Aligned to DBE
Revised ATPs

Mathematics

Navigation pack

FET PHASE
GRADE 12

Platinum

X-kit Achieve!

CLASSROOM MATHEMATICS

Pearson South Africa (Pty) Ltd

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Mathematics Grade 12 Navigation Pack

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Dear Teacher

The National State of Disaster due to the COVID-19 pandemic has resulted in the disruption of Education in South Africa and the loss of valuable teaching time and disruption of the school calendar.

As a result of this, the DBE has created and released revised Annual Teaching Plans (ATPs) to assist schools and teachers in ensuring the 2021 school year is completed. The 2021 ATPs are based on the revised ATPs that were developed in 2020. It is important to note that fundamental and core topics are retained in the 2021 ATPs. Some of the strategies that have been used in the process of developing the 2021 DBE ATPs are:

- reduction of content covered in certain topics
- merging of topics
- deleting topics
- revising the assessment guidelines
- reduction in teaching time for certain topics
- resequencing of topics/concepts

At Pearson South Africa, we believe that education is the key to every individuals' success. To ensure that despite the challenges, teachers and learners can meet all the necessary learning outcomes for the year, we have created the Navigation Guide, a free resource to support teachers and learners during this challenging time.

The Navigation Pack aims to summarise and highlight the changes in the 2021 DBE ATP and provide teachers and learners with worksheets that focus on impacted topics in the curriculum.

Due to resequencing of topics, the order of topics in the textbook that is currently used in the classroom may not be aligned to the new sequence of topics in the ATP. The Navigation Pack has a set of assessments based on the Section 4 changes and the revised assessment guidelines.

COVID-19 safety guidelines for teachers and learners

Gatherings at school

Where schools are open for learning, it is up to management to take decisive action to ensure sites are not simultaneously used for other functions such as shelters or treatment units in order to reduce the risk.

Implement social distancing practices that may include:

- A staggered timetable, where teachers and learners do not arrive/leave at the same time for the beginning and end of the school day.
- Cancelling any community meetings/events such as assemblies, cake sales, market d y, tuckshop, after-care classes, matric dance, Eisteddfod and other events.
- Cancelling any extra-mural activities such as ballet classes, swimming lessons, sport games, music class and other events that create a crowd gathering.
- Teaching and modeling creating space and avoiding unnecessary touching.
- Limiting movement and interaction between classes.
- Schools with an established feeding scheme plan are to ensure that hygiene and social distancing is always implemented. Teachers and staff members assisting with food
- distribution are to wear masks, sanitise prior to issuing food items and learners are to stand 1,5m apart in the queue.

Wear a mask at all times.

1. Restrooms/toilets

Hand washing

Washing hands with soap and water or using alcohol-based hand sanitisers is one of the most important ways to help everybody stay healthy at school. Critical to this is preparing and maintaining handwashing stations with soap and water at the toilet and in each classroom.

Teachers and learners should always wash their hands after:

- eating
- entering the classroom
- using the toilet
- blowing your nose or coughing
- touching tears, mucous, saliva, blood or sweat.

2. Premises and Classroom setting

When schools open, classroom settings should be altered in order to promote hygiene, safety and social distancing.

Changed classroom settings may include:

- Cleaning and disinfecting school buildings, classrooms and especially sanitation of facilities at least once a day, particularly surfaces that are touched by many people (railings, lunch tables, sports equipment, door and window handles, toys, teaching and learning tools etc.).
- Ensure the proper ventilation and fresh flow of air through classrooms.
- Providing learners with vital information about how to protect themselves by incorporating the importance of hygiene, handwashing and other measures of protecting themselves, into the lessons.
- Promoting best handwashing and hygiene practices and providing hygiene supplies.

- Prepare and maintain handwashing stations with soap and water, and if possible, place alcohol-based hand sanitisers in each classroom, at entrances and exits, and near lunchrooms and toilets.

- Ensure teachers and learners wear a mask at all times.

Social distancing

- Space the learners out in the classroom (or outdoors) – try to keep learners separated by a minimum of 1,5m.

- Create space for learner's desks to be at least 1,5m apart
- Learners are not to exceed 30 per class or 50% of original class size

- Learners should not share cups, eating utensils, or food
- Do not let learners eat items that fall on the floor or chew on pencils or other objects
- Avoid close contact, like shaking hands, hugging or kissing

3. Social behaviour

It is extremely vital during a pandemic that focus is not only directed towards optimal physical health and hygiene but fi nding ways to facilitate mental health support.

- Treat everybody with respect and empathy – no teasing about COVID-19.
- Encourage kindness towards each other and avoid any stereotyping when talking about the virus.
- Stay home if you have a temperature or are ill.
- Do not touch people who are ill, but be empathetic.

Wear a mask at all times.

How to use this Navigation Pack

Revised DBE Teaching Plan: Comprehensive summary of the CAPS topics according to the revised ATPs.

Navigation Plan: Link to the resources in the Navigation Pack.

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
FINANCE, GROWTH AND DECAY	Derivation and use of formulae for annuities	Annuities: Apply knowledge of geometric series to solve problems involving present value and future value annuities.*1 Make use of logarithms to calculate the value of n, the time period, in the equations $A = P(1 + i)^n$ or $A = P(1 - i)^n$.	2 weeks		
	Annuity application and problem solving				
	Calculate time periods using logarithms				
	Analyse investments and loan options Investments and loan options	Critically analyse different loan options.			
REVISION					
ASSESSMENTS				Navigation Guide: Targeted Worksheet 1	Page 23 – 24
	Task: Test			Navigation Pack: Term 2 Control Test	Page 36 – 39
TOTAL WEEKS = 10					

*1 Use of the simple and compound decay formulae: $A = (1 - in)$ and $A = (1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance) to be treated in Grade 11. However, related questions should still be expected in Grade 12.

Assessments for the Term as per the revised ATPs and the Section 4 amendments.

Link to a targeted worksheet in the Navigation Pack, that focus on impacted or challenging topics in the curriculum.

Footnotes provide any additional information.

Link to an exemplar assessment in the Navigation Pack, that was created with Section 4 and curriculum changes in mind.

Navigation Guide

GRADE 12

Mathematics**[1]

TERM	TOPIC	TIME (WEEKS)
TERM 1	Number patterns, sequences and series	4
	Euclidean Geometry	3
	Trigonometry	3
TERM 2	Analytical Geometry	2
	Functions: Formal definitions; inverses, exponential and logarithms	1
	Differential Calculus including Polynomials	4
	Finance, growth and decay	2
TERM 3	Finance, growth and decay (continuation)	1
	Statistics	3
	Counting and Probability	2
TERM 4	Revision	4
	Final Examination	6

Programme of Assessment**[2]

TERM 1	TERM 2	TERM 3	TERM 4
Task 1 Assignment (**15%**)	Task 4 Test (**10%**)	Task 5 Test (**10%**)	**Final Examination**
Task 2 Investigation/Project (**15%**)		Task 6 Test (**25%**)	
Task 3 Test (**10%**)			

**[1] No important aspect in Mathematics curriculum is compromised.
**[2] The amended School Based Assessment (SBA) is aligned to the content and time available. Informal tasks and activities should be used as assessment for learning, to prepare for formal assessment.

Term 1

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
PATTERNS, SEQUENCES AND SERIES	Quadratic number patterns	Patterns: Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	4 weeks		
	Arithmetic sequences	Number patterns, including arithmetic and geometric sequences and series.			
	Geometric sequences				
	The sum of arithmetic series	Derivation and application of the formulae for the sum of arithmetic and geometric series: $S_n = \frac{n}{2}(a + l)$ $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$ $S_n = \frac{a}{1 - r}; -1 < r < 1; r \neq 1$			
	The sum of geometric series				
	Sigma notation	Sigma notation			
	Practical applications				
REVISION					
ASSESSMENTS					

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
EUCLIDEAN GEOMETRY*1	Revision: Grade 11 Geometry	Revision	3 weeks		
	Similar polygons				
	Proportionality theorem	Prove (accepting results established in earlier grades): That a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem); That equiangular triangles are similar; That triangles with sides in proportion are similar.			
	Equiangular triangles and similarity				
	Triangles with proportional sides and similarity				
	Pythagoras' Theorem and similarity	The Pythagorean Theorem by similar triangles.			
REVISION					
ASSESSMENTS					

*1 Euclidean Geometry has been moved from Term 3 to Term 1.

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
TRIGONOMETRY: COMPOUND AND DOUBLE ANGLE IDENTITIES	Derive the compound and double angle identities	Compound angle identities $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$	3 weeks		
		Double angle identities $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $\cos 2\alpha = 1 - 2\sin^2\alpha$			
		Compound and double angle identities for $\tan(\alpha \pm \beta)$ and $\tan 2\alpha$			
	Prove identities using compound and double angle identities				
REVISION					
ASSESSMENTS					
TRIGONOMETRY: PROBLEM SOLVING IN TWO AND THREE DIMENSIONS	Problems in two dimensions	Solving two- and three-dimensional problems 1. Prove and apply the sine, cosine, and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules.			
	Problems in three dimensions				
REVISION					
ASSESSMENTS					
TOTAL WEEKS = 10					

Term 2

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
ANALYTICAL GEOMETRY	Revision: Equation of a line	Derive and apply: 1. The equation of a line through two given points. 2. The equation of a line through one point and parallel or perpendicular to a given line. 3. The inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line ($0° \leq \theta \leq 180°$)	2 weeks		
	Equation of a circle	The equation that defines a circle with radius r and centre $(a; b)$.			
	Equation of a tangent to a circle	Determination of the equation of a tangent to a given circle.			
REVISION					
ASSESSMENTS					
FUNCTIONS AND INVERSE FUNCTIONS	Functions and inverse functions	1. Definition of a function. 2. General concept of the inverse of a function and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine and sketch graphs of the inverses of the functions defined by Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases /decreases.	2 weeks		
	Inverse functions				
REVISION					
ASSESSMENTS					

Term 2

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
EXPONENTIAL AND LOGARITHMIC FUNCTIONS	Revision of exponential laws and function	1. Revision of the exponential function and the exponential laws and graph of the function defined by $y = a^x$ where $b > 0$; and $b \neq 0$. 2. Understand the definition of a logarithm: $y = \log_b x \Leftrightarrow x = b^y$ and $b > 0$; and $b \neq 0$. 3. The graph of the function defined by $y = \log_b x$ for both the cases $0 < b < 1$ and $b > 1$.	2 weeks		
	Logarithms and logarithmic functions				
REVISION					
ASSESSMENTS					
DIFFERENTIAL CALCULUS	Limits	Revision: • Factorise third-degree polynomials. Apply the Remainder and Factor Theorems to polynomials of degree at most 3 (no proofs required). • An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point.	4 weeks		

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
DIFFERENTIAL CALCULUS	Use limits to define the derivative of a function f	Use limits to define the derivative of a function f. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ Generalise to find the derivative of f at any point x in the domain of f, i.e., define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the gradient of the tangent to the graph of f at the point with x-coordinate is a.			
	Differentiation of functions from first principles	Using the definition (first principle), find the derivative, $f'(x)$ for a, b and c constants: $f(x) = ax^2 + bx + c$; $f(x) = ax^3$; $f(x) = \frac{a}{x}$; $f(x) = c$.			
	Use the specific rules for differentiation	Use the formula (for any real number n) together with the rules $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ and $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$, ($k$ a constant).			
	Find the equations of tangents to functions	Find equations of tangents to graphs of functions.			
	The second derivative	Introduce the second derivative of $f(x)$ and how it determines the concavity of a function.			
	Sketch cubic graphs	Sketch graphs of cubic polynomial functions using differentiation to determine the coordinates of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques.			

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
DIFFERENTIAL CALCULUS	Optimisation and rate of change	Solve practical problems concerning optimisation and rate of change, including calculus of motion.			
REVISION					
ASSESSMENTS					
FINANCE, GROWTH AND DECAY	Derivation and use of formulae for annuities	Annuities: Apply knowledge of geometric series to solve problems involving present value and future value annuities.[*2] Make use of logarithms to calculate the value of n, the time period, in the equations $A = P(1 + i)^n$ or $A = P(1 - i)^n$.	2 weeks		
	Annuity application and problem solving				
	Calculate time periods using logarithms				
	Analyse investments and loan options Investments and loan options	Critically analyse different loan options.			
REVISION					
ASSESSMENTS				Navigation Pack: Targeted Worksheet 1	Page 24 – 25
	Task: Test			Navigation Pack: Term 2 Control Test	Page 37 – 40
TOTAL WEEKS = 10					

[*2] Use of the simple and compound decay formulae: $A = (1 - in)$ and $A = (1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance) to be treated in Grade 11. However, related questions should still be expected in Grade 12.

