Chapter 12 Mensuration 1: Plane shapes

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use appropriate formulae to calculate the perimeter of plane shapes.
2. Recall and use appropriate formulae to calculate the area of plane shapes.
3. Solve problems relating to parallelograms and triangles drawn between parallels.
4. Calculate the lengths of arcs and perimeters of shapes in circles.
5. Calculate the area of sectors and segments or a circle.

Teaching and learning materials

Students: Textbook, exercise book, pencil and ruler and Mathematical instruments and coloured pencils or highlighters.

Teacher: Posters on perimeter and area of shapes (Fig. 12.1), parts of a circle (Fig. P18 on p. 10), cardboard cut-out shapes; string to demonstrate perimeter.

Teaching notes

- When proving that the areas of plane shapes are equal in area, let students use coloured pencils or highlighters to shade figures with equal areas. Let them always concentrate seeing triangles or parallelograms on the same base and between the same two parallel lines.
- Students have to remember that the diagonal of a parallelogram bisects its area into two triangles with equal areas.
- Make sure that students always give a reason for each statement that they make.
- When letting your students do the problems of this chapter, choose problems according to their ability.
  - It does not help them to pass Mathematics, if you choose all the most difficult problems and they cannot do any of them. This would only serve to give them a lower self-esteem where Mathematics is concerned.
  - Rather choose problems that let students practise the basic principles.
- In the JSS course, a formula was derived to determine the area of a triangle where the altitude of the triangle was not used (See Fig. 12.1).

You can remind the students that the formula was derived as follows:

$\frac{AD}{c} = \sin B$, $\therefore AD = c \cdot \sin B$

$\frac{AD}{b} = \sin C$, $\therefore AD = b \cdot \sin C$

In the same way, it can be proven that:

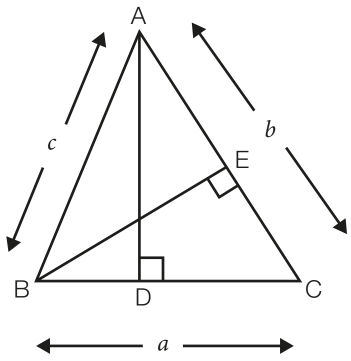
$BE = c \cdot \sin A$ and $BE = a \cdot \sin C$

So,

$$\text{Area } \triangle ABC = \tfrac{1}{2}BC \times AD = \tfrac{1}{2}ac \cdot \sin B = \tfrac{1}{2}ab \cdot \sin C$$

And

$$\text{Area } \triangle ABC = \tfrac{1}{2}AC \times BE = \tfrac{1}{2}bc \cdot \sin A = \tfrac{1}{2}ab \cdot \sin C$$



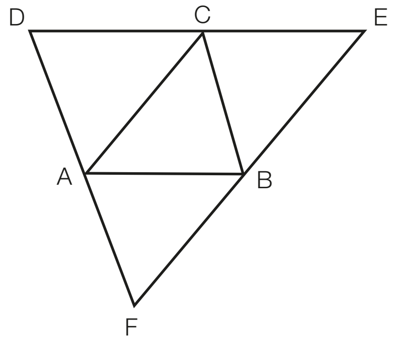
This formula is used when you have two sides, and the angle between the two sides.

Areas of difficulty and common mistakes

- Students cannot see what side of a triangle or parallelogram to use as base when they have a certain altitude. Teach them, that if they are in doubt about this, to draw a line parallel to the base of the triangle or parallelogram and to take the perpendicular distance between those parallel lines as the altitude of the figure.

- Students find it difficult to prove that areas of plane shapes are equal by adding or subtracting shapes with equal areas.
  - It could help if the students shade certain of the shapes.
  - It could also help, if the students let a certain area be equal to *x* (say), and then write all the areas of the other figures in terms of *x*.

Supplementary worked examples



In the figure, ABCD and ABEC are parallelograms; and EBF and DAF are straight lines. Prove that:

a) $\triangle BAF = \triangle ADC$
b) Area of quad FACE = area of quad ADEB.

Solution

a) $BE \parallel AC$ (opp sides $\parallel^m$)
$FB \parallel AC$ (FBE is a straight line given)
In the same way, $CB \parallel AF$.
AFBC is a parallelogram (both pairs opp sides $\parallel$)
Let $\triangle BAF = x$
$\therefore$ Area $\triangle ABC = x$ (diagonals bisect area $\parallel^m$ AFBC)
$\therefore$ Area $\triangle ACD = x$ (diagonals bisect area $\parallel^m$ ADCB)
$\therefore$ Area $\triangle BAF = \triangle ADC$

b) Area $\triangle BEC = x$ (diagonal $\parallel^m$ ABEC bisects its area)
Area quad FACE = $3x$ = Area quad ADEB.