will vary.

Assessment

Pupils should be able to use a scale on a map to find distances from the map.

Assess their performance in Exercise 2.

Extension activity

Interactive learning diagram: Planets

Support activity

Interactive learning diagram: River width

Homework activity

Complete WB Worksheet 31, Question 1.

Lesson 5 *Pupil's Book page 217*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Answers

Revision exercise page 217

Answers will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Interpret pictograms and bar graphs

- Use pictograms and bar graphs in representing population of people or data.



Suggested resources

pupils' data



Frequently asked questions

Q *What are the mathematical understandings?*

A Pictograms, bar and column graphs represent frequencies – the number of cases of a particular result. A pictogram may involve rounded numbers.



Evaluation guide

Pupils to:

1. Prepare a given population data on pictogram or bar graph.

Lesson 1

*Pupil's Book page 218;
Workbook page 61*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Collect data from each pupil: their age, their birth month, their height to the nearest centimetre



Lesson focus

Pictogram of ages

The first step is to choose a suitable icon. The second is to choose a suitable ratio. Presuming you have a large class, you will want to have at least 2 to 10 people shown on each icon. (This depends on the number at each age.)

Immediately the difficulty with pictograms becomes clear; there may not be a suitable common factor of all the numbers, so you may have to round some of the numbers to the nearest (such as the nearest 5.).



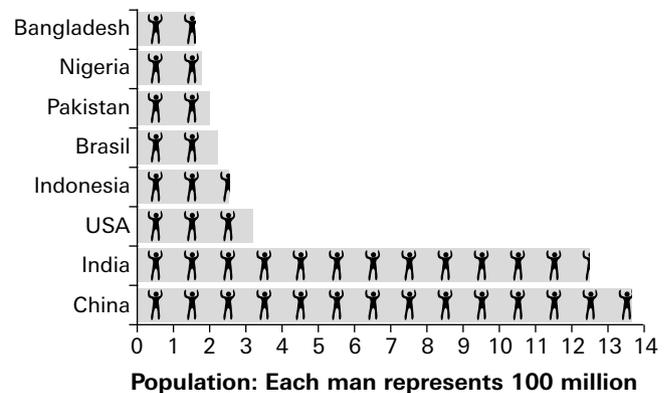
Answers

Exercise 1

Class activity.

Worksheet 32

2.



Assessment

Pupils should be able to draw a pictogram for suitable data, or interpret a pictogram.

Assess their performance in Exercise 1 and class work.

Extension activity

Pupils can graph these results for the Nigerian football team in 2014:

Nigeria beat Ethiopia 2–1

lost to Mali 2–1

beat Mozambique 4–2

beat South Africa 3–1

beat Morocco 4–3

drew with Ghana 0–0

beat Zimbabwe 1–0

drew with Mexico 0–0

drew with Scotland 0–0

drew with Greece 0–0

lost to United States 2–1

drew with Iran 0–0

beat Bosnia/Herz'na 1–0

lost to Argentina 2–3

Support activity

Pupils can toss a coin 100 times, and construct a pictogram to show the numbers of head and tails.

Homework activity

Complete WB Worksheet 32, Question 2.

Lesson 2 *Pupil's Book page 219; Workbook page 60*



Preparation

You will need to have: Pupil's Book; Workbook.



Starter activity

Use the data you gathered from your pupils in Lesson 1. Make a table from the data of birth months, showing the numbers who were born in each month.

Show them how to draw a bar graph of the birth month data.



Lesson focus

Put a list of the heights of pupils on the board. Pupils create their own table and draw a column graph of heights.



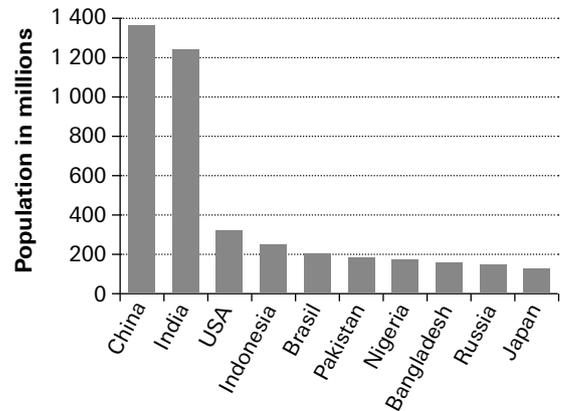
Answers

Exercise 2

Class activities.