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
FINANCE, GROWTH AND DECAY (CONTINUATION)	Analyse investments and loan options	Delayed payments. Critically analyse investment and loan options and make informed decisions as to best option(s) (including pyramid).	1 week		
REVISION					
ASSESSMENTS					
STATISTICS	Bivariate data: scatter plots, regression lines and correlation	Bivariate data – Identification of outliers.*3	3 weeks		
		Correlation coefficient: Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.			
		Correlation coefficient. Use of available technology to calculate the correlation co-efficient of a set of bivariate numerical data and make relevant deductions.			
REVISION					
ASSESSMENTS				Navigation Pack: Targeted Worksheet 2	Page 26 – 28 Page 33 – 34

*3 The addition rule for mutually exclusive events, $P(A \text{ or } B) = PA + P(B)$. The complementary rule, $P(\text{not } A) = 1 - P(A)$ and the identity, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
The identification of dependents and independents events and the product rule for independent events, $P(A \text{ and } B) = P(A) \times P(B)$.
The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S.
Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.
Probability problems using Venn diagrams, tree diagrams, two-way contingency tables and other techniques to solve probability problems (where events are not necessarily independent) to be treated in Grade 11. However, related questions should still be expected in Grade 12.

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	**UNIT**	**CONTENT SPECIFIC CONCEPTS**	**TIME**	**LINKS TO PEARSON NAVIGATION PACK**	**PAGE REFERENCE**
COUNTING PRINCIPLES AND PROBABILITY	The fundamental counting principle	Generalise and use the fundamental counting [*4] (multiplication) principle.	2 weeks		
	Applications of the counting principle to solve probability problems	Probability problems using the fundamental counting principle and other techniques.			
REVISION					
ASSESSMENTS				Navigation Pack: Targeted Worksheet 3	Page 29 – 30 Page 35
	Task: Test Task: Trial Examinations		3 weeks	Navigation Pack: Control Test Navigation Pack: Trial Exemplars	Page 41 – 42 Page 58 – 59 Page 43 – 53 Page 60 – 70
TOTAL WEEKS = 9					

[*4] Histograms, Frequency polygons, Ogives (cumulative frequency curves), Variance and standard deviation of ungrouped data, Symmetric and skewed data to be treated in Grade 11, however, related questions should still be expected in Grade 12.

Term 4

REVISED DBE ANNUAL TEACHING PLAN				NAVIGATION PLAN	
TOPIC	UNIT	CONTENT SPECIFIC CONCEPTS	TIME	LINKS TO PEARSON NAVIGATION PACK	PAGE REFERENCE
SBA		Revision	4 weeks		
FINAL EXAMINATION		Final Examination	6 weeks		
TOTAL WEEKS = 10					

Targeted Worksheets

TARGETED WORKSHEET	TOPIC IN CAPS
1	Financial Mathematics
2	Statistics
3	Probability

Content summary

Due to the loss of teaching and learning time as a result of adhering to the health precautions of Covid-19, the content of the following topics was trimmed by the Department of Basic Education in Grade 12. The affected topics are:

- Finance, growth and decay
- Statistics,
- Counting principles and probability.

Finance, growth and decay

- Calculate the value of n in the formulae.
- Apply knowledge of geometric series to solve annuity and bond repayment problems.
- Critically analyse different loan options.

Statistics

- Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data.
- Use of available technology to calculate the regression line which best fits a given set of bivariate numerical data.
- Use of available technology to calculate the correlation co-efficient of a set of bivariate numerical data and make relevant deductions.

Counting principles and probability

- Generalise and use of the fundamental counting (multiplication) principle.
- Probability problems using the fundamental counting principle and other techniques.

Teachers are advised to take note of the content trimmed from the Grade 12 curriculum which should, in addition to the Grade 11 content be covered extensively in Grade 11.

Grade 12 assessments will continue to be examined on the complete syllabus as set out in the CAPS document.

Targeted Worksheet 1

Time: 40 minutes

Name: **Surname:**

Topic: Finance, growth and decay

This worksheet consists of 5 questions.

Instructions

Read the following instructions carefully before answering the questions

1. Answer ALL the questions.
2. Clearly show ALL calculations.
3. You may use a non-programmable scientific calculator.
4. Write neatly and legibly.

Question 1

1.1 On their saving accounts, a bank offers an interest rate of 18% nominal, paid monthly. If you save R15 000 in such an account now, how much would the amount have accumulated to in four and half years' time? (3)

[3]

Question 2

2.1 Kabelo received R180 000 on her investment of R100 000 in five years. Calculate the interest rate per annum compounded quarterly. (3)

[3]

Question 3

3.1 Calculate the effective rate equivalent to a nominal interest rate of 8,75% p.a. compounded monthly. (3)

[3]

Targeted Worksheet 1

Question 4

Akhona pays R15 000 into a savings account after every 3 months. She made her first payment on the 1st of April 2020 and she intends to continue doing so until the 31st of December 2023. The bank pays an interest of 8% p.a. compounded quarterly. Determine:

4.1 how much she will have in the account at the end of her investment on the 31st of December 2023. (4)

4.2 how much she will have in the account at the end of 2023 if she withdrew R100 000 from the account at the end of 2022 for personal reasons. (3)

[7]

Question 5

Mark borrowed R600 000 from a bank. The loan was to be paid back over a period of 25 years at an interest rate of 10% p.a. compounded monthly. Mark made monthly repayments starting exactly one month after the loan was granted.

5.1 If Mark made monthly payments of R8 000 instead of the required R5 452,20, how many payments will he need to settle the loan? (5)

5.2 Calculate the value of the last payment. (3)

5.3 After making monthly repayments of R8 000 for 8 years, Mark required money to fund his business. He approached the bank for another loan. Instead of a loan, the bank advised Mark that the extra amount repaid every month could be regarded as an investment and that he could withdraw the full amount to fund his business. Calculate the maximum amount that Mark may withdraw from the loan account. (4)

[12]

Total: [28]

Targeted Worksheet 2

Time: 45 minutes

Name: **Surname:**

Topic 2: Statistics

This worksheet consists of 3 questions.

Instructions

Read the following instructions carefully before answering the questions

1. Answer ALL the questions.
2. Clearly show ALL calculations.
3. You may use a non-programmable scientific calculator.
4. Write neatly and legibly.

Question 1

1.1 The following data represents term marks obtained by 10 Grade 12 learners in Mathematics.

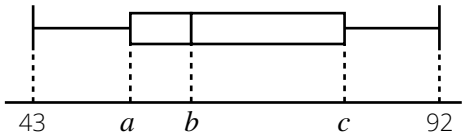
43; 55; 58; 60; 62; 65; 70; 75; 85; 92

1.1.1 Calculate the mean mark. (2)

1.1.2 Calculate the standard deviation of the marks. (2)

1.1.3 How many learners got a mark one standard deviation above the mean mark? (3)

1.2 The diagram below shows a box-and-whisker plot of the term marks in Question 1.1 above.



1.2.1 Determine the values of a, b and c. (3)

1.2.2 Comment on the skewness of the data. (1)

[11]

Targeted Worksheet 2

Question 2

The data below represents the percentage marks obtained by 10 Grade 12 learners for English Home Language (HL) and Afrikaans First Additional Language (FAL).

English HL	42	54	85	32	63	71	92	62	58	66
Afrikaans FAL	50	58	80	45	60	65	98	75	71	58

2.1 Draw a scatter plot of the data above and hence use it to draw the line of best fit. (4)

2.2 Calculate the equation of the least squares regression line. (3)

2.3 Calculate the correlation coefficient. (2)

2.4 Describe the correlation between English HL and Afrikaans FAL. (1)

2.5 Predict the final English HL mark for the learner who obtained 74% in the Afrikaans FAL. (2)

[12]

Question 3

Mr Mokoena, a street vendor, kept record of his daily sales during the months of May and June. The information is represented in the frequency table below.

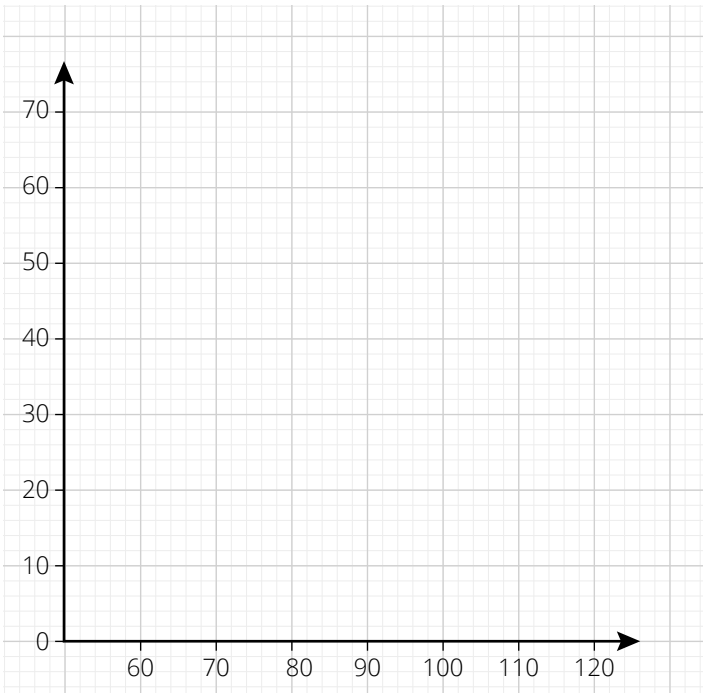
Daily sales	Frequency	Cumulative frequency
$60 < x \leq 70$	5	a
$70 < x \leq 80$	11	16
$80 < x \leq 90$	b	38
$90 < x \leq 100$	13	51
$100 < x \leq 110$	7	c
$110 < x \leq 120$	d	61

3.1 For how many days did Mr Mokoena keep record of his daily sales? (1)

3.2 Determine the value of the unknowns (a, b, c, d). (4)

Targeted Worksheet 2

3.3 Use the grid provided to draw an Ogive for the daily sales. (4)



3.4 Use the graph drawn to estimate the median value for the daily sales. (2)

3.5 Estimate the interval of the upper 25% of the daily sales. (1)

[12]

Total: [35]

Targeted Worksheet 3

Time: 40 minutes

Name: **Surname:**

Topic 3: Counting principles and probability

This worksheet consists of 3 questions.

Instructions

Read the following instructions carefully before answering the questions

1. Answer ALL the questions.
2. Clearly show ALL calculations.
3. You may use a non-programmable scientific calculator.
4. Write neatly and legibly.

Question 1

The events A and B are independent. $P(A) = 0{,}4$ and $P(B) = 0{,}5$.

Determine:

1.1 $P(A \text{ and } B)$ (3)

1.2 $P(A \text{ or } B)$ (3)

1.3 $P(A' \text{ and } B')$ (3)

[9]

Question 2

A survey done in a certain country showed that most people are carriers of a deadly disease. Out of the 1 200 people tested, findings were that 270 out 480 adults tested positive and 480 out of 720 children tested negative.

2.1 Determine:

2.1.1 The values of a, b, c and d, and hence, copy and complete the table below. (4)

	Positive	Negative	Total
Adults	270	a	480
Children	b	480	720
Total	c	d	1 200

2.1.2 The probability that an adult tested positive. (1)

2.1.3 The probability that a person randomly selected tested positive. (1)

2.1.4 The number of children who are positive if the estimated population of country is 40 million people. (4)

2.2 Are the events of testing positive and being an adult independent? (4)

Targeted Worksheet 3

[14]

Question 3

The Gauteng province uses a coding system of number plates with two letters followed by two digits then two letters. All codes end with GP as shown below:

DW68RHGP

The vowels (A, E, I, O, U) and Q may not be used and digits 1 to 9 are used. The letters and digits may be repeated.

