Learning objectives
By the end of this chapter, the students should be able to:
1. Determine the sine, cosine and tangent ratios of any angle between 0° and 360°.
2. Derive the sine rule.
3. Use the sine rule to solve triangles.
4. Apply the sine rule to real-life situations (such as bearings and distances, angles of elevation).

Teaching and learning materials
Students: 4-figure tables (provided on pages 237 and 248 of the Student’s Book), calculator.
Teacher: Poster showing the sine rule and its relation to ΔABC with sides \( a, b, c \). Computer-assisted instructional materials where available. Chalk board compass and protractor.

Areas of difficulty
• Students find applying the sine rule difficult, if the cosine and area rules are also taught at the same time. Students simply have to remember that, if there is a side and an angle opposite each other, they can use the sine rule.
• To make using the sine rule easy:
  • Use this version of the sine rule, if you have to work out a side:
    \[
    \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.
    \]
  • Use this version of the sine rule, if you have to work out an angle:
    \[
    \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.
    \]
    Remember that the angle could also be an obtuse angle, because the sines of obtuse angles are positive.
• When you are given side, side, angle of a triangle that you have to solve, the possibility exists that there are two possible triangles. Students find it difficult to understand why this is the case.
  You could help them to understand this by actually constructing such a triangle. \textit{Given: In any } \triangle ABC, \angle B = 40° \text{ and } AB = 6 \text{ cm. Now depending on the length of } AC, \text{ you could get 1 or 2 triangles.}
  To find point C, you measure the required length on a ruler with your compass. Then you put the sharp, metal point of the compass on A and chop off the required length through the line through B by drawing an arc with the pencil of your compass:
  a) If \( AC > AB \), then the sketch will look something like this:
  
  \[
  \begin{align*}
  \triangle ABC: & \\
  \angle B & = 40° \text{ cm} \\
  AB & = 6 \text{ cm} \\
  AC & > AB
  \end{align*}
  \]
  The arc drawn will intersect the line at one point only. So, only one triangle is possible.
  b) If \( AC = AB \), then the sketch will look something like this:
  
  \[
  \begin{align*}
  \triangle ABC: & \\
  \angle B & = 40° \text{ cm} \\
  AB & = 6 \text{ cm} \\
  AC & = AB
  \end{align*}
  \]
  The arc drawn will intersect the line at one point only. So, only one triangle is possible.
  c) \( AC < 6 \text{ cm} \) (and not so short that the arc will not intersect the base line).
  The arc drawn will intersect the line in two places. So, two triangles are possible.
This is probably why, when angle, side, side of a triangle are given, we say that it is an ambiguous case. If we do not know the length of the side opposite the given angle, we can have one triangle, two triangles or no triangle if the side is too short (that is when the sum of the two sides is less than the third side). Students do not have to construct the triangle. You only have to teach them to follow this procedure:

1. Draw a sketch of the given triangle showing all the information given.
2. If the side opposite the given angle is equal or longer than the side adjacent to the given angle, there is only one possible triangle.
3. If the side opposite the given angle is shorter than the side adjacent to the given angle, there are two possible triangles.
4. If the given angle is obtuse, the side opposite this angle is obviously longer than the side adjacent to the given angle. The longest side of a triangle is always opposite the biggest angle of the triangle. In a triangle, there is only one obtuse angle possible, because the sum of the angles of a triangle is equal to 180°.

Supplementary worked examples

Solve \( \triangle ABC \).

\[ \sin C = \frac{6 \sin 40°}{4} \]

\[ \therefore \angle C = 74.62° \]

But this is the size of \( \angle BC_1A \).

\[ \angle BC_2A = 180° - 74.62° = 105.38° \] (the sum of the angles on straight line \( AC_1 = 180° \))

\[ \angle ABC_1 = 180° - 40° - 74.62° = 65.38° \] (sum \( \angle s \) \( \triangle ABC_1 = 180° \))

\[ \angle ABC_2 = 180° - 40° - 105.38° = 34.62° \] (sum \( \angle s \) \( \triangle ABC_2 = 180° \))

\[ AC_1 = \frac{4 \sin 65.38°}{\sin 40°} = 5.66 \] (multiply both sides by \( \sin 65.38° \))

\[ AC_2 = \frac{4 \sin 34.62°}{\sin 40°} = 3.54 \] (multiply both sides by \( \sin 34.62° \))