Worksheet 32

1.



3. Lagos: 8 000 000
Kano: 3 250 000
Ibadan: 3 100 000
Kaduna: 1 500 000
Port Harcourt: 1 100 000

Assessment

Pupils should be able to draw a bar or column for suitable data, or interpret a bar or column graph.

Assess their performance in: Exercise 2 and class work.

Extension activity

Interactive learning diagram: National areas

Support activity

Interactive learning diagram: Column graph

Homework activity

Complete WB Worksheet 32, Questions 1 and 3.



Preparation

You will need to have: Pupil's Book.



Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.



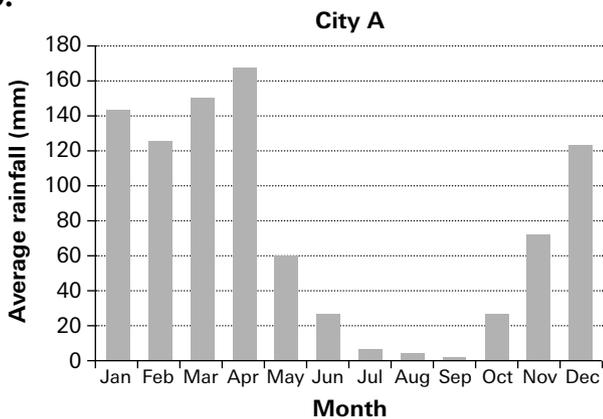
Answers

Revision exercise page 221

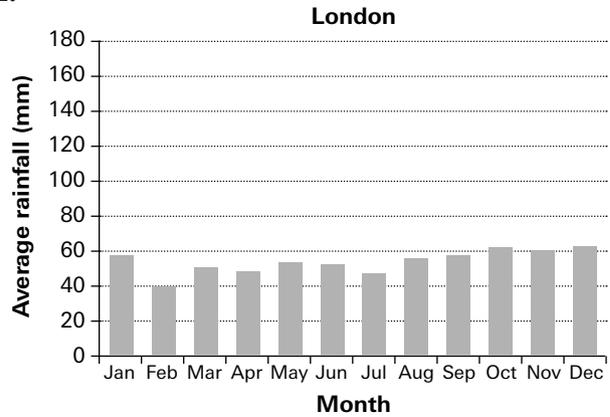
- a) 140. D; b) 360 pots. C.
-

Day	Number of fish caught
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Scale:	= 20 fish

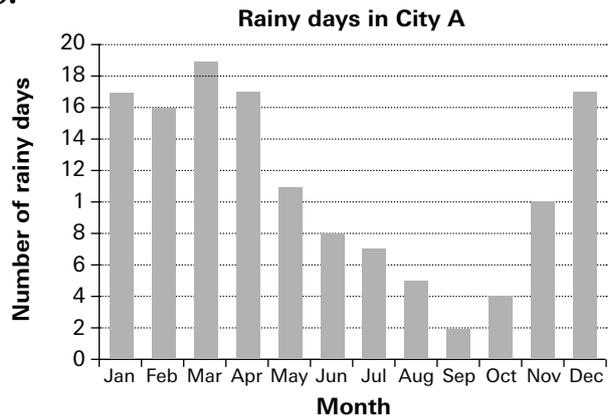
3.



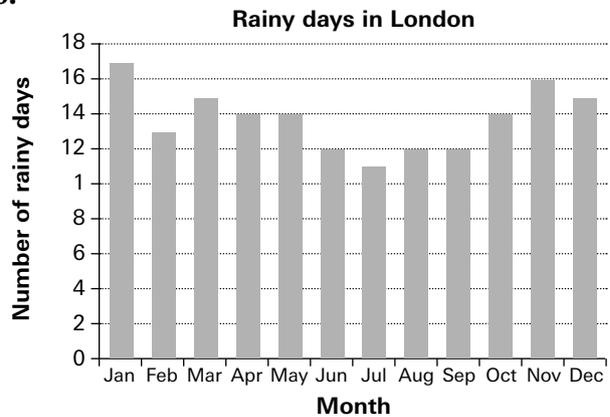
4.



5.



6.