3.1 Determine how many number plates with different codes can be made. (4)

3.2 Determine the probability that a code randomly selected will consist of even digits which are not the same. (4)

[8]

Total: [31]

Targeted Worksheet 1 Answers

Time: 40 minutes

Topic 1: Finance, growth and decay

1.1 $A = P\left(1 + \frac{i_{nom}}{m}\right)^{m \times n}$

$A = 15\,000\left(1 + \frac{0{,}18}{12}\right)^{4{,}5 \times 12}$ ✓✓

$A =$ R33 516,41 ✓ (3)

[3]

2.1 $A = P\left(1 + \frac{i_{nom}}{m}\right)^{m \times n}$

$180\,000 = 100\,000\left(1 + \frac{i}{4}\right)^{5 \times 4}$ ✓

$1{,}8 = \left(1 + \frac{i}{4}\right)^{20}$

$1{,}8 = \left(1 + \frac{i}{4}\right)^{20}$ ✓

$i = 11{,}93\%$ ✓ (3)

[3]

3.1 Conversion formula: $(1 + i_{eff}) = \left(1 + \frac{i_{nom}}{m}\right)^{m}$ ✓

$(1 + i_{eff}) = \left(1 + \frac{0{,}0875}{12}\right)^{12}$ ✓

$1 + i_{eff} = 0{,}0911$

$i_{eff} = 9{,}11\%$ ✓ (3)

[3]

4.1 $F_v = \frac{x[(1 + i)^n - 1]}{i}$

$F_v = \frac{15\,000\left[\left(1 + \frac{0{,}08}{4}\right)^{16} - 1\right]}{\frac{0{,}08}{4}}$ ✓

$F_v =$ R279 589,28 ✓ (4)

4.2 $A = 100\,000\left(1 + \frac{0{,}08}{4}\right)^{4}$ ✓

$F_v = 279\,589{,}28 - 100\,000\left(1 + \frac{0{,}08}{4}\right)^{4}$ ✓

$F_v =$ R171 346,06 ✓ (3)

[7]

Targeted Worksheet 1 Answers

5.1 $P_v = \frac{x[1-(1+i)^{-n}]}{i}$

$600\,000 = \frac{8\,000\left[1-\left(1+\frac{0,1}{12}\right)^{-n}\right]}{\frac{0,1}{12}}$ ✓

$\frac{5}{8} = 1-\left(\frac{121}{120}\right)^{-n}$

$\left(\frac{121}{120}\right)^{-n} = \frac{3}{8}$ ✓

$-n = \log_{\left(\frac{121}{120}\right)}\left(\frac{3}{8}\right) = -118,19$ ✓

$n = 118,19$

He will need to make 119 payments ✓ (5)

5.2 $OB = \frac{x[1-(1+i)^{-n}]}{i}$

$OB = \frac{8\,000\left[1-\left(1+\frac{0,1}{12}\right)^{-0,19}\right]}{\frac{0,1}{12}}$ ✓

$OB =$ R1 512,51 ✓

$A = 1\,512,51\left(1+\frac{0,1}{12}\right)$

$A =$ R1 525,11 ✓ (3)

5.3 Extra amount $= 8\,000 - 5\,452,20 = 2\,547,80$ ✓

$A = \frac{2\,547,80\left[\left(1+\frac{0,1}{12}\right)^{96}-1\right]}{\frac{0,1}{12}}$ ✓

$A =$ R372 440,14 ✓ (4)

[12]

Total: [28]

Targeted Worksheet 2 Answers

Time: 45 minutes

Topic 2: Statistics

1.1.1 $\bar{x} = \frac{665}{10}$ ✓ = 66,5 ✓ (2)

1.1.2 SD = 13,78 ✓✓ (2)

1.1.3 $x > \bar{x} + 1SD$

66,5 + 1 × 13,78 ✓ = 80,28 ✓

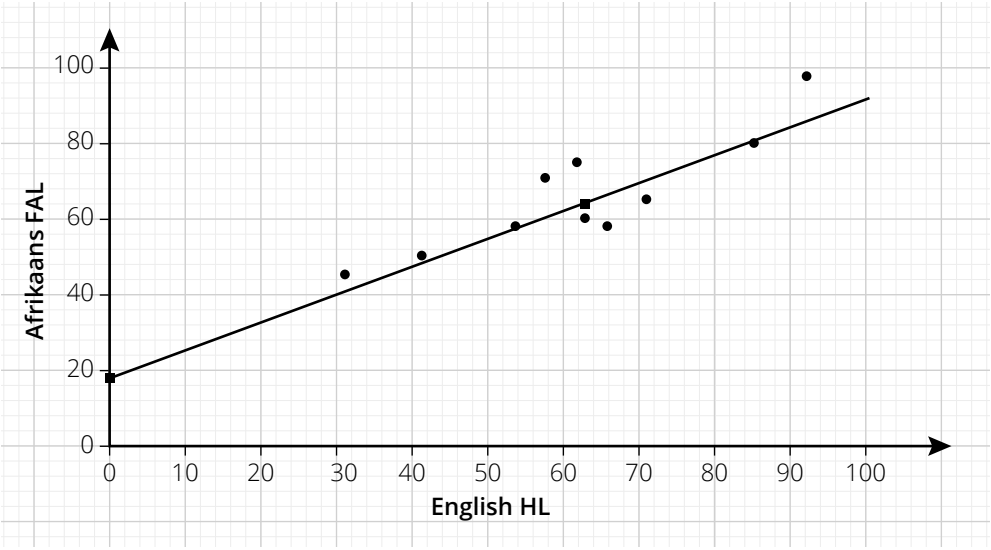
2 learners ✓ (3)

1.2.1 a = 58; ✓ $b = \frac{62 + 65}{2}$ = 63,5; ✓ c = 75 ✓ (3)

1.2.2 The data is skewed to the right. ✓ (1)

[11]

2.1



✓✓ For all points correctly plotted

✓ For plotting $(\bar{x}; \bar{y})$

✓For y-intercept = 18 (4)

2.2 A = 18,04 ✓

B = 0,77 ✓

y = 18,04 + 0,77x ✓ (3)

2.3 r = 0,88 ✓✓ (2)

2.4 Strong positive correlation✓ (1)

2.5 y = 18,04 + 0,77x

74 = 18,04 + 0,77x ✓

x = 73% ✓ (Accept 73% - 75%) (2)

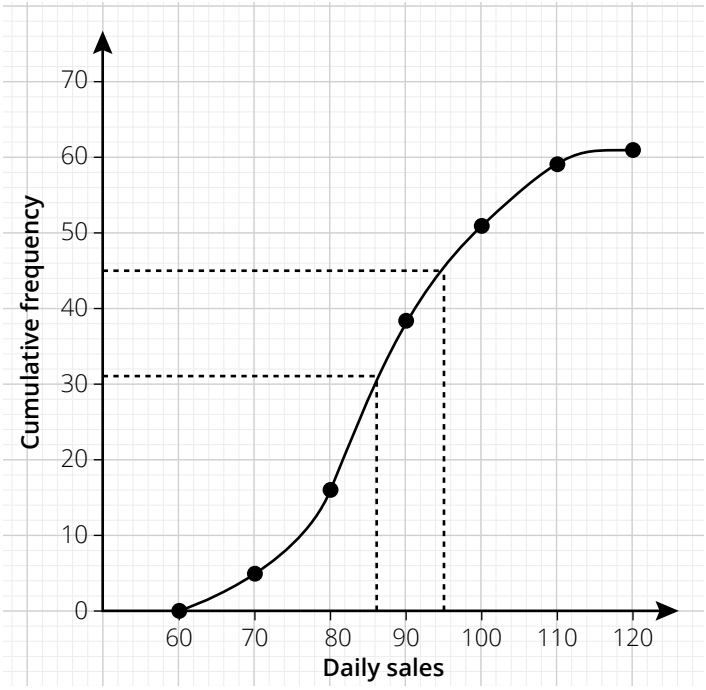
[12]

Targeted Worksheet 2 Answers

3.1 61 ✓ (1)

3.2 $a = 5$; ✓ $b = 22$; ✓ $c = 58$; ✓ $d = 3$ ✓ (4)

3.3



✓ grounding
✓ upper limits
✓ cumulative frequency values
✓ smooth curve (4)

3.4 median = 87 ± 1 ✓
Locating median from graph ✓ (2)

3.5 R96 to R120 ✓ (accept R94 to R120) (1)

[12]

Total: [35]

Targeted Worksheet 3 Answers

Time: 40 minutes

Topic 3: Counting principles and probability

1.1 $P(A \text{ and } B) = P(A) \times P(B)$ ✓
$= 0{,}4 \times 0{,}5$ ✓ $= 0{,}2$ ✓ (3)

1.2 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ ✓
$= 0{,}4 + 0{,}5 - 0{,}2$ ✓ $= 0{,}7$ ✓ (3)

1.3 $P(A' \text{ and } B') = 1 - P(A \text{ or } B)$ ✓
$= 1 - 0{,}7$ ✓ $= 0{,}3$ ✓ (3)

[9]

2.1.1 $a = 210$; ✓ $b = 240$; ✓ $c = 510$; ✓ $d = 690$ ✓ (4)

2.1.2 $\frac{270}{1\,200} = \frac{9}{40}$ ✓ (1)

2.1.3 $\frac{510}{1\,200} = \frac{17}{40}$ ✓ (1)

2.1.4 Children who positive in the sample = 240 ✓

$P(\text{children positive}) = \frac{240}{1\,200} = \frac{1}{5}$ ✓

Number of children positive in population $\frac{1}{5} \times 40\,000\,000$ ✓ $= 8\,000\,000$ ✓ (4)

2.2 A = Testing positive; B = Being an adult

$P(A) = \frac{17}{40}$; $P(B) = \frac{480}{1200} = \frac{2}{5}$; $P(A \text{ and } B) = \frac{9}{40}$ ✓

$P(A) \times P(B) = \frac{17}{40} \times \frac{2}{5} = \frac{17}{100}$ ✓

$P(A \text{ and } B) \neq P(A) \times P(B)$ ✓

Therefore, the events of testing positive and being an adult are NOT independent. ✓ (4)

[14]

3.1

20	20	9	9	20	20

20^2 ✓ $\times 9^2$ ✓ $\times 20^2$ ✓ $= 20^4 \times 9^2 = 12\,960\,000$ ✓ (4)

3.2 Even digits = {2; 4; 6; 8} ✓

20	20	4	3	20	20

$20^4 \times 4 \times 3$ ✓ $= 1\,920\,000$ ✓

Prob $= \frac{1\,920\,000}{12\,960\,000} = \frac{4}{27}$ ✓ (4)

[8]

Total: [31]

Exemplar Assessments

Exemplar Assessments

Time: 2 hours

Name: **Surname:**

Term 2: Control Test

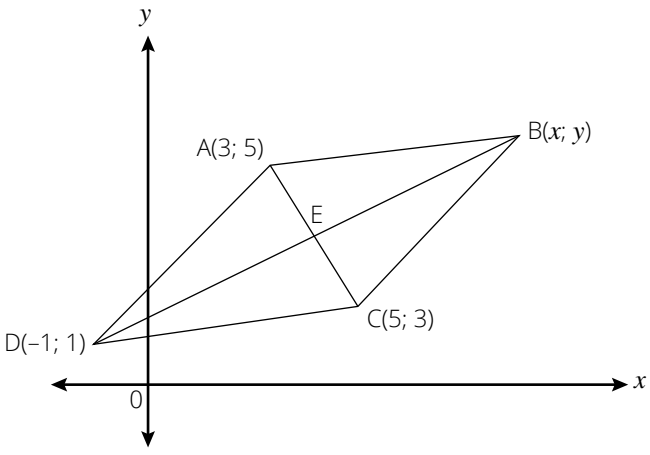
Instructions

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

Question 1

In the figure below, A(3; 5), B(x; y), C(5; 3) and D(-1; 1) are the vertices of a parallelogram ABCD. AC and BD, the diagonals of the parallelogram intersect at E.



1.1 Determine:

1.1.1 The coordinates of E. (2)

1.1.2 The coordinates of B. (3)

1.1.3 The coordinates of F, the midpoint of CD and hence the equation of the line passing through F parallel to AD. (5)

Exemplar Assessments

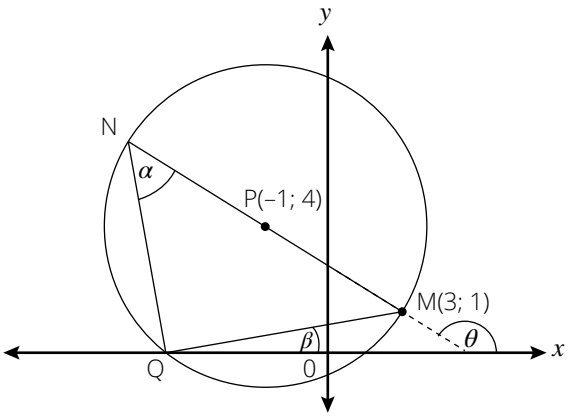
1.2 If the points G$\left(x+1;\frac{5}{2}\right)$, D(−1; 1) and E(4; 4) are collinear. Calculate the value of x. (4)

1.3 Determine, by calculation whether ABCD is a rhombus or not. Give a reason for your answer. (5)

[19]

Question 2

In the diagram below, M(3; 1), Q and N lie on the circumference of the circle with centre P(−1; 4) and form △MQN. NPM is a straight line.



2.1 Determine the equation of the circle. (4)

2.2 Give a reason why is $N\widehat{Q}M = 90°$. (1)

2.3 Show that the coordinates of Q are (−4; 0). (3)

2.4 Calculate the gradient of MN. (2)

2.5 Hence calculate the size of α. (5)

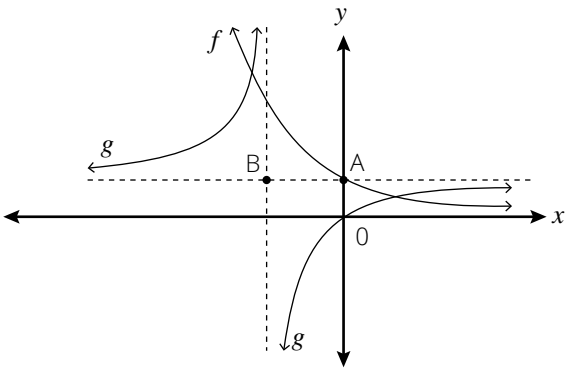
2.6 Determine the equation of the tangent to the circle at M. (5)

[20]

Question 3

The sketch below shows the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \frac{a}{x+p} + q$.

