Chapter 15 Mensuration 2: Solid shapes

Learning objectives

By the end of this chapter, the students should be able to:

1. Recall and use appropriate formulae to calculate the volume and surface area of cubes, cuboids, prisms and cylinders.
2. Recall and use appropriate formulae to calculate the surface area and volume of cones and pyramids.
3. Use addition and subtraction methods for the mensuration of composite solids and hollow shapes.
4. Use similar triangles and subtraction to calculate the volume of the frustum of a cone or a pyramid.

Teaching and learning materials

Students: Textbook, exercise book, writing materials.
Teacher: Collection of solid shapes (use familiar objects such as boxes, tin cans, tyres, buckets and lampshades) and stiff paper (the kind found in new shirts) or cardboard (the kind find in shoe boxes).

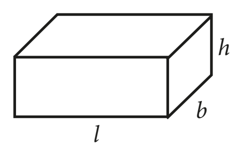
Glossary of terms

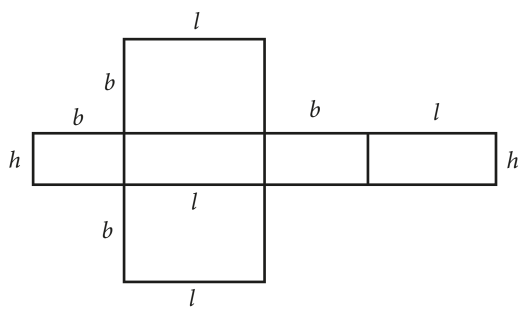
Prism: If you cut through a solid parallel to its base and the cross section is identical or congruent to the base, the solid is a prism.
Or
A solid object with two identical ends and flat sides: The sides are parallelograms (4-sided shapes with opposites sides parallel). The cross section is the same all along its length. The shape of the ends gives the prism its name, such as "triangular prism".

Teaching notes

- If we look at the net of the figure, the total surface area can also be calculated as follows:

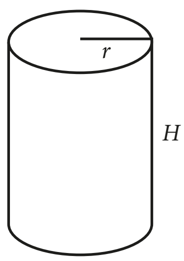
A cuboid

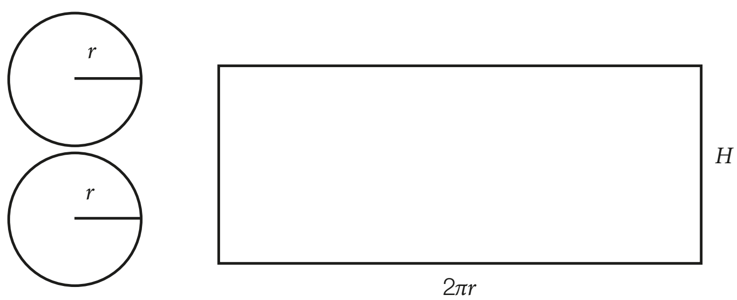




Total surface area = $2(l \times b) + 2(l + b) \times h$

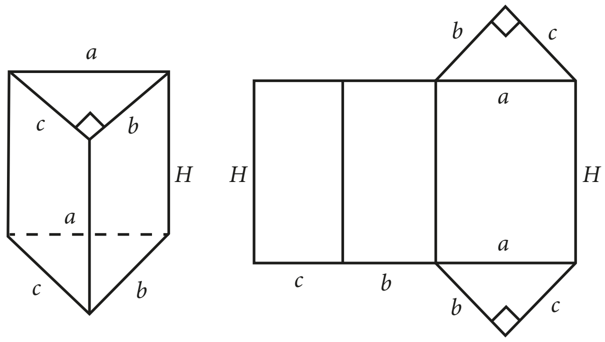
A cylinder





Total surface area = $2\pi r^2 + 2\pi rH$

A triangular prism



Total surface area = $2(\frac{1}{2}bc) + (c + b + a)H$

The total surface area in general for a prism
= 2 × base area + perimeter base × height of prism

The students would understand this method better, if the nets above were cut out and shown to them. You can then fold the nets to form the 3D prism.

- If you explain how the formula for the surface area of the cone is found, you could illustrate by cutting out a sector of a circle as shown in Fig. 15.4 and then folding this sector to form the required cone.
- Remind the students to always use the same units when they calculate the volume or total surface area. To avoid working with fractions they should always convert all units to the smallest unit. For example, if the units are given as cm and m, they should convert all units to cm.
- If the volume is required to be in m^3 or the total surface area is required to be in m^2, for example, convert the lengths to m from the beginning. In this way errors are avoided, because it is much more difficult to convert cm^3 or cm^2 to m^3 or m^2, than it is to convert cm to m.

Areas of difficulty and common mistakes

- The students do not always convert all the units to the same unit. Emphasise that they can only work with length units that are the same.
- If the volume is required to be in m^3 or the total surface area is required to be in m^2, for example, convert the lengths to m from the beginning. In this way errors are avoided, because it is much more difficult to convert cm^3 or cm^2 to m^3 or m^2, than it is to convert cm to m. So, to make the work easier and to avoid errors, the length units should always be converted to same length unit as the area or volume units.
- Students may find it difficult to work out the total surface area or volume of a prism, if it is of another form than the four basic prisms shown on p. 182 in Fig. 15.1.
  The reason for this is that they cannot identify the bases of the prisms. They could overcome this difficulty if they imagine that they have a knife and that they cut through the solid, like they would cut through bread to obtain identical slices.
  In this way, they could identify the base and apply the general formula for working out the volume or the total surface area of the solid.
- When working with composite solids, students may find it difficult to visualise the 3D forms. Their work can be made less complicated by drawing the separate solids and then working out the required volume or area and adding them.
- When calculating the volume of the frustum of a cone or a pyramid, students may find it difficult to identify the two similar triangles that are necessary to calculate the altitude of the remainder of the cone or pyramid.
  Teach them to always start with the length they want to work out.
  This length is part of a right-angled triangle of which the length of another side is known.
  Then they must look for another right-angled triangle of which the lengths of the corresponding sides are both known.