7. Answers will vary.

Objectives

By the end of this unit, each pupil should be able to:

- Find the mode of data

- Calculate the mean of given data.

**Suggested resources**

playing cards; age data from Unit 32; frequency data from Unit 32

**Common errors that pupils make**

Pupils will be surprised when there is no mode – when no one number is more common than any others.

**Evaluation guide**

Pupils to:

1. Organise a given data and find the mode.
2. Organise a given data and calculate the mean.

Lesson 1

*Pupil's Book page 223;
Workbook page 62*

**Preparation**

You will need to have: Pupil's Book; Workbook; age data from Unit 32; frequency data from unit 32.

**Starter activity**

Use the age data collected in the previous unit. Show the ages in a table. It will be clear which age(s) is/are the most common.

**Lesson focus**

In the last unit we used the frequency table of birth month and height (to the nearest cm). Using these, pupils determine the most common birth month and height.

**Answers****Exercises 1****Class activities****Worksheet 33**

1. a) 90; 93; 93; 94; 94; 94; 95; 95; 95; 95; 95; 95; 96; 96; 96; 98; 99; 99; 100; 100; 101; 101; 101; 101; 102; 102; 102; 103; 107; 107
b) 95
2. Stephanie

Assessment

Pupils should be able to collect data and find the mode from a frequency table.

Assess their performance in Exercise 1 and class work.

Homework activity

Complete WB Worksheet 33, Questions 1 and 2.

Lesson 2

*Pupil's Book page 223;
Workbook page 63*

**Preparation**

You will need to have: Pupil's Book; Workbook; playing cards.

Starter activity

Silent sharing

Each group needs a pack of cards.

Give five people these numbers of cards: 6, 3, 16, 10, 5. Put the rest aside.

These people should then share them silently until all have the same number. There should be no talking, asking or taking, only giving.

When everyone has the same number (8) this will be the mean (or average).

Average (mean) means having equal shares.

Lesson focus

Determine the mean of age, birth month and height (nearest cm).

Card game – Mean and mode

Use the numbers 0 to 10, and let the picture cards all show 0. Shuffle the cards.

Deal each player five cards. (We use five so the averages will be simple terminating decimals.)

Each player finds the *mean* of their own cards.

The biggest mean gets one point.

Each player finds the *mode* of their cards if there is one. There may be none or many. The biggest mode gets one point.

Then deal out another five cards to all players and repeat.

Answers

Exercise 2

Class activities.

Worksheet 33

3. mode = 10, mean = $9\frac{1}{3}$

It would suggest that size 9 and 10 are the best sellers.

4. mean = $9\frac{1}{3}$, mode = 10

5. 3 373 740

6. own data

7. The mean is calculated as the mathematical average of values. Therefore it can't be applied to names but can be used for a number of items like the nuts.

Assessment

Pupils should be able to find and interpret the mean of data.

Assess their performance in Exercise 2 and class work.

Extension activity

Interactive learning diagram: Estimating means 0–100

Support activity

Interactive learning diagram: Estimating averages 0–10

Homework activity

Complete WB Worksheet 33, Questions 3–7.

Lesson 3 *Pupil's Book page 226*

Preparation

You will need to have: Pupil's Book.

Lesson focus

This lesson is dedicated to maximising each pupil's chances of success in the unit assessment. It could form the assessment itself, by observing whether or not each pupil has met the objectives.

Revision exercise page 226

- | | |
|-------------------|---------------|
| 1. a) 25 children | b) 151 people |
| c) 6 | d) 6 |
| 2. a) 17 | b) 14 |
| 3. a) 11 days | b) 13.75 days |
| 4. a) March | b) January |
| 5. a) September | b) July |
| 6. a) 75.92 | b) 54.33 |

Pupil's Book page 227

Objectives

This project requires pupils to work with perimeter and area, and applications of these concepts in the world. They also make a neat scale drawing of a room.

**Guidelines**

You will need to have the following resources:
Pupil's Book, scissors, a big piece of paper or card and glue.

Pupils need to follow the steps, as mentioned on page 227 of the PB.