B is the point of intersection of the asymptotes of g. A is the y-intercept of f. The graph of g passes through the origin. AB is parallel to the x-axis.



3.1 Write down the equation of f^{-1} in the form $y = \ldots$ (2)

3.2 Write down the domain of f^{-1}. (1)

3.3 Calculate the value(s) of x if $4f(x + 1) = \sqrt{2}$. (3)

3.4 Determine the range of g. (2)

3.5 If $h(x) = x + 3$ is the equation of one of the axis of symmetry of g, determine the coordinates of B. (2)

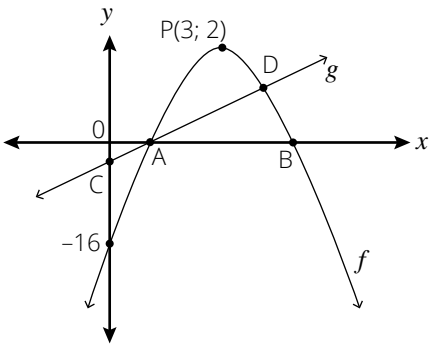
3.6 Hence determine the equation of g. (4)

3.7 For which value(s) of x is $g'(x) > 0$? (1)

[15]

Question 4

Sketched below are the graphs of: $g(x) = x - 2$ and $f(x) = a(x + p)^2 + q$



4.1 Determine the equation of f in the form $y = a(x + p)^2 + q$. (4)

4.2 Calculate the coordinates of A and D. (5)

4.3 Write down the values of x for which:

4.3.1 $f'(x) > 0$. (3)

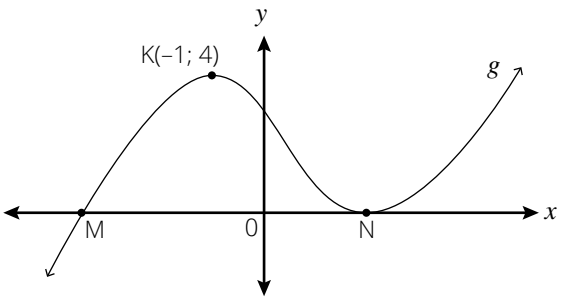
4.3.2 $f'(x) \cdot g(x) \leq 0$. (3)

[15]

Exemplar Assessments

Question 5

Sketched below is the graph of $g(x) = x^3 + px^2 + qx + 1$. K(−1; 4) is a turning point of g.
N is a local minimum of g.



5.1 Show that $p = -1$ and $q = -5$. (5)

5.2 Determine the coordinates of N. (3)

5.3 Calculate the x-value of the point of inflection of g. (2)

5.4 Determine the equation of the tangent to the graph of g at the point where $x = -2$. (3)

5.5 Use the graph to determine the value(s) of k for which the equation $g(x) = k$ will always have three real roots. (2)

5.6 For which value(s) of x is $x \cdot g'(x) > 0$? (2)

[17]

Question 6

6.1 Determine the nominal interest rate if the investment received r % p.a. compounded monthly whereas effectively it receives 8.3 % p.a. (3)

6.2 Mpho takes a loan of R400 000 at an interest rate of 11 % p.a. compounded monthly. Mpho must pay off the loan within 5 years with equal monthly repayments starting in one month's time. If he agrees to pays the loan over 5 years, his monthly repayments would be R8 696,97.

6.2.1 Determine the interest Mpho would pay if he were to sign this agreement. (1)

6.2.2 How long would it take Mpho to pay off the loan if he were to increase his monthly repayments by R303,03? (4)

6.2.3 What would be the value of Mpho's final payment with this new arrangement? (4)

6.2.4 How much interest will he save based on the decision he took in 6.2.2? (2)

[14]

Total: [100]

Exemplar Assessments

Time: 1 hour

Name: **Surname:**

Term 3: Control Test

Instructions

Read the following instructions carefully before answering the questions.

1. This question paper consists of 4 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

Question 1

Consider the following set of marks.

45; 53; 35; 66; 38; 40; 49; 22; 63; 57; 74; 42

Determine:

1.1 The median mark. (3)

1.2 The mean mark. (3)

1.3 The standard deviation. (2)

1.4 How many marks fall outside one standard deviation from the mean? (2)

1.5 The inter-quartile range. (3)

[13]

Exemplar Assessments

Question 2

The following data shows the marks obtained in the trial and final examinations by 12 learners.

Trial Exam	55	68	95	88	34	28	62	58	90	72	49	75
Final Exam	48	72	87	93	39	35	70	55	84	75	45	62

2.1 Draw a scatter plot for the marks. (3)

2.2 Determine the equation of the least squares regression line. (3)

2.3 Draw the least squares regression line. (2)

2.4 Estimate the final mark for a student who scored 83% in the trial examination. (2)

2.5 Determine the correlation coefficient for the data and interpret the result. (2)

[12]

Question 3

Consider the word LETTER.

3.1 How many different letter arrangements can be made out of this word? (2)

3.2 What is the probability that such a word will start with the letter E? (3)

3.3 What is the probability that such a word will start and end with the letter E? (3)

3.4 What is the probability that such a word will start and end with the same letter? (3)

[11]

Question 4

During summer in a certain city in South Africa, the probability of a sunny day is $\frac{4}{7}$ and the probability of a rainy day is $\frac{3}{7}$.

- If it is a sunny day, the probability that Vusi cycles to work is $\frac{7}{10}$, the probability that he drives to work is $\frac{1}{5}$ and the probability that he takes the train to work is $\frac{1}{10}$.
- If it is a rainy day, the probability that Vusi cycles to work is $\frac{1}{9}$, the probability that he drives to work is $\frac{5}{9}$ and the probability that he takes the train to work is $\frac{1}{3}$.

4.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes. (5)

4.2 For a day selected at random, what is the probability that:

4.2.1 It is rainy and Vusi will cycle to work? (2)

4.2.2 Vusi takes the train to work? (3)

4.3 If Vusi works 245 days in a year, on approximately how many occasions does he drive to work? (4)

[14]

Total: [50]

Exemplar Assessments

Time: 3 hours

Name: **Surname:**

Term 4: Examination Paper 1

Instructions

R**ead the following instructions carefully before answering the questions.**

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

Question 1

1.1 Solve for x:

1.1.1 $x^2 - 6x + 2 = 0$ (correct to 2 decimal places) (3)

1.1.2 $8^{3x-2} = 16^{2x+1}$ (3)

1.1.3 $x + 5 - \sqrt{x-5} = 12$ (5)

1.1.4 $x = \frac{a^2-1}{a-1}$ if $a = 889$. (2)

1.2 Given $f(x) = (x + 2)(x - 3) - 6$.

1.2.1 Solve for x if $f(x) = 0$. (4)

1.2.2 Determine the values of x for which $f'(x) \geq 0$. (2)

1.3 Solve the following equations simultaneously:

$2x + 3 - y = 0$ and $x^2 - 2x = y + 9$. (6)

[25]

Exemplar Assessments

Question 2

2.1 Consider the following arithmetic series: $-4; -1; 2; \ldots; 98$.

2.1.1 Derive a formula for the nth term of the sequence. (3)

2.1.2 Calculate the value of the 20th term. (2)

2.1.3 Calculate the sum of all the terms in the sequence that are divisible by 2. (5)

2.2 Given the pattern $1; \frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; \ldots$ assume the pattern continues consistently:

2.2.1 Write down the next two terms of the sequence. (2)

2.2.2 Determine the 33rd term of the sequence. (3)

2.2.3 Calculate the sum of the first 50 terms of the sequence. (5)

2.3 Consider the series: $\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n$

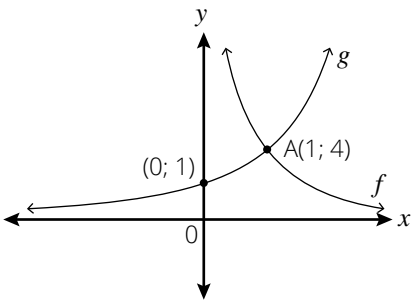
2.3.1 For which values of x will the series converge? (3)

2.3.2 If $x = \frac{1}{2}$, calculate the sum to infinity of the series. (3)

[26]

Question 3:

The diagram below shows the graph of $f(x) = \frac{a}{x}$ where $x > 0$ and $g(x) = 4^x$.



3.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

3.2 Sketch the graph of g^{-1}. (3)

3.3 Is g^{-1} a function? Give a reason for your answer. (2)

3.4 Write an equation for the asymptote of g. (1)

3.5 Determine the value of a in the equation of f. (2)

3.6 Determine the equation of $h(x)$, the graph formed when f is shifted 1 unit to the left and 2 units vertically down. (2)

[12]

Question 4

Consider the function $h(x) = \frac{x+1}{x-1}$.

4.1 Show that $\frac{x+1}{x-1} = \frac{2}{x-1} + 1$. (2)

4.2 Hence calculate the coordinates of:

4.2.1 The y-intercept of h. (1)

4.2.2 The x-intercept of h. (2)

4.3 Sketch the graph of h, showing clearly the asymptotes and intercepts with the axes. (3)

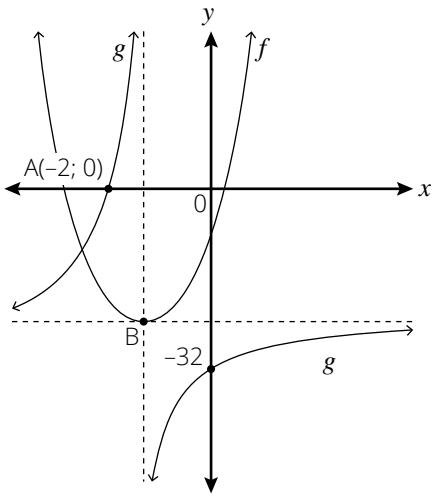
4.4 For which values of x is $h(x) > 0$? (2)

[10]

Question 5

Sketched below is the parabola f, with equation $f(x) = (x + 5)(x - 3)$ and a hyperbola g, with equation $y = \frac{-16}{(x+p)} - q$.

- A$(-2; 0)$ is the x-intercept of g.
- B, the turning point of f, lies at the point of intersection of the asymptotes of g.



5.1 Determine the coordinates of B. (4)

5.2 Write down the range of f. (1)

5.3 For which value(s) of x will $g \leq 0$? (2)

5.4 Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x - 4)$. (2)

5.5 Determine the values of p and q. (3)

5.6 Write down the value(s) of x for which $f(x) \cdot g'(x) \geq 0$. (2)

[14]

Exemplar Assessments

Question 6

6.1 Exactly five years ago Mpume bought a new car for R 145 000. The current book value of this car is R 72 500. If the car depreciates at a fixed annual rate according to the reducing balance method. Calculate the rate of depreciation. (3)

6.2 Samuel took out a home loan for R500 000 at an interest rate of 12% p.a. compounded monthly. He plans to repay this over 20 years and his first payment is made one month after the loan is granted.

6.2.1 Calculate the value of Samuel's instalment. (4)

6.2.2 Melissa took out a loan for the same amount and the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

6.2.3 Who pays more interest, Samuel or Melissa? Justify your answer. (2)

[13]

Question 7

7.1 Determine $f'(x)$ from first principles given $f(x) = 3x^2 + 2x$. (5)

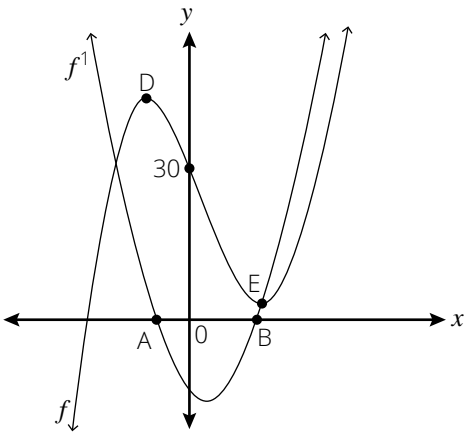
7.2 Determine $\frac{dy}{dx}$ if $y = (1 + \sqrt{x})^2$. (3)

7.3 Calculate $D_x\left[\frac{x^3 + 8}{3x + 6}\right]$. (4)

[12]

Question 8

The graph of $f'(x) = 3x^2 - 6x - 9$ meets the x-axis at A and B as shown below. D and E are the stationary points of the cubic graph $f(x) = ax^3 + bx^2 + cx + d$ and it cuts the y-axis at (0; 30).



Determine the:

8.1 x-coodinates of A and B. (4)

8.2 equation of the cubic graph $f(x)$. (5)

8.3 equation of the tangent to the graph of f at the point where $x = -2$. (4)

8.4 the values of x for which the graph of f is concave upwards. (2)

[15]

Question 9

The flow Q m^3/s of a river t hours after midnight is monitored after a storm, and is given by the equation:

$Q(t) = t^3 - 8t^2 + 14t + 10; 0 \leq t \leq 5$

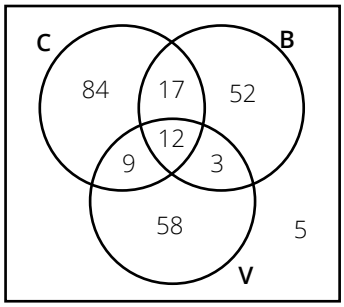
9.1 Determine the rate at which the river is rising 1 hour after midnight. (2)

9.2 Determine the maximum flow of water and the time when this occurs. (6)

[8]

Question 10

A survey was carried out with 240 customers who bought food from a fast- food outlet on a particular day. The outlet sells cheeseburgers (C), bacon burgers (B), and vegetarian burgers (V). The Venn diagram below shows the number of customers who bought different types of burgers on the day.



10.1 How many customers did NOT buy burgers on the day? (1)

10.2 Are events C and B mutually exclusive? Give a reason for your answer. (2)

10.3 A customer from this group is selected at random, determine the probability that he/she:

10.3.1 bought only a vegetarian burger. (1)

10.3.2 bought a cheeseburger and bacon burger. (1)

10.3.3 did not buy a cheeseburger. (1)

10.3.4 bought a bacon or vegetarian burger. (1)

[7]

Question 11

Paballo has a bag containing 80 marbles that are either green, yellow or red in colour. $\frac{3}{5}$ of the marbles are green and 10% of the marbles are yellow. Paballo picks two marbles out of the bag, one at a time and without replacing the first one.

11.1 How many red marbles are in the bag? (2)

11.2 Draw a tree diagram to represent the above data. (3)

11.3 What is the probability that Paballo will choose a GREEN and YELLOW marble? (3)

[8]

Total: [150]

Exemplar Assessments

Time: 3 hours

Name: **Surname:**

Term 4: Examination Paper 2

Instructions

R**ead the following instructions carefully before answering the questions.**

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

Question 1

The time taken, in seconds, to complete a task and the number of errors made on the task were recorded for a sample of 10 secondary school learners. The data is shown in the table below.

Time taken to complete tasks (in seconds)	23	21	19	9	15	22	17	14	21	18
Number of errors made	2	4	5	9	7	3	7	8	3	5

1.1 Draw a scatter plot of this data. (3)

1.2 What is the influence of more time taken to complete the task on the number of errors made? (1)

1.3 Determine the equation of the least squares regression line. (3)

1.4 Calculate the correlation coefficient. (2)

1.5 Predict the number of errors that will be made by a learner who takes 13 seconds to complete this task. (2)

1.6 Comment on the strength of the relationship between the variables. (1)

Question 2

In the grid below a, b, c, d, e, f and g represent values in a data set written in an increasing order. No value in the data set is repeated.

a	b	c	d	e	f	g

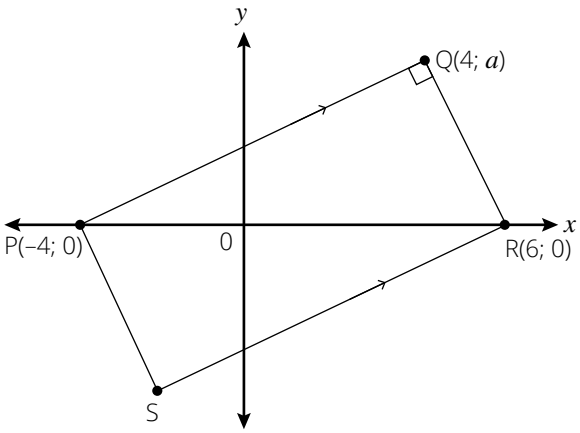
2.1 Determine the value of a, b, c, d, e, f and g if: (7)

- The maximum value is 42.
- The range is 35.
- The median is 23.
- The difference between the median and the upper quartile is 14.
- The interquartile range is 22.
- e = 2c.
- The mean is 25.

[7]

Question 3

In the diagram below, PQRS is a rectangle with vertices P(-4; 0), Q(4; a), R(6; 0) and S. Q lies in the first quadrant.



3.1 Show that $a = 4$. (4)

3.2 Determine the equation of the straight line passing through the points S and R in the form $y = mx + c$. (4)

3.3 Calculate the coordinates of S. (4)

3.4 Calculate the length of PR. (2)

3.5 Determine the equation of the circle that has diameter PR. Give the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

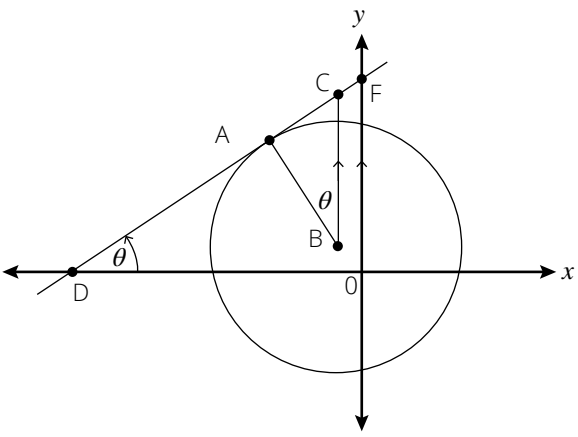
3.6 Show that Q is a point on the circle in Question 3.5. (2)

[19]

Exemplar Assessments

Question 4

The circle with centre B(−1; 1) and radius $\sqrt{20}$ is shown. BC is parallel to the y-axis and CB = 5. The tangent to the circle at A passes through C. $A\widehat{B}C = A\widehat{D}O = \theta$.



4.1 Determine the coordinates of C. (2)

4.2 Calculate the length of AC. (5)

4.3 Determine the angle of inclination of the line DAC. (3)

4.4 Show that the gradient if AB is −2. (2)

4.5 Calculate the ratio of the area of △ABC to the area of △ODF. Simplify your answer. (6)

[19]

Question 5

5.1 Given: $\sin\alpha = \frac{3}{5}$ and $90° < \alpha < 270°$. With the aid of a sketch diagram and without the use of a calculator, determine:

5.1.1 $\tan\alpha$ (3)

5.1.2 $\sin(90° - \alpha)$ (2)

5.1.3 $\cos 2\alpha$ (4)

5.2 Simplify the following (without the use of a calculator): $\frac{\sin 150° \cdot \tan 225°}{\sin 30° \cdot \sin 420°}$ (6)

5.3 Prove the following identity: $\frac{1 + \sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$ (5)

5.4 Hence, or otherwise, calculate the following without the use of a calculator, giving your answer in simplified surd form: $\frac{\cos 15° + \sin 15°}{\cos 15° - \sin 15°}$ (3)

[23]

Question 6

6.1 Use the system of axes provided on the diagram sheet to sketch the following graphs:
$f(x) = \sin 2x$ and $g(x) = \cos(-x - 60°)$ if $-180° \leq x \leq 180°$
Clearly indicate all turning points as well as intercepts with the axes. (6)

6.2 Write down the period of g. (1)

6.3 Solve for x if $\sin 2x - \cos(-x - 60°) = 0$ for $-180° \leq x \leq 180°$. Show all your working. (7)

6.4 Use the graphs to determine the value(s) of x for which: $g(x) > f(x)$ for $-180° \leq x \leq 90°$. (2)

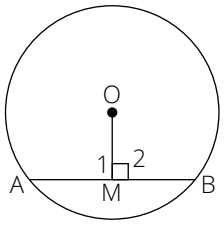
6.5 Describe the transformation that maps g to $h(x) = \cos(2x + 40°)$. (2)

[18]

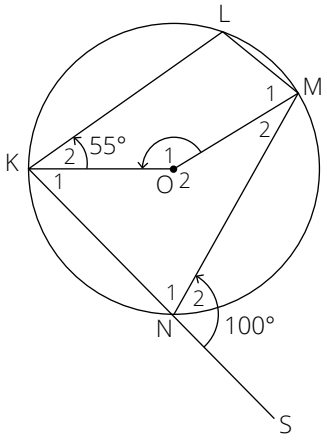
Question 7

7.1 Complete the statement:
A line drawn from the centre of a circle perpendicular to a chord. (1)

7.2 Prove that OM ⊥ AB, then AM = MB. (8)



7.3 In the diagram, O is the centre of the circle KLMN; KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55°$ and $\hat{N}_2 = 100°$.



Determine with reasons, the size of:

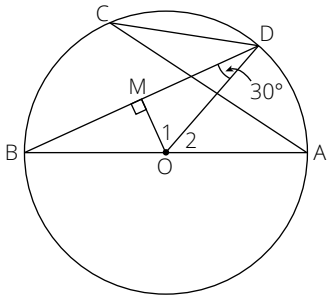
7.3.1 $\hat{L}$ (2)

7.3.2 $\hat{O}_1$ (4)

7.3.3 $\hat{M}_1$ (2)

Exemplar Assessments

7.4 In the figure below, $B\widehat{D}O = 30°$. AB is a diameter of the circle with centre O. A, B, C and D are points on the circle. OM ⊥ BD.



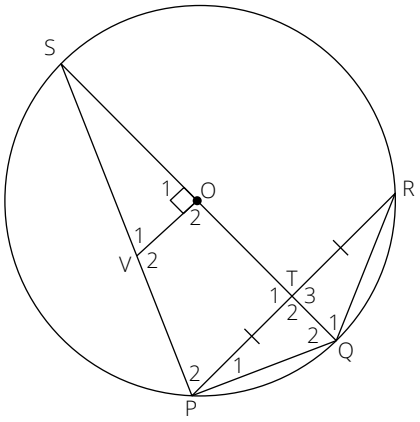
7.4.1 Write down, with reasons, TWO more angles that are equal to 30°. (4)

7.4.2 Calculate the size of $\widehat{O}_2$. (2)

[23]

Question 8

In the diagram, O is the centre of the circle PQRS and QOS is a diameter. T is the midpoint of PR, lies on QOS. V is a point on PS such that VO ⊥ SQ.



8.1 Give a reason why:

8.1.1 $S\widehat{P}Q = 90°$ (1)

8.1.2 PVOQ is cyclic quadrilateral (1)

8.2 Prove that:

8.2.1 PR ∥ VO (4)

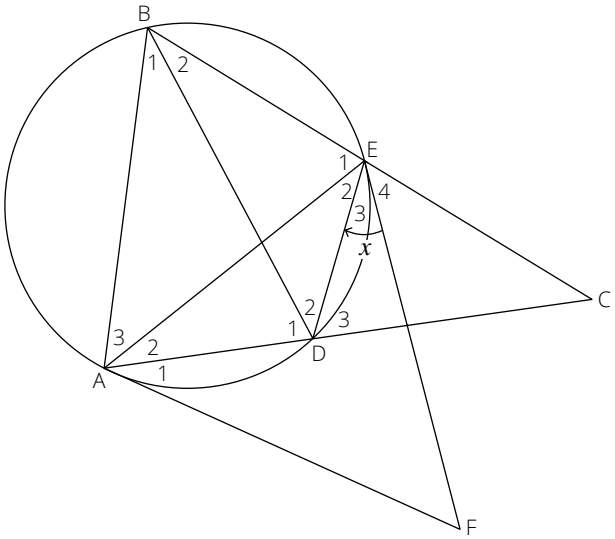
8.2.2 $\widehat{V}_1 = \widehat{Q}_1$ (4)

8.3 If it is given that $TQ = \frac{2}{5}r$, wher r is the radius of the circle, determine with reasons, the ratio of $\frac{SV}{SP}$. (4)

[14]

Question 9

In the diagram, FA and FE are tangents to a circle at A and E respectively. B and D are points on the circle such that BE produced and AD produced meet at C and BC = AC. BD, AE and ED are drawn. $\hat{E}_3 = x$.