For the first part of this project pupils need to convert all the measurements of the rough drawing to millimetres.

$$60 \text{ cm} = 600 \text{ mm}$$

$$50 \text{ cm} = 500 \text{ mm}$$

$$50 \text{ cm}^2 = 25\,000 \text{ mm}^2$$

$$150 \text{ cm} = 1\,500 \text{ mm}$$

$$1.9 \text{ cm} = 19 \text{ mm}$$

$$95 \text{ cm} = 950 \text{ mm}$$

$$1.2 \text{ m} = 1\,200 \text{ mm}$$

For the next part of this investigation, pupils must choose a suitable scale and make rough drawings of all the components in the drawing.

Remind pupils that the scale factor is: reduced length/real length.

Pupils must then cut out the scale drawings of the furniture and arrange the pieces on the drawing of the room to make sure that the layout given is the best layout. If the layout is not suitable, pupils must change it to suit their ideas. Remind pupils that the cupboard will be a built-in cupboard.

Pupils then make a neat scale drawing of the room with the (new) layout of the furniture.

For the next part of this investigation, pupils calculate the perimeter of the room to find the length of the skirting board that will be needed. Remind pupils that the skirting board will not be needed where the door is positioned. Pupils then calculate the area of the floor that will be covered in carpet.

Remind pupils of the following:

Perimeter is the distance around the edge of a shape.

There are three formulae that can be applied for working out the perimeter of something:

$$P = W + W + H + H$$

$$P = 2 \times W + 2 \times H$$

$$P = 2 \times (W + H)$$

Area is the space inside the edges of the shape.

Pupils' work must be neat and as accurate as possible, especially when working with their rough scale drawings. Assist pupils where necessary, especially in terms of rearranging the furniture and positioning things; some pupils may struggle with this part of the project.

Assessment

Because the pupils' findings will be different, assess pupils on:

- drawing plans according to a given scale
- applying and using scale drawing in converting lengths and distances of objects to any scale.

Pupil's Book page 228

Objectives

For this investigation pupils are required to work with three-dimensional objects and their different properties.



Guidelines

You will need to have the following resource:
Pupil's Book.

Pupils work on their own, and answer the questions of the Investigation on page 228 of the PB.

Their answers should look like these below.

1.

A	0	1	1	1
B	5	0	5	0
C	1	2	2	2
D	5	0	5	0
E	6	0	6	0
F	6	0	6	0

2.

Name of this group of objects: 3-D objects					
Object	A	B	C	D	E
Name of shape of base	Triangle	Square	Parallelogram	Hexagon	Pentagon
Number of side faces	4	5	5	6	7
Name of shape of side face	Rectangle	Square	Rectangle	Parallelogram	Rectangle
Number of right angles in a side face	4	4	4	0	4
Number of right angles in base	0	4	4	0	0

3.

Name of this group of objects:						
Object	A	B	C	D	E	F
Name of shape of base	Triangle	Parallelogram	Parallelogram	Hexagon	Pentagon	Octagon
Number of side faces	3	4	4	5	6	8
Number of right angles in base	0	4	4	0	0	0

Pupils must represent their answers in tables like the ones provided above. Remind pupils that they need to work neatly and as accurately as possible.

Assessment

Because the pupils' findings will be different, assess pupils on:

- solving problems with two-dimensional shapes
- identifying faces and bases of three-dimensional shapes
- solving problems on three-dimensional shapes.

Pupil's Book page 229

Objectives

This assessment is a summative assessment of work that pupils have covered in Units 23 to 33. It has been designed to assess the pupils' mathematical understanding. It is also important that it is completed by individuals and not with the support of other pupils as this would not uncover any difficulties a pupil may be having with particular concepts.

 **Guidelines**

Pupils must work through the questions on their own. However, observing pupils while they are completing the assessment provides further information about each pupil and his or her level of understanding.

 **Answers**

1. a)

	Length	Breadth	Area	Perimeter
A	12	2	24	28
B	8	3	24	22
C	6	4	24	20

b)

	Length	Breadth	Area	Perimeter
A	16	1	16	34
B	8	2	16	20
C	4	4	16	16

c) The rectangle that measures 4 by 4

2. a) 22; b) 34; c) 56

3. a) 125; b) 96

4. a) 1 000 litres; b) 20 square metres

5. a) 90 km/hr; b) 720 km

6. a)

Property	Rectangle	Square
Two pairs equal sides	P	
Two pairs parallel sides	P	P
All four sides equal		P
All four angles right angles	P	P
Diagonals are equal	P	P
Diagonals bisect each other	P	P

b) Yes, because both have two pairs of equal sides, as well as two sets of parallel lines. Furthermore, both have four right angles.