9.1 Give a reason why $\hat{E}_3 = \hat{A}_2 = x$. (1)

9.2 Prove that:

9.2.1 AB ∥ DE (5)

9.2.2 $\hat{E}_4 = \hat{A}_1$ (6)

9.2.3 CEAF is a cyclic quadrilateral. (2)

[14]

Total: [150]

Exemplar Assessments Memorandum

Time: 2 hours

Term 2: Control Test

- This memorandum serves as a guide for the allocation of marks.
- Any acceptable / valid method can be used.
- Do not forget to mark with CA in mind. (Corrective Accuracy)

1.1.1 E(4; 4) ✓✓ (2)

1.1.2 $\frac{x-1}{2} = 4$ ✓ and $\frac{y+1}{2} = 4$ ✓

B(9; 7) ✓ (3)

1.1.3 F(2; 2) ✓✓

$m_{AD} = \frac{5-1}{3+1} = 1$ ✓

$y - 2 = 1(x - 2)$ ✓

$y = x$ ✓ (5)

1.2 $\frac{2,5-1}{x+2} = \frac{4-1}{4+1}$ ✓✓

$7,5 = 3x + 6$ ✓

$x = 0,5$ ✓ (4)

1.3 $m_{AC} = \frac{5-3}{3-5} = -1$ ✓

$m_{BD} = \frac{7-1}{9+1} = \frac{6}{10}$ ✓

$m_{AC} \times m_{BD} = -\frac{6}{10} \neq -1$ ✓

The diagonal are not perpendicular to each other. ✓ Therefore, the parallelogram is not a rhombus. ✓ (5)

[19]

2.1 $r^2 = (3 + 1)^2 + (1 - 4)^2 = 25$ ✓✓

$(x + 1)^2 + (y - 4)^2 = 25$ ✓✓ (4)

2.2 Angle in a semi-circle. ✓ (1)

2.3 $(x + 1)^2 + (y - 4)^2 = 25$

At Q: $y = 0$

$(x + 1)^2 + (0 - 4)^2 = 25$ ✓

$\sqrt{(x+1)^2} = \sqrt{9}$

$(x + 1) = \pm 3$ ✓

$x = -4$ or $x = 2$ ✓

Q$(-4; 0)$ (3)

2.4 $m_{MN} = \frac{4-1}{-1-3}$ ✓ $= -\frac{3}{4}$ ✓ (2)

2.5 $\theta = \tan^{-1}\left(-\frac{3}{4}\right) = -36{,}87 + 180 = 143{,}13°$ ✓

$m_{MQ} = \frac{1-0}{3+4} = \frac{1}{7}$ ✓

$\beta = \tan^{-1}\left(\frac{1}{7}\right) = 8{,}13°$ ✓

$\alpha + 90° = \theta - \beta$ ✓

$\alpha = 143{,}13° - 8{,}13° - 90°$

$\alpha = 45°$ ✓ (5)

2.6 $m_{MN} \times m_{tan} = -1$ ✓

$m_{tan} = \frac{4}{3}$ ✓

$y - 1 = \frac{4}{3}(x - 3)$ ✓✓

$y = \frac{4}{3}x - 3$ ✓✓ (5)

[20]

3.1 $y = \left(\frac{1}{2}\right)^x$

$x = \left(\frac{1}{2}\right)^y$ ✓

$y = \log_{\left(\frac{1}{2}\right)} x$ ✓ (2)

3.2 $x > 0$ ✓ (1)

3.3 $4\left[\left(\frac{1}{2}\right)^{x+1}\right] = \sqrt{2}$ ✓

$(2)^{-(x+1)} = \frac{2^{\frac{1}{2}}}{2^2} = 2^{-\frac{3}{2}}$

$-(x+1) = -1{,}5$ ✓

$x = 0{,}5$ ✓ (3)

3.4 A(0; 1) ✓

$y \in \mathbb{R}; y \neq 1$ ✓ (2)

3.5 $y = (x + p) + 1 = x + 3$

$p + 1 = 3$

$p = 2$ ✓

B(−2; 1) ✓ (2)

3.6 $y = \frac{a}{x+2} + 1$ ✓

$0 = \frac{a}{0+2} + 1$ ✓

$a = -2$ ✓

$y = \frac{-2}{x+2} + 1$ ✓ (4)

3.7 $x \in \mathbb{R}; x \neq -2$ ✓ (1)

[15]

Exemplar Assessments Memorandum

4.1 $y = a(x-3)^2 + 2$ ✓

$-16 = a(0-3)^2 + 2$ ✓

$-18 = 9a$

$a = -2$ ✓

$y = -2(x-3)^2 + 2$ ✓ (4)

4.2 $-2(x-3)^2 + 2 = x - 2$ ✓

$-2(x^2 - 6x + 9) = x - 4$

$-2x^2 + 12x - 18 = x - 4$

$2x^2 - 11x + 14 = 0$ ✓

$(x-2)(2x-7) = 0$ ✓

$x = 2$ or $x = 3{,}5$

A(2; 0) ✓ and D(3,5; 1,5) ✓ (5)

4.3.1 $x < 3$ ✓✓✓ (3)

4.3.2 $x \leq 2$ or $x \geq 3$ ✓✓✓ (3)

[15]

5.1 Show that $p = -1$ and $q = -5$

$g'(x) = 3x^2 + 2px + q$ ✓

$3(-1)^2 + 2p(-1) + q = 0$ ✓

$2p - q = 3$ ①

$g(-1) = (-1)^3 + p(-1)^2 + q(-1) + 1 = 4$ ✓

$p - q = 4$ ____ ②

$p = (4 + q)$ ✓

$2(4 + q) - q = 3$ ✓

$8 + 2q - q = 3$

$q = -5; p = -1$ (5)

5.2 $3x^2 + 2px + q = 0$

$3x^2 - 2x - 5 = 0$ ✓

$(x+1)(3x-5) = 0$ ✓

$x = -1; x = \frac{5}{3}$

$N\left(\frac{5}{3}; 0\right)$ ✓ (3)

5.3 $x = \frac{-1 + \frac{5}{3}}{2}$ ✓ or $g''(x) = 6x - 2 = 0$

$x = \frac{1}{3}$ ✓ (2)

5.4 $m = g'(-2)$

$m = 3(-2)^2 - 2(-2) - 5 = 11$ ✓

$y = g(-2) = -1$ ✓

$y + 1 = 11(x + 2)$

$y = 11x + 21$ ✓ (3)

5.5 $0 < k < 4$ ✓✓ (2)

5.6 $-1 < x < 0$ or $x > \frac{5}{3}$ ✓✓ (2)

[17]

6.1 $1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{m}\right)^m$

$1 + 0{,}083 = \left(1 + \frac{r}{12}\right)^{12}$ ✓

$\sqrt[12]{1{,}083} = 1 + \frac{r}{12}$ ✓

$r = 0{,}08$ ✓

$r = 8\%$ (3)

6.2.1 $8\,696{,}97 \times 60 - 400\,000 = 121\,818{,}20$ ✓ (1)

6.2.2 $P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$

$i = \frac{0{,}11}{12}$; $n = 30$

$400\,000 = \frac{9\,000\left[1 - \left(1 + \frac{0{,}11}{12}\right)^{-n}\right]}{\frac{0{,}11}{12}}$ ✓

$\frac{11}{27} = 1 - \left(\frac{1\,211}{1\,200}\right)^{-n}$ ✓

$\left(\frac{1\,211}{1\,200}\right)^{-n} = \frac{16}{27}$

$-n = \log_{\left(\frac{1\,211}{1\,200}\right)}\left(\frac{16}{27}\right)$ ✓

$n = 57{,}3428$

It would take him 58 months ✓ (4)

6.2.3 OB $= \frac{9\,000\left[1 - \left(1 + \frac{0{,}11}{12}\right)^{-0{,}3428}\right]}{\frac{0{,}11}{12}}$ ✓ $= 3\,066{,}35$ ✓

$A = 3\,066{,}35\left(1 + \frac{0{,}11}{12}\right)$ ✓ $= 3\,094{,}46$ ✓ (4)

6.2.4 Total payment after $= 9\,000 \times 57 + 3\,094{,}46 = 516\,094{,}46$ ✓

6.2.5 Total payment before $= 8\,696{,}97 \times 60 = 521\,818{,}20$

6.2.6 Saving $= 521\,818{,}20 - 516\,094{,}46 = 5\,723{,}74$ ✓ (2)

[14]

Total: [100]

Exemplar Assessments Memorandum

Time: 1 hour

Term 3: Control Test

- This memorandum serves as a guide for the allocation of marks.
- Any acceptable / valid method can be used.
- Do not forget to mark with CA in mind. (Corrective Accuracy)

1.1 22 35 38 40 42 45 49 53 57 63 66 74 ✓

Median $= \frac{45 + 49}{2}$ ✓ $= 47$ ✓ (3)

1.2 Mean $= \frac{584}{12}$ ✓✓ $= 48{,}67$ ✓ (3)

1.3 Standard deviation = 14,09 ✓✓ (2)

1.4 $(48{,}67 - 14{,}09) < x < 48{,}67 + 14{,}09$

$34{,}58 < x < 62{,}76$ ✓

$x \in [22; 63; 66; 74]$

4 learners ✓ (2)

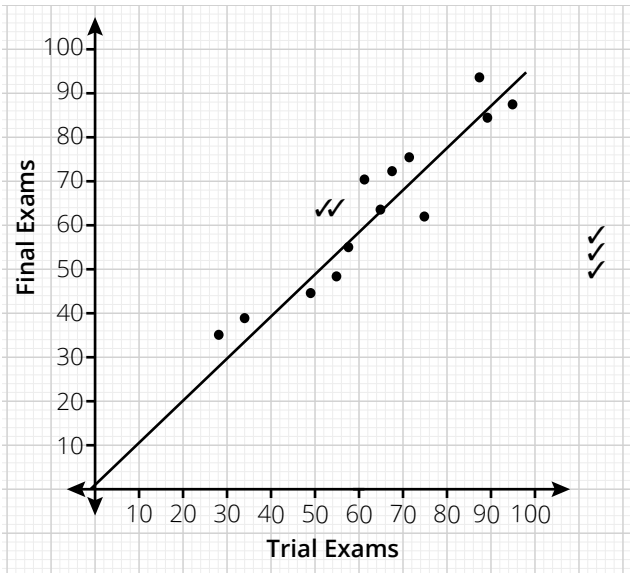
1.5 Upper quartile $= \frac{57 + 63}{2} = 60$ ✓

Lower quartile $= \frac{38 + 40}{2} = 39$ ✓

1.6 $IQR = 60 - 39 = 21$ ✓ (3)

[13]

2.1 (3)



2.2 A = 2,156 ✓

B = 0,817 ✓

$y = 2{,}156 + 0{,}817x$ ✓ (3)

2.3 See diagram (2)

2.4 $y = 2{,}156 + 0{,}817(83)$ ✓ = 70% ✓ (2)

2.5 $r = 0{,}95$ ✓

It means there is a very strong positive correlation between the trial exam mark and the final exam mark. ✓ (2)

[12]

3.1 $\frac{6!}{2! \times 2!}$ ✓ $= 180$ ✓ (2)

3.2 $n(S) = \frac{6!}{2! \times 2!} = 180$ ✓

$n(E) = \frac{5!}{2!} = 60$ ✓

$P(E) = \frac{60}{180} = \frac{1}{3}$ ✓ (3)

3.3 $n(S) = \frac{6!}{2! \times 2!} = 180$ ✓

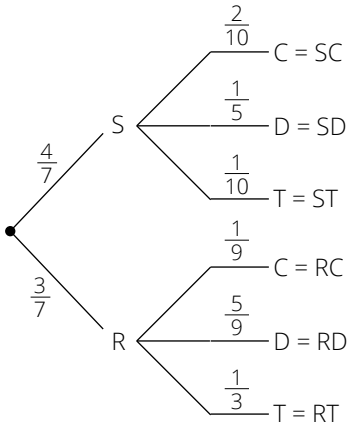
$n(E) = \frac{4!}{2!} = 12$ ✓

$P(E) = \frac{12}{180} = \frac{1}{15}$ ✓ (3)

3.4 P(start and end with E) + P(start and end with T) ✓$= \frac{1}{15} + \frac{1}{15}$ ✓$= \frac{2}{15}$ ✓ (3)

[11]

4.1 (5)



✓1st break
✓2nd break
✓3rd break
✓Correct probabilities
✓All outcomes

4.2.1 It is rainy and Vusi will cycle to work $= \frac{3}{7} \times \frac{1}{9}$ ✓ $= \frac{3}{63} = \frac{1}{21}$ ✓ (2)

4.2.2 Either it is sunny and Vusi takes a train or it rainy and Vusi takes a train:

P(ST or RT) $= \frac{4}{7} \times \frac{1}{10}$ ✓ $+ \frac{3}{7} \times \frac{1}{3}$ ✓ $= \frac{1}{5}$ ✓ (3)

4.3 P(driving to work) = P(SD or RD)

P(SD) + P(RD) $= \frac{4}{7} \times \frac{1}{5} + \frac{3}{7} \times \frac{5}{9}$ ✓ $= \frac{37}{105}$ ✓

Number of days $= \frac{37}{105} \times 245$ ✓ $= 8$ (Approximately) ✓ (4)

[14]

Total: [50]

Exemplar Assessments Memorandum

Time: 3 hours

Term 4: Examination Paper 1

- This memorandum serves as a guide for the allocation of marks.
- Any acceptable / valid method can be used.
- Do not forget to mark with CA in mind. (Corrective Accuracy)

1.1.1 $x = -\frac{(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$ ✓

$x = 5{,}65$ ✓ or $x = 0{,}35$ ✓ (3)

1.1.2 $(2^3)^{(3x-2)} = (2^4)^{(2x+1)}$

$(2)^{3(3x-2)} = (2)^{4(2x+1)}$ ✓

$9x - 6 = 8x + 4$ ✓

$x = 10$ ✓ (3)

1.1.3 $x + 5 - \sqrt{x-5} = 12$

$x - 7 = \sqrt{x-5}$ ✓

$(x-7)^2 = x - 5$ ✓

$x^2 - 15x + 54 = 0$ ✓

$(x-6)(x-9) = 0$ ✓

$x = 6$ or $x \neq 9$ ✓ (5)

1.1.4 $x = \frac{a^2-1}{a-1}$ if $a = 889$

$x = \frac{(a-1)(a+1)}{a-1}$ ✓

$x = a + 1$

$x = 889 + 1 = 890$ ✓ (2)

1.2.1 $(x+2)(x-3) - 6$

$x^2 - x - 12 = 0$ ✓

$(x-4)(x+3) = 0$ ✓

$x = 4$ ✓ or $x = -3$ ✓ (4)

1.2.2 $2x - 1 \geq 0$ ✓

$x \geq \frac{1}{2}$ ✓ (2)

1.3 $x^2 - 2x - 9 = 2x + 3$ ✓

$x^2 - 4x - 12 = 0$ ✓

$(x-6)(x+2) = 0$ ✓

$x = 6$ or $x = -2$ ✓

If $x = 6$; $y = 15$ ✓

If $x = -2$; $y = -1$ ✓ (6)

[25]

Exemplar Assessments Memorandum

2.1.1 $T_n = a + d(n-1)$ ✓

$T_n = -4 + 3(n-1)$ ✓ $= 3n - 7$ ✓ (3)

2.1.2 $T_{20} = 3(20) - 7$ ✓ $= 53$ ✓ (2)

2.1.3 $-4; 2; 8; \ldots; 98$ ✓

$T_n = a + d(n-1)$

$98 = -4 + 6(n-1)$ ✓

$98 = -4 + 6n - 6$

$108 = 6n$

$n = 18$ ✓

$s_{18} = \frac{18}{2}(-4 + 98)$ ✓

$s_{18} = 846$ ✓ (5)

2.2.1 $10; \frac{1}{16}$ ✓✓ (2)

2.2.2 T_{33} is a whole number. Whole numbers are in odd positions.

T_{33} is in 17th position of 1; 4; 7 ✓

$T_n = a + d(n-1)$

$T_{17} = 1 + 3(16)$ ✓ $= 49$ ✓ (3)

2.2.3 A.S: $S_{25} = \frac{25}{2}[2(1) + 3(25-1)]$ ✓ $= 925$ ✓

G.S: $S_{25} = \frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^{25}\right]}{1 - \frac{1}{2}}$ ✓ $= 1$ ✓

$S_{50} = 925 + 1 = 926$ ✓ (5)

2.3.1 $\sum_{n=1}^{\infty}\left(\frac{x}{2}\right)^n = \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \ldots$

$r = \frac{x}{2}$ ✓

Series converges for $-1 < r < 1$ ✓

$-1 < \frac{x}{2} < 1$

$-2 < x < 2$ ✓ (3)

2.3.2 $a = \frac{1}{4}; r = \frac{1}{4}$ ✓

$S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{4}}$ ✓

$S_\infty = \frac{1}{3}$ ✓ (3)

[26]

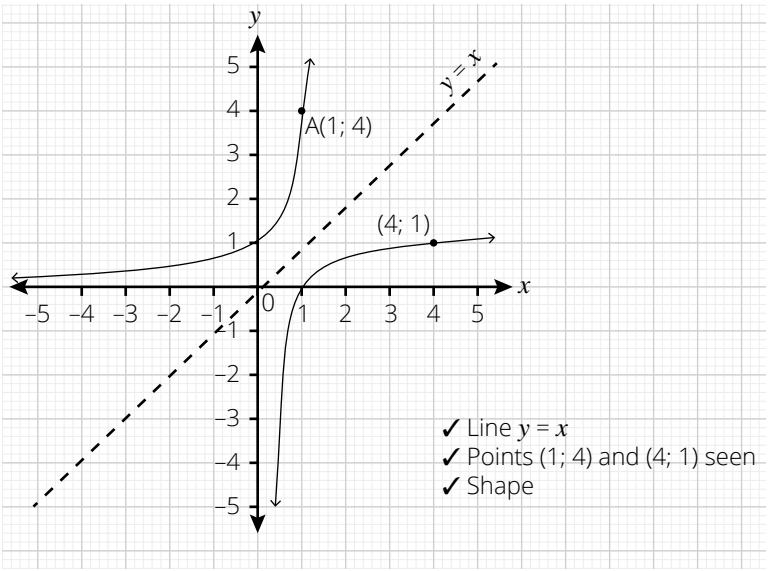
3.1 $y = 4^x$

$x = 4^y$ ✓

$y = \log_4 x$ ✓ (2)

Exemplar Assessments Memorandum

3.2 (3)



3.3 Yes. ✓ It passes the vertical line test. ✓ (2)

3.4 $y = 0$ ✓ (1)

3.5 $y = \frac{a}{x}$

At A(1; 4): $4 = \frac{a}{1} = a$ ✓✓ (2)

3.6 $h(x) = \frac{4}{x+1} - 2$ ✓✓ (2)

[12]

4.1 $\frac{x+1}{x-1} = \frac{2+(x-1)}{x-1}$ ✓ $= \frac{2}{x-1} + 1$ ✓ (2)

4.2.1 $y = \frac{2}{0-1} + 1 = -1$

$(0; -1)$ ✓ (1)

4.2.2 $y = \frac{2}{x-1} + 1$

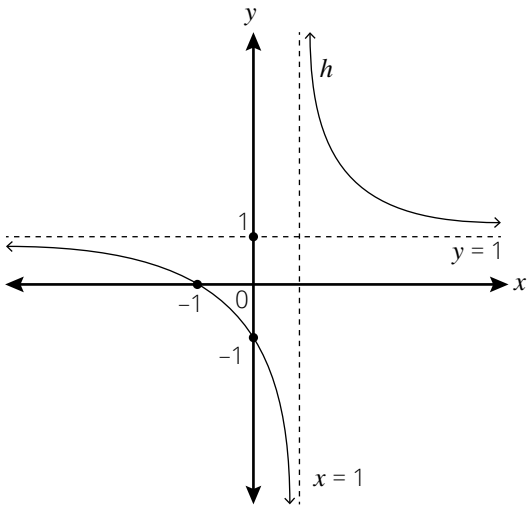
$0 = \frac{2}{x-1} + 1$ ✓

$-1 = \frac{2}{x-1}$

$x = -1$ ✓

$(-1; 0)$ (2)

4.3 (3)



4.4 $x < -1$ ✓ and $x > 1$ ✓ (2)

[10]

5.1 $(x + 5)(x - 3) = 0$ ✓

$x = -5$ or $x = 3$ ✓

$x_B = -\frac{5+3}{2} = -1$ ✓

$y_B = f(-1) = -16$ ✓ (4)

5.2 $y \geq -16$ ✓ (1)

5.3 $x \leq -2$ ✓ and $x > -1$ ✓ (2)

5.4 $x = -1 + 4 = 3$ ✓✓ (2)

5.5 $y = \frac{-16}{x+1} - 16$ ✓

$p = 1$; ✓ $q = 16$ ✓ (3)

5.6 $x \leq -5$ ✓ and $x \geq 3$ ✓ (2)

[14]

6.1 $A = P(1 - i)^n$ ✓

$72\,000 = 145\,000\,(1 - i)^5$ ✓

$i = 12{,}94\%$ ✓ (3)

6.2.1 $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ ✓

$500\,000 = \frac{x\left[1-\left(1+\frac{0{,}12}{12}\right)^{-240}\right]}{\frac{0{,}12}{12}}$

$x = 5\,505{,}43$ ✓ (4)

6.2.2 $500\,000 = \frac{6\,000\left[1-\left(1+\frac{0{,}12}{12}\right)^{-n}\right]}{\frac{0{,}12}{12}}$ ✓

$\left(\frac{101}{100}\right)^{-n} = \frac{1}{6}$ ✓

$-n = \log_{\left(\frac{101}{100}\right)}\frac{1}{6}$ ✓

$n = 180{,}07$ months ✓ (4)

6.2.3 Melissa: $6\,000 \times 180{,}07 = 1\,080\,420$

Samuel: $5\,505{,}43 \times 240 = 1\,321\,303{,}20$ ✓

Samuel pays more interest. ✓ (2)

[13]

Exemplar Assessments Memorandum

7.1 $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f(x) = 3x^2 + 2x$

$f(x+h) = 3(x+h)^2 + 2(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$ ✓

$f'(x) = \lim_{h\to 0}\frac{[3x^2 + 6xh + 3h^2 + 2x + 2h] - [3x^2 + 2x]}{h}$ ✓

$f'(x) = \lim_{h\to 0}\frac{h(6x + 3h + 2)}{h}$ ✓

$f'(x) = \lim_{h\to 0}(6x + 3h + 2)$ ✓ $= 6x + 2$ ✓ (5)

7.2 $y = 1 + 2x^{\frac{1}{2}} + x$ ✓

$\frac{dy}{dx} = x^{-\frac{1}{2}}$ ✓ $+1$ ✓ (3)

7.3 $D_x\left[\frac{x^3+8}{3x+6}\right] = D_x\left[\frac{(x+2)(x^2-2x+4)}{3(x+2)}\right]$ ✓ $= D_x\left[\frac{(x^2-2x+4)}{3}\right]$ ✓

$D_x\left[\frac{x^3+8}{3x+6}\right] = \frac{1}{3}(2x-2) = \frac{2x}{3}$ ✓ $-\frac{2}{3}$ ✓ (4)

[12]

8.1 $3x^2 - 6x - 9 = 0$ ✓

$(x-3)(x+1) = 0$ ✓

$x_B = 3$; ✓ $x_A = -1$ ✓ (4)

8.2 y-intercept: $f(0) = d = 30$ ✓

$f'(x) = 3ax^2 + 2bx + c$ ✓

$3ax^2 + 2bx + c = 3x^2 - 6x - 9$

$a = 1$; ✓ $b = -3$; ✓ $c = -9$ ✓

$f(x) = x^3 - 3x^2 - 9x + 30$ (5)

8.3 $m = f'(-2) = 15$ ✓

$f(-2) = 28$ ✓

$y - 28 = 15(x+2)$ ✓

$y = 15x + 58$ ✓ (4)

8.4 $f''(x) > 0$ ✓

$6x - 6 > 0$

$x > 1$ ✓ (2)

[15]

Exemplar Assessments Memorandum

9.1 $Q(1) = 17$ ✓✓ (2)

9.2 $Q'(t) = 0$ ✓

$3t^2 - 16t + 14 = 0$ ✓

$t = -\frac{(-16) \pm \sqrt{(-16)^2 - 4(3)(14)}}{2(3)}$ ✓

$t = 4{,}23$ or $t = 1{,}10$ ✓

$Q(4{,}23) = 1{,}76$ and $Q(1{,}10) = 17{,}05$ ✓

max. flow= 17,05 and time = 1,10 hours after midnight ✓ (6)

[8]

10.1 5 ✓ (1)

10.2 No ✓

P(B and C) ≠ 0 ✓ (2)

10.3.1 $\frac{58}{240}$ ✓ (1)

10.3.2 $\frac{29}{240}$ ✓ (1)

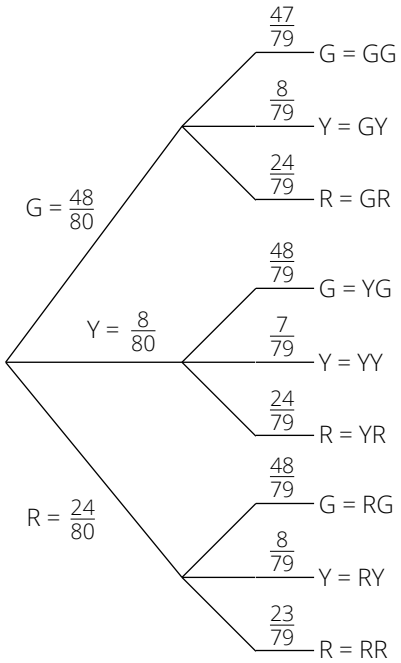
10.3.3 $\frac{118}{240}$ ✓ (1)

10.3.4 $\frac{151}{240}$ ✓ (1)

[7]

11.1 $\frac{30}{100} \times 80$ ✓ $= 24$ ✓ (2)

11.2 (3)



✓ Correct branches
✓ Correct probabilities
✓ Correct outcomes

11.3 P(GY) or P(YG) $= \frac{48}{80} \times \frac{8}{79}$ ✓ $+ \frac{8}{80} \times \frac{48}{79}$ ✓ $= \frac{48}{395}$ ✓ (3)

[8]

Total: [150]