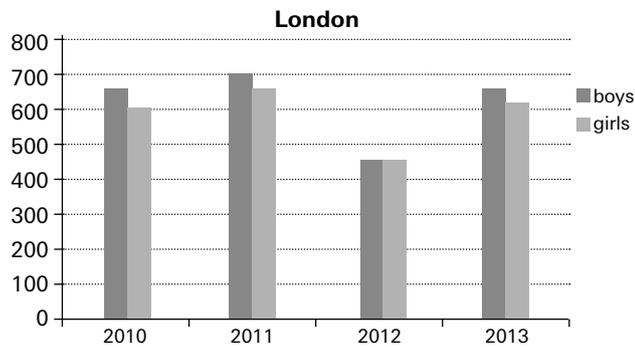
7. a) 80°; b) 100°

8. a = 52°; b = 90°; c = 80°; d = 180°; e = 140°; f = 110°; g = 65°; h = 74°

9.

	Name	Lines of Symmetry	Number of sides	Number of equal sides	Number of angles	Number of equal angles	Number of parallel pairs of sides
A	Rhombus	2	4	2 pairs	4	2 pairs	2
B	Isosceles Trapezium	1	4	none	4	2 pairs	1
C	Rectangle	3	4	2 pairs	4	4	2
D	Irregular polygon	none	4	none	4	none	none
E	Square	4	4	4	4	4	2
F	Trapezium	none	4	none	4	2	1
G	Rectangle	3	4	2 pairs	4	4	2
H	Rhombus	2	4	2 pairs	4	2 pairs	2
I	Square	4	4	4	4	4	2

10. a)



b) 560; c) 2012

11. a) 12; b) 5; c) 7.58

d) It tells you the average of all the data by adding together all of the data points and dividing this by the total number of scores.

Assessment

On completion of the assessment, teachers should look for correct answers and mistakes made by pupils. They should also check to see if there is a pattern in terms of any particular question causing a significant number of pupils' difficulties. By analysing the results of an assessment, they can identify weaknesses in individuals and provide the necessary support, and also strengths of individuals and provide them with more challenging activities. They will also be able to identify any weaknesses in their teaching programme and make adjustments where, or if, necessary.

Objectives

This practice examination is a summative assessment of work covered throughout the year.

It is important that it is completed by individuals and not with the support of other pupils as this would not uncover any difficulties a learner may be having with particular concepts.

Encourage pupils to not spend too much time on one problem. They should rather move onto the next problem and return to the difficult ones if they have time at the end of the examination.



Guidelines

Simulate examination conditions: tell the pupils that they have to work on their own and may not discuss questions or answers with other pupils; ensure quiet in the classroom while pupils work; write the start and end time of the examination on the board, with 10 minute intervals – cross out the time interval as the session progresses to help pupils keep track on time.

Complete the practice examination over two class sessions. Complete questions 1–12 in session 1 and then questions 13–26 in session 2.

Have pupils write their answers and workings out on loose sheets of paper so that you can take them in for marking.



Answers

- a)** 797 981 462; **b)** 276 602 350;
c) 966 199 372; **d)** 734 2114;
e) 257 935; **f)** 10 110.8; **g)** 10 735.2;
h) $\frac{7}{3} \times \frac{4}{9} = \frac{28}{27}$; **i)** 36.8387; **j)** 22.103; **k)** 44.032;
l) 80.8176; **m)** 196; **n)** 361; **o)** 441;
p) 625; **q)** 8; **r)** 11; **s)** 17; **t)** 22
- a)** $96: 2^5 \times 3 = 128:2^3$
 23: This is a prime number; it is not possible to write this in index form
 67: This is a prime number; it is not possible to write this in index form

- b)** 1 541; **c)** 16
- a)** $\frac{2}{3}$; .05; .03; $\frac{2}{9}$; .01
b) 2.6; $\frac{5}{3}$; 1.4; $\frac{7}{5}$; $\frac{14}{10}$
- a)** $((\frac{7}{9} \div \frac{2}{9}) \times 8) + 5 = 33$
b) It is not possible to make this equation true.
- a)** 512; **b)** 640; **c)** 48; **d)** 44.4%
- a)** 1 602 g; **b)** 1.602 kg
- a)** 18; **b)** 2:3; **c)** 2:5; **d)** 3:5
- 15:22
- a)** 2:3; **b)** 5; **c)** 33
- a)** 86 191 000; **b)** 89 709 000
- a)** 80%; **b)** 73%; **c)** 70%
- a)** $\frac{7}{10}$; **b)** $\frac{1}{4}$; **c)** $\frac{13}{20}$
- 2.3%; **14.** 5%; **15.** 24 357 000
- a)** TV stand: ₦20 000
 Bed: ₦132 640
 Bedside table: ₦16 960
 Sofa: ₦400 000
 Dining room set: ₦515 20
 Microwave: ₦88 000

b)

	Value (₦)	Profit/Loss	Percentage
TV stand	₦35 399 – ₦20 000 = ₦15 399	PROFIT	$15\,399 \div 20\,000 \times 100 = 76.995\%$
Bed	₦180 695 – ₦132 640 = ₦48 055	PROFIT	$48\,655 \div 132\,640 \times 100 = 36.235\%$
Bedside table	₦24 129 – ₦16 960 = ₦7 169	PROFIT	$7\,169 \div 16\,960 \times 100 = 42.27\%$
Sofa	₦375 599 – ₦400 000 = ₦335 599	LOSS	$335\,599 \div 400\,000 \times 100 = 83.89\%$
Dining room set	₦49 950 – ₦51 520 = ₦1 570	LOSS	$1\,570 \div 51\,520 \times 100 = 3.05\%$
Microwave	₦92 229 – ₦88 000 = ₦4 229	PROFIT	$4\,229 \div 88\,000 \times 100 = 4.81\%$

c) 31.73%

17. QR = 5; 18. 10.99 kg

19. a) 4:45; b) 7:30; c) 7:20; d) 3:10

e) 2:45; f) 12:57

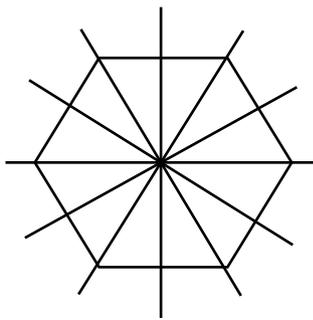
20. a) 4.9 km; b) 1 425 000 m²; c) 142.5 ha

21. 48.6 min

22. a) 73 584 kl; b) 292 kl

23. a) hexagon

b)



c) Right Angle Triangle: Has one right angle

24. a) 96

b) Triangle:

Three equal sides

Three equal angles

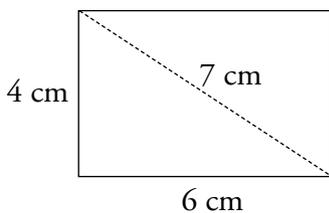
Rectangle:

Four right angles

Two pairs of parallel lines

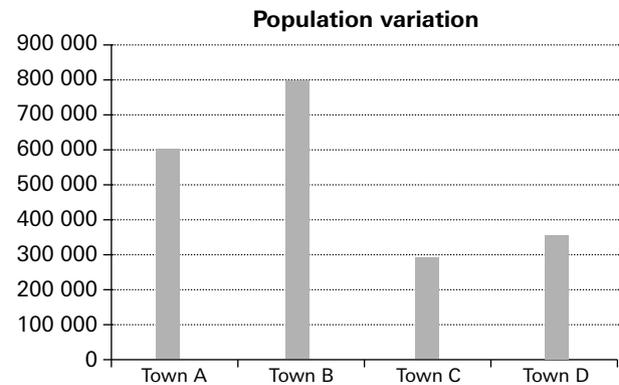
25. a) if 1 cm = 4 m

∴ Width = 4 cm; Length = 6 cm



b) See drawing; c) 28 m

26. a)



b) 502 331

c) There is no mode, as each town has a unique population size.

Assessment

On completion of the assessment, teachers should look for correct answers and mistakes made by pupils. They should also check to see if there is a pattern in terms of any particular question causing a significant number of pupils' difficulties. By analysing the results of an assessment, they can identify weaknesses in individuals and provide the necessary support, and also strengths of individuals and provide them with more challenging activities. They will also be able to identify any weaknesses in their teaching programme and make adjustments where, or if, necessary.

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