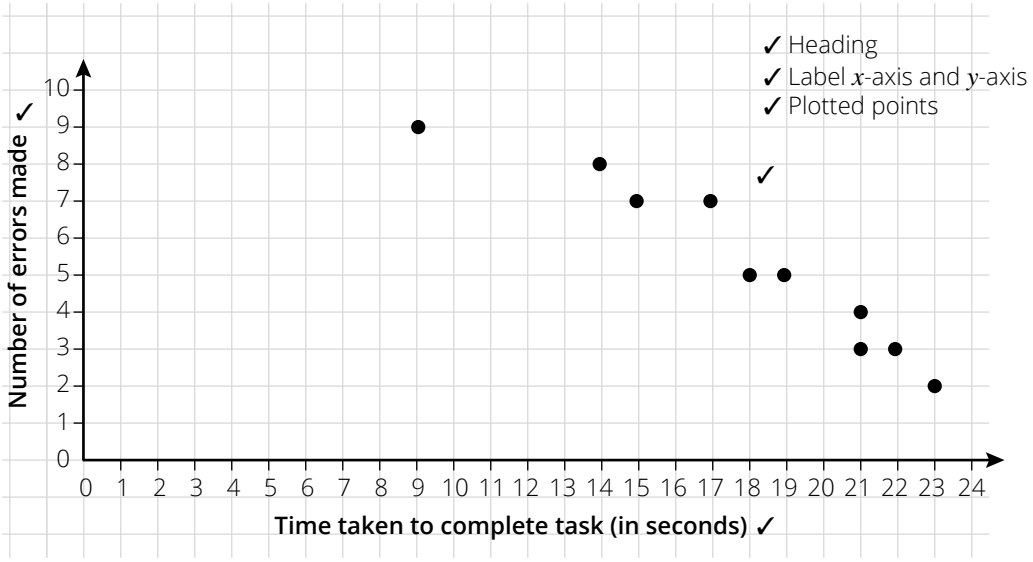
Exemplar Assessments Memorandum

Time: 3 hours

Term 4: Examination Paper 2

- This memorandum serves as a guide for the allocation of marks.
- Any acceptable / valid method can be used.
- Do not forget to mark with CA in mind. (Corrective Accuracy)

1.1



$\frac{3}{3}$ If all points are plotted correctly

$\frac{2}{3}$ If not more than 3 point are wrong

$\frac{1}{3}$ If not more than 5 points are wrong (3)

1.2 The more the time taken, the less the number of errors made. ✓ (1)

1.3 A = 14,71 ✓

B = −0,53 ✓

$y = 14{,}71 - 0{,}53x$ ✓ (3)

1.4 $r = -0{,}96$ ✓✓ (2)

1.5 $y = 14{,}71 - 0{,}53(13)$ ✓ = 7,82

Approximately 8 errors ✓ (2)

1.6 Very strong negative correlation. ✓ (1)

[12]

2.1 $a = 7$; ✓ $b = 15$; ✓ $c = 17$; ✓ $d = 23$; ✓ $e = 34$; ✓ $f = 37$; ✓ $g = 42$ ✓ (7)

[7]

3.1 $m_{PQ} = \frac{a-0}{4+4} = \frac{a}{8}$ ✓

$m_{QR} = \frac{a-0}{4-6} = -\frac{a}{2}$ ✓

$m_{PQ} \times m_{QR} = -1$

$\frac{a}{8} \times -\frac{a}{2} = -1$ ✓

$a^2 = 16$

$a = \pm 4$ ✓

$a = 4$ (4)

3.2 $m_{SR} = m_{PQ} = \frac{1}{2}$ ✓

$y - 0 = \frac{1}{2}(x - 6)$ ✓✓

$y = \frac{1}{2}x - 3$ ✓ (4)

3.3 mid-point PR = mid-point QS

$\frac{x+4}{2} = 1$ ✓ and $\frac{y+4}{2} = 0$ ✓

$x = -2; y = -4$ ✓

S$(-2; -4)$ ✓ (4)

3.4 PR $= 6 - (-4) = 10$ units ✓✓ (2)

3.5 Centre $= (1; 0)$; ✓ $r = 5$ ✓

$(x-1)^2 + y^2 = 25$ ✓ (3)

3.6 LHS $= (4-1)^2 + (4-0)^2$ ✓ $= 25 =$ RHS ✓ (2)

[19]

4.1 C$(-1; 6)$ ✓✓ (2)

4.2 $B\hat{A}C = 90°$ ✓

AC $= \sqrt{5^2 - (\sqrt{20})^2}$ ✓✓ Pythagoras' Theorem ✓ $= \sqrt{5}$ ✓ (5)

4.3 $\tan\theta = \frac{\sqrt{5}}{\sqrt{20}}$ ✓

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$ ✓ $= 26{,}57°$ ✓ (3)

4.4 $m_{AB} \times m_{AC} = -1$ ✓

$m_{AB} \times \frac{1}{2} = -1$ ✓

$m_{AB} = -2$ ✓ (3)

4.5 Area $\triangle ABC = \frac{1}{2}(\sqrt{20})(\sqrt{5}) = 5$ ✓

y-intercept of CD : F$\left(0; \frac{13}{2}\right)$ ✓

x-intercept of CD : D$(-13; 0)$ ✓

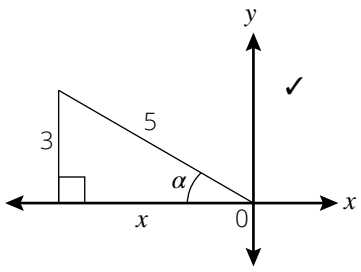
Area $\triangle ODF = \frac{1}{2}(13)\left(\frac{13}{2}\right) = \frac{169}{4}$ ✓✓

$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle ODF} = \frac{5}{\frac{169}{4}} = \frac{20}{169}$ ✓ (6)

[19]

Exemplar Assessments Memorandum

5.1.1



✓

$x = \sqrt{25 - 9} = -4$ ✓

$\tan\alpha = -\frac{3}{4}$ ✓ (3)

5.1.2 $\sin(90° - \alpha) = \cos\alpha$ ✓ $= -\frac{4}{5}$ ✓ (2)

5.1.3 $\cos 2\alpha = 1 - 2\sin^2\alpha$ ✓ $= 1 - 2\left(\frac{3}{5}\right)^2$ ✓ $= \frac{7}{25}$ ✓✓ (4)

5.2 $\sin 150° = \sin 30°$ ✓

$\tan 225° = \tan 45°$ ✓

$\sin 420° = \sin 60°$ ✓

$\frac{\sin 150° \cdot \tan 225°}{\sin 30° \cdot \sin 420°} = \frac{\sin 30° \cdot \tan 45°}{\sin 30° \cdot \sin 60°} = \frac{\frac{1}{2} \times 1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}}$ ✓✓ $= \frac{2}{\sqrt{3}}$ ✓ (6)

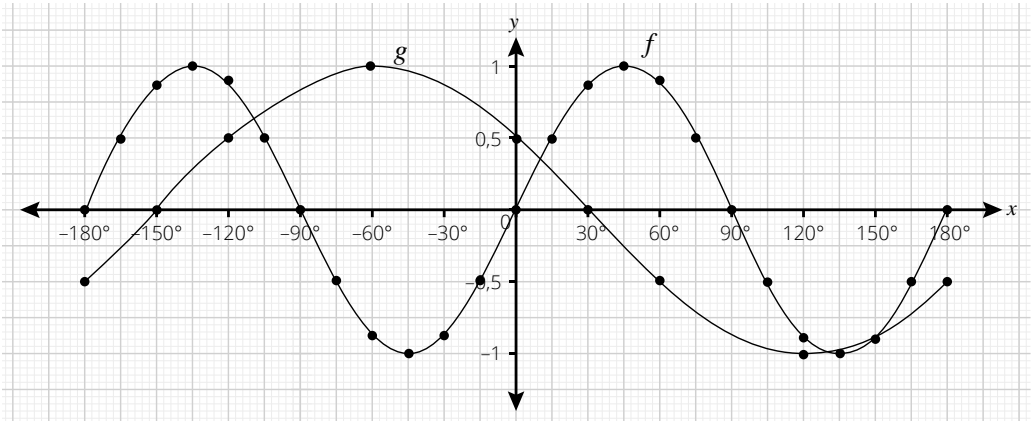
5.3 LHS $= \frac{1 + \sin 2x}{\cos 2x} = \frac{1 + 2\sin x \cdot \cos x}{\cos^2 x - \sin^2 x}$ ✓✓

$= \frac{\cos^2 x + \sin^2 x + 2\sin x \cdot \cos x}{\cos^2 x - \sin^2 x}$ ✓ $= \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$ ✓ $= \frac{\cos x + \sin x}{\cos x - \sin x}$ ✓ = RHS (5)

5.4 $\frac{\cos 15° + \sin 15°}{\cos 15° - \sin 15°} = \frac{1 + \sin 30°}{\cos 30°}$ ✓ $= \frac{1 + \frac{1}{2}}{\frac{\sqrt{3}}{2}}$ ✓ $= \sqrt{3}$ ✓ (3)

[23]

6.1



f: ✓ Correct intercepts; ✓ Turning points; ✓ Correct shape

g: ✓ Correct intercepts; ✓ Turning points; ✓ Correct shape (6)

6.2 360° ✓ (1)

6.3 $\sin 2x = \cos(x + 60°)$

$\cos(90° - 2x) = \cos(x + 60°)$ ✓

$90° - 2x = \pm(x + 60°) + k \cdot 360°;\ k \in \mathbb{Z}$

$x = 10° + k \cdot 120°$ ✓ or $x = 150° + k \cdot 360°$ ✓

$x = -110°$; ✓ 10°; ✓ 130°; ✓ 150° ✓ (7)

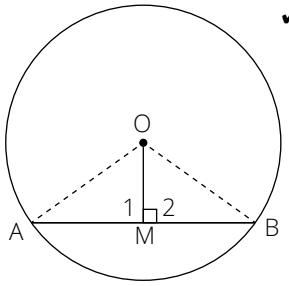
6.4 $-110° < x < 10°$ ✓✓ (2)

6.5 Reducing the period by half ✓ and shifting the graph by 20° to the right. ✓ (2)

[18]

7.1 Bisects the chord ✓ (1)

7.2 ✓



Using △OAM and △OBM

OM = OM	(common) ✓	
OA = OB	(radii) ✓	
$M_1 = M_2 = 90°$ ✓	(given) ✓	
△OAM = △OBM ✓	(RHS) ✓	
AM = MB	(=△s) ✓	(8)
7.3.1 $\hat{L} = 100°$ ✓	(ext. ∠ of cyclic quad) ✓	(2)
7.3.2 $\hat{O}_2 = 200°$ ✓	(∠ at centre=2× ∠ at circumference) ✓	
$\hat{O}_1 = 160°$ ✓	(∠ in a rev.) ✓	(4)
7.3.3 $\hat{M}_1 = 45°$ ✓	(sum of ∠s in a quad.) ✓	(2)
7.4.1 $\hat{B} = 30°$ ✓	(∠s opp = sides; OB = OD) ✓	
$\hat{C} = \hat{B} = 30°$ ✓	(∠s in same seg.) ✓	(4)
7.4.2 $\hat{O}_2 = 60°$ ✓	(∠ at centre = 2 × ∠at circumference) ✓	(2)

[23]

8.1.1 ∠ in semi-circle ✓ (1)

8.1.2 opp. ∠s of quad are supplementary ✓ (1)

8.2.1 $\hat{T}_2 = 90°$ ✓	(∠ line from centre to midpt. of chord) ✓	
$\hat{O}_2 = \hat{T}_2 = 90°$ ✓		
PR ‖ VO	(corr. ∠s =) ✓	(4)
8.2.2 $\hat{Q}_1 = \hat{P}_2$ ✓	(∠s in same seg.) ✓	
$= \hat{V}_1$ ✓	(corr. ∠s; PR ‖ VO) ✓	(4)

Exemplar Assessments Memorandum

8.3	$ST = 2r - \frac{2}{5}r = \frac{8}{5}r$ ✓		
	$\frac{SV}{SP} = \frac{SO}{ST}$ ✓	(prop. theorem; VO ∥ PT) ✓✓	(4)
			[14]
9.1	tan chord theorem ✓		(1)
9.2.1	$\hat{B}_1 + \hat{B}_2 = \hat{A}_2 + \hat{A}_3$ ✓	(∠s opp. = sides; AC = BC) ✓	
	$\hat{A}_2 + \hat{A}_3 = \hat{E}_3 + \hat{E}_4$	(ext. ∠ of cyclic quad) ✓	
	$\hat{B}_1 + \hat{B}_2 = \hat{E}_3 + \hat{E}_4$		
	AB ∥ DE	(corr. ∠s =) ✓	(5)
9.2.2	$\hat{B}_1 + \hat{B}_2 = \hat{E}_3 + \hat{E}_4$ ✓	(proven)	
	$\hat{B}_1 = \hat{A}_1$	(tan chord theorem) ✓	
	$\hat{E}_3 = \hat{B}_2$ ✓	(tan chord theorem) ✓	
	$\hat{B}_2 + \hat{E}_4 = \hat{A}_1 + \hat{B}_2$ ✓		
	$\hat{E}_4 = \hat{A}_1$	(proven) ✓	(6)
9.2.3	CEAF is a cyclic quadrilateral ✓	(converse of ∠s in same seg.) ✓	(3)
			[15]
			Total: [150